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BRANES AND BLACK HOLES IN SUPERGRAVITY

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INTRODUCTION

- *Supergravity* is a field theory combining GR and supersymmetry.
- *Branes* are the generalisations of particles to higher spatial dimensions. *P-branes*, a specific class of branes, appear as solutions in supergravity.
- *The Hierarchy problem* is the discrepancy between Higgs mass $m_h \approx 126\ GeV$ and gravitational scale $M_{Planck} \sim \sqrt{G} \sim 10^{19} GeV$. Extra spatial dimensions and supersymmetry are used to tackle this problem.
- In this project, we derive some brane solutions from $11D\ \mathcal{N} = 1$ supergravity, discuss some of their properties and explore Brane/Brane orbits.

SUPERGRAVITY MODEL

Supergravity arises as a gauge field theory of supersymmetry (Chamseddine, A. H. and West, P. C., 1977; Kibble, 1961). We start with the bosonic sector of $11D\ \mathcal{N} = 1$ supergravity action, containing a 3-form gauge field $A_{[3]}$ and a 4-form field strength $F_{[4]} = dA_{[3]}$.

$$I_{11} = \int d^{11}x \left\{\sqrt{-g}\left(R - \frac{1}{48}F_{[4]}^2\right)\right\} - \frac{1}{6}\int F_{[4]} \wedge F_{[4]} \wedge A_{[3]}$$

After applying *Kaluza-Klein dimensional reduction*, a 10d action can be derived. Through *consistent truncation*, we generalise it to a toy model in D dimension(Stelle, 1998).

$$I = \int d^Dx\sqrt{-g}\left[R - \frac{1}{2}\nabla_M\nabla^M\phi - \frac{1}{2n!}e^{a\phi}F_{[n]}^2\right]$$

Here, ϕ is *dilaton field* and a is the scalar which controls the coupling. They both appear during the process of dimensional reduction.

P-BRANE ANSATZ

In order to find the solution, we assume Poincaré invariance along d-dimensional worldvolume and rotational symmetry along transverse space, i.e. the symmetry group of the space is $ISO(d-1,1) \times SO(D-d)$.

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(r)}\delta_{mn}dy^m dy^n$$

where $r = \sqrt{y_m y^m}$. The scalars A and B are assumed to have the linear condition

$$dA + \tilde{d}B = 0$$

where $\tilde{d} = D - d - 2$. This condition makes the brane satisfy the *Bogomol'nyi-Prasad-Sommerfield (BPS) bound* (Mass = Charge) and preserves half of the supersymmetries. An extra condition, either the "electric" or "solitonic" ansatze, is made on the field strength. Neither of these contribute to the *Chern-Simons term* which can therefore be consistently truncated from the model action.

P-BRANE SOLUTIONS

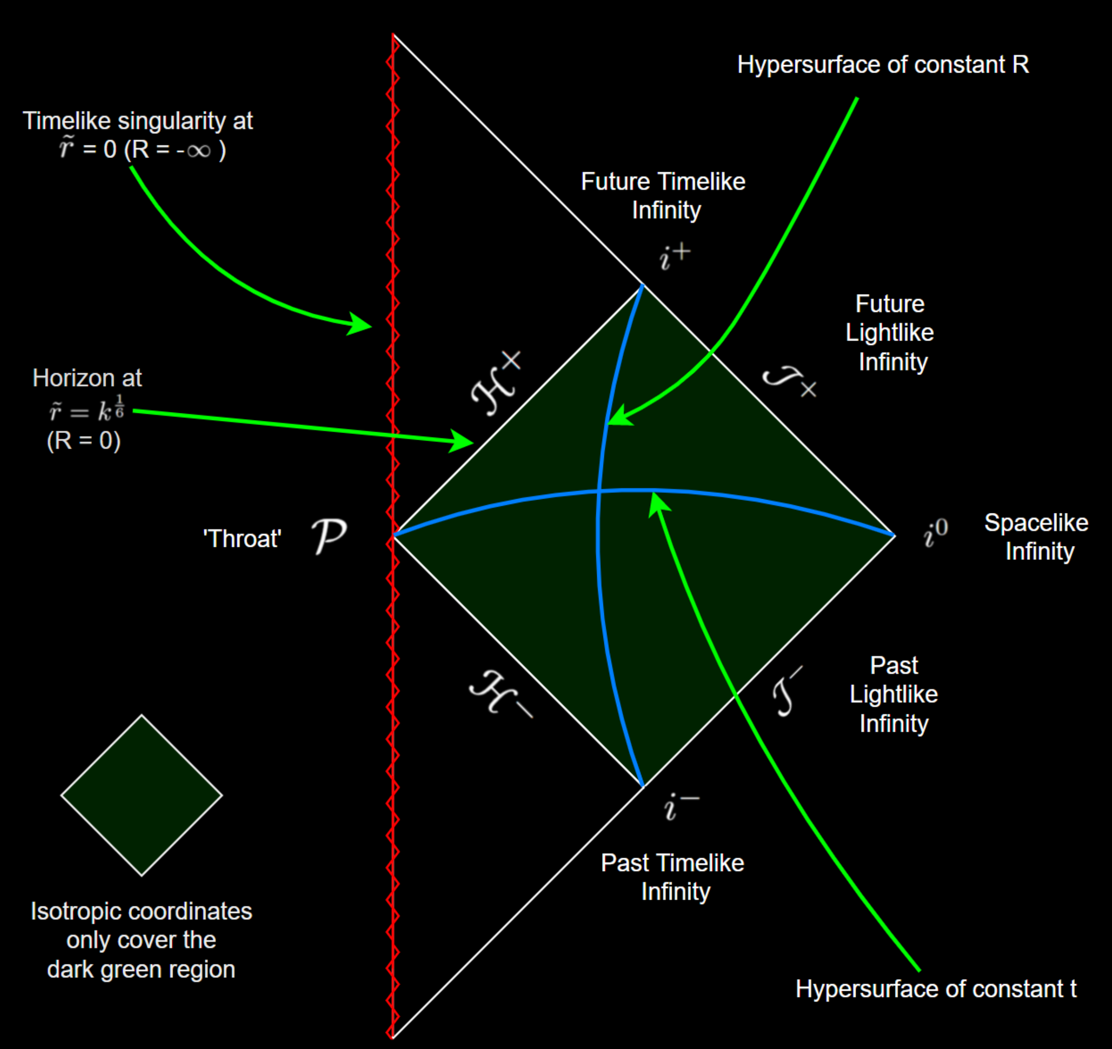
We work with *vielbein* to derive the equations of motion for the fields from the model action. Using the above ansatze, the brane solution metrics are obtained in $D = 11$. The following figures are the Carter-Penrose diagrams for both solutions.

- "Electric" 2-brane solution

$$ds^2 = \left(1 + \frac{k}{r^6}\right)^{-2/3}\eta_{\mu\nu}dx^\mu dx^\nu + \left(1 + \frac{k}{r^6}\right)^{1/3}\delta_{mn}dy^m dy^n$$

$$A_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda}\left(1 + \frac{k}{r^6}\right)^{-1},\ \textit{other components zero.}$$

FIGURE 1



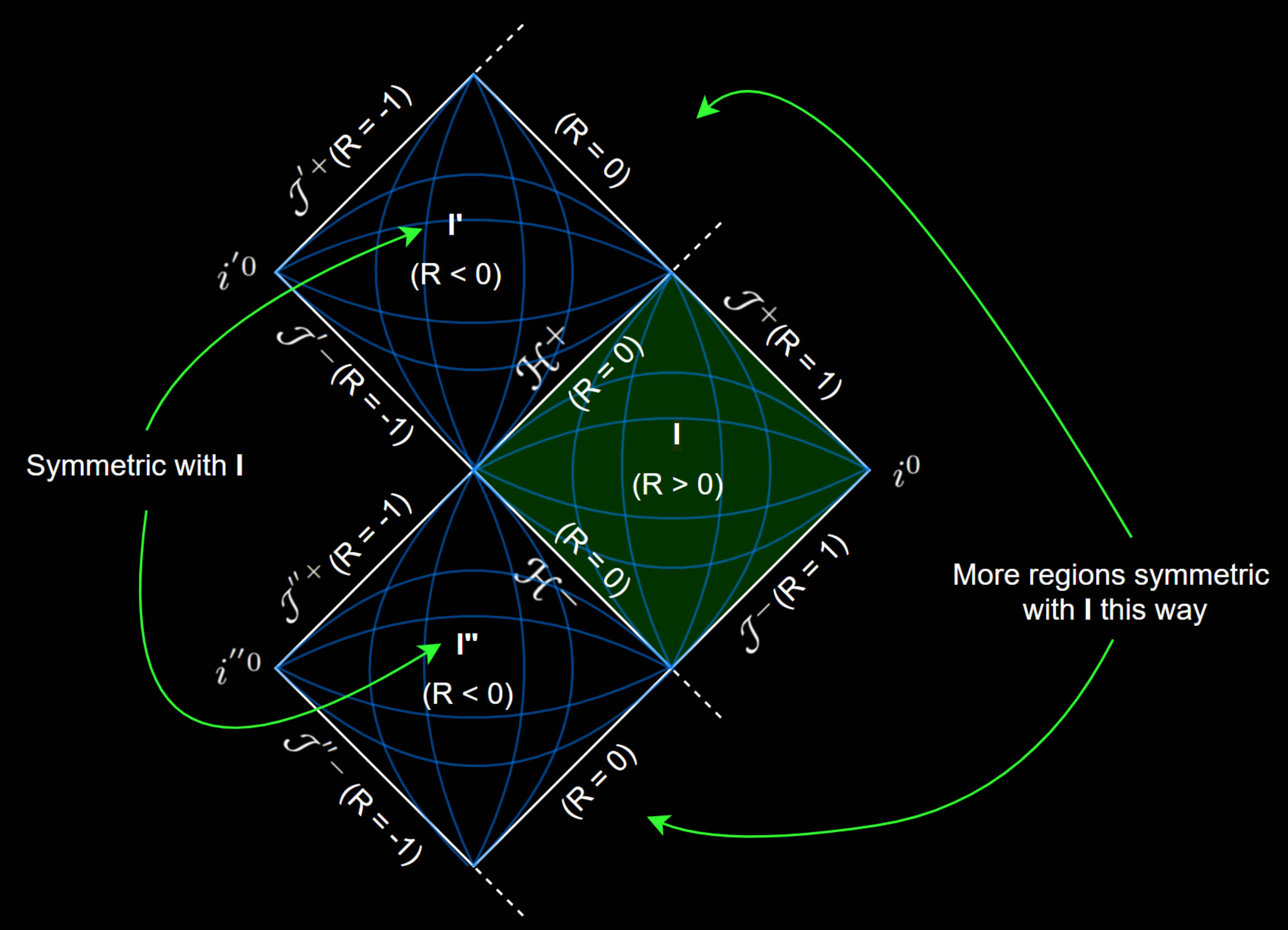
"Electric" 2-brane has a *timelike* singularity at $\tilde{r} = 0$. $\tilde{r}$ is a new coordinate with the transformation $r = (\tilde{r}^6 - k)^{1/6}$. This is because the isotropic coordinate used above only covers the shaded area shown in the figure, which leaves out the singularity. The solution can be mapped to *Reissner–Nordström black hole* in classical GR. The geometry on the horizon ("throat") is $(AdS)_4 \times \mathcal{S}^7$, and as such, this solution interpolates between $(AdS)_4 \times \mathcal{S}^7$ and $\mathcal{M}^{11}$. The singularity is labelled with the pointed red line.

- "Solitonic" 5-brane solution

$$ds^2 = \left(1 + \frac{k}{r^3}\right)^{-1/3}\eta_{\mu\nu}dx^\mu dx^\nu + \left(1 + \frac{k}{r^3}\right)^{2/3}\delta_{mn}dy^m dy^n$$

$$F_{m_1 \dots m_4} = 3k\epsilon_{m_1 \dots m_4 p}\frac{y^p}{r^6},\ \textit{other components zero.}$$

FIGURE 2



"Solitonic" 5-brane has a horizon on $r = 0$, which has a $(AdS)_7 \times \mathcal{S}^4$ Structure. In a similar way to the "electric" 2-Brane, this solution interpolates between $(AdS)_7 \times \mathcal{S}^4$ and $\mathcal{M}^{11}$. A symmetry of this solution is revealed by making a coordinate transformation $r \to R$: $r = k^{1/3}R^2/(1 - R^6)^{1/3}$. The symmetry is then apparent through the isometry $R \to -R$.

EXTENDED WORK

- Kaluza-Klein reduction

$$ds^2 = e^{2\alpha\phi}g_{\mu\nu}dx^\mu dx^\nu + e^{2\beta\phi}(dz + \mathcal{A}_\mu dx^\mu)^2$$

ϕ is a scalar dilaton field, $\mathcal{A}_\mu$ is a vector field, and $g_{\mu\nu}$ is the metric on the D-1 remaining dimensions. The constants α and β are set such that the kinetic term of the scalar field in the resulting action is normalised. The z dimension is then curled up, so that all dependencies on z can be expanded in a Fourier series. The size of this dimension is often taken to 0, so that nothing depends on z (The cylinder condition). When Kaluza first did this with a 5 dimensional Einstein-Hilbert Action, the $\mathcal{A}_\mu$ field ended up being equivalent to that of classical EM (Kaluza, 1921).

- Orbits

In order to consider the orbits of these branes, one can extend the same principle used to calculate possible types of orbits for the Reissner-Nordstrom Black Hole. By looking at the Killing vectors of the appropriate metrics, and constraining the motion to a specific plane, one can create an effective potential, from which the stability of different orbits can be determined. These calculations can be enormously simplified by assuming that the worldvolumes of both branes are parallel, and then performing diagonal dimensional reduction on the corresponding directions.

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