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Bosonic String Theory, Geometries and T-duality

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String Theory: what is it and why do we need it?

String theory involves understanding the properties of one-dimensional objects, called strings, which trace out a world-sheet in space-time [1]. The string can be closed, forming a loop, or open, with the geometry of the string determining the spectrum of particles. Our work focuses on bosonic string theory, which was first introduced to explain the strong interaction, but was later proposed as a Grand Unified Theory.

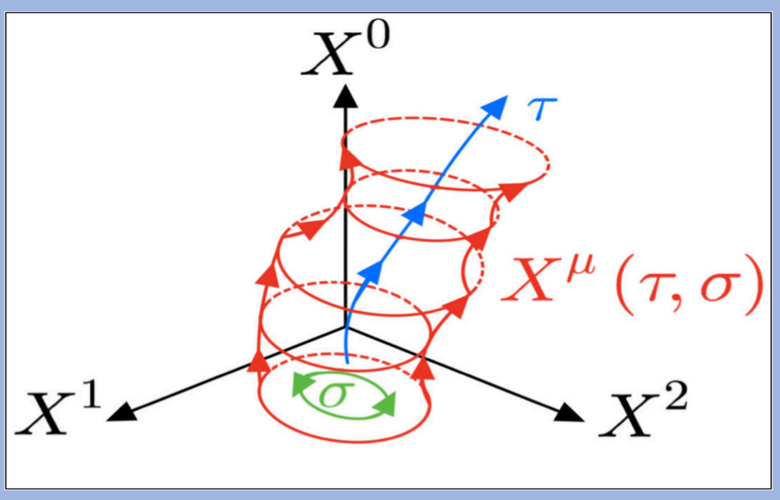

Fig 1: The world-sheet of a closed string, parameterised by τ and σ.

Essentially, bosonic string theory describes bosons – integer spin particles – with no supersymmetry. The action that describes the dynamics of the free bosonic string is called the Polyakov action:

$$S[X,\gamma] = -\frac{1}{4\pi\ell_s^2}\int d\tau d\sigma\sqrt{-\gamma}\gamma^{ab}h_{ab}(X),$$

where:

- h_{ab} is the induced world-sheet metric
- γ_{ab} is an auxiliary metric
- $\gamma = det(\gamma_{ab})$
- ℓ_s is the length scale of the string and is brought in for dimensional reasons.

The Standard Model has been successful at describing the strong, weak and EM interactions, however, it fails to unify them or describe gravity.

Conversely, in string theory the massless state of the closed string contains the graviton — the force carrier of gravity — as well as two other particles known as the B-field and dilaton [2]. This means that string theory has gravity in-built, so it should be interpreted as a theory that unifies gravity with the other fundamental interactions [1].

The Dimension Problem: what is it and how do we solve it?

String theory is not without its flaws, for example to remove negative probability states from the theory the number of space-time dimensions is constrained to be 26. However it is currently thought that our universe is 4-dimensional, so it seems unlikely that bosonic string theory describes our universe [2].

This conflict can be resolved via compactification [1]. A compactified dimension has a finite length, meaning that the additional 22 dimensions could be "wrapped" up in such a way that they are undetectable at low energy [2]. We have found that theories with different compactified geometries have different properties.

T-Duality

Our work focuses on Target-Space Duality (T-duality), which is an important equivalence between string theories. We are interested in how T-duality relates string theories with different compact geometries.

A simple example of compactification is to take the 25th spatial dimension and wrap it into a circle of radius R.

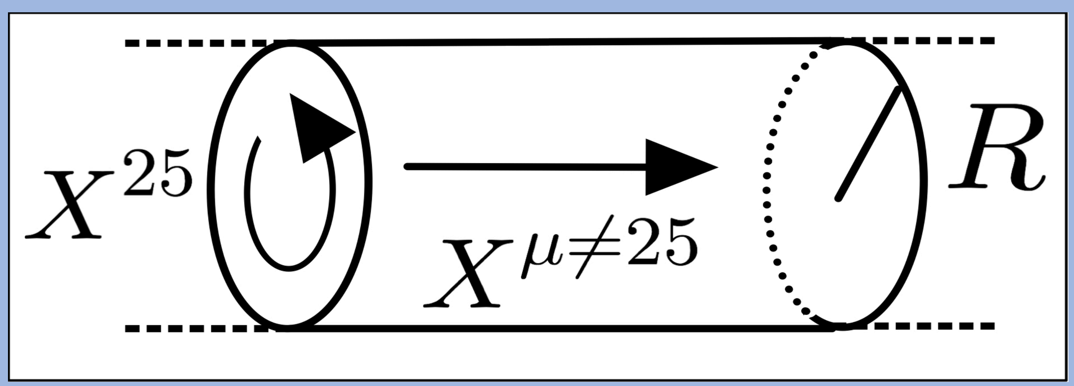

Fig 2: A representation of the geometry $\mathbb{R}^{1,24}\times S^1_R$.

This has the following effects:

- The string is free to wrap around this compact dimension W times, $W\in\mathbb{Z}$.
- The momentum along the compact direction becomes quantised: $p^{25}=\frac{K}{R}$, $K\in\mathbb{Z}$.

Consequently, the particle spectrum now includes winding and momentum modes.

The T-duality transformation in this geometry is defined as:

$$T:\mathbb{R}^{1,24}\times S^1_R \longleftrightarrow \mathbb{R}^{1,24}\times S^1_{R'},\ W\longleftrightarrow K,\ R'=\frac{\ell_s^2}{R}.$$

Under this transformation the compact dimension changes radius and the winding and momentum excitations swap, but the particle spectrum remains the same.

If the universe had this geometry there would be no way for us to measure the radius of the compact dimension, making the T-dual theories indistinguishable.

The Buscher Rules

We can considered a string moving in a background of a large number of massless closed strings, which can be viewed as a classical field. To do so we generalise the action to a non-linear sigma model:

$$S[X,\gamma,g,B] = -\frac{1}{4\pi\ell_s^2}\int d\tau d\sigma\sqrt{-\gamma}(\gamma^{ab}g_{\mu\nu}+\varepsilon^{ab}B_{\mu\nu})\partial_a X^\mu\partial_b X^\nu,$$

with a general metric $g_{\mu\nu}$ and B-field $B_{\mu\nu}$. For any such action, if there is a translational symmetry in the nth direction then there is a T-dual theory with background fields given by [3]:

$$\hat{g}_{\alpha\beta}=g_{\alpha\beta}-\frac{\left(g_{\alpha n}g_{n\beta}+B_{\alpha n}B_{n\beta}\right)}{g_{nn}},\ \hat{g}_{\alpha n}=\frac{\ell_s^2B_{\alpha n}}{g_{nn}},\ \hat{g}_{nn}=\frac{\ell_s^4}{g_{nn}},$$

$$\hat{B}_{\alpha\beta}=B_{\alpha\beta}+\frac{\left(g_{\alpha n}B_{\beta n}-B_{\alpha n}g_{\beta n}\right)}{g_{nn}},\ \hat{B}_{\alpha n}=\frac{\ell_s^2 g_{\alpha n}}{g_{nn}},\ \hat{B}_{nn}=0.$$

Phenomenological Implications

- String theory has gravity in-built, so it is better interpreted as a unified theory which could lead to a comprehensive Theory of Everything.
- The extra dimensions required for the consistency of string theory may be resolved using compactification.
- Investigating compactification might lead to a string theory with 4-spacetime dimensions, better reflecting the universe we live in.
- T-duality implies that some large compact dimensions are physically indistinguishable from small compact dimensions.

References

[1] J. H. Schwarz, "Introduction to Superstring Theory," arXiv:hep-ex/0008017, Aug. 2000. [Online]. Available: http://arxiv.org/abs/hep-ex/0008017

[2] K. Wray, "University of Berkeley - An Introduction to String Theory," May 2011. [Online]. Available: https://math.berkeley.edu/~kwray/papers/string_theory.pdf

[3] T. Buscher, "*A symmetry of the string background field equations*", Phys. Lett. **B194** (1987) 59-62. [Online]. Available: https://www.sciencedirect.com/science/article/pii/0370269387907696