Theoretical Physics Group
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Bosonic String Theory and T-Duality

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1. What is String Theory?

Our most successful descriptions of the universe are **General Relativity** and the **Standard Model of Particle Physics.** Despite their strong predictive powers, the two theories have incompatibilities that have not yet been resolved [1]. In particular, we believe the universe to be quantum in nature. However, we have been unable to find a consistent model of **quantum gravity**. **String theory** is a model of the universe that aims to provide a description of quantum gravity.

Firstly, we look at the **action** of the theory. We then present some results found by quantising the theory. This leads to some unexpected requirements for spacetime. We lastly show how **geometry** can be used to resolve these issues – but this leads to an additional phenomenon known as **T-duality**.

2. Particle dynamics

In the standard model, particles are treated as **0-dimensional points** in spacetime. To investigate point particle dynamics, the simplest action to consider is the **worldline** path length in D-dimensional **Minkowski** space $\mathbb{R}^{1,D-1}$,

$$S = -m\int ds\,,$$

where m is the **mass** of the particle and ds is the infinitesimal path length [1].

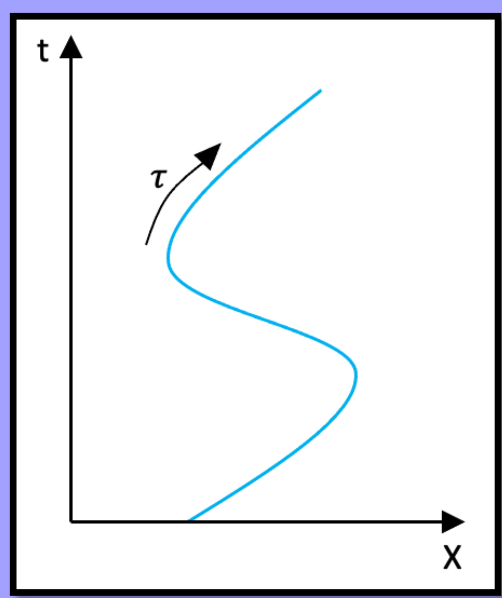


Figure 1: worldline of a point particle

3. What is a String?

The string is the **1-dimensional** analogue of the point particle. Instead of a worldline, the string sweeps out a 2-dimensional **worldsheet** in spacetime. The simplest action to consider is the area of the worldsheet,

$$S = -T\int dA,$$

where T is the **tension** of the string, and dA is an infinitesimal area element [2].

The string has a **length scale** derived from fundamental constants. It is

$$l_s = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}\text{m},$$

which is far smaller than current measurements are able to probe [3].

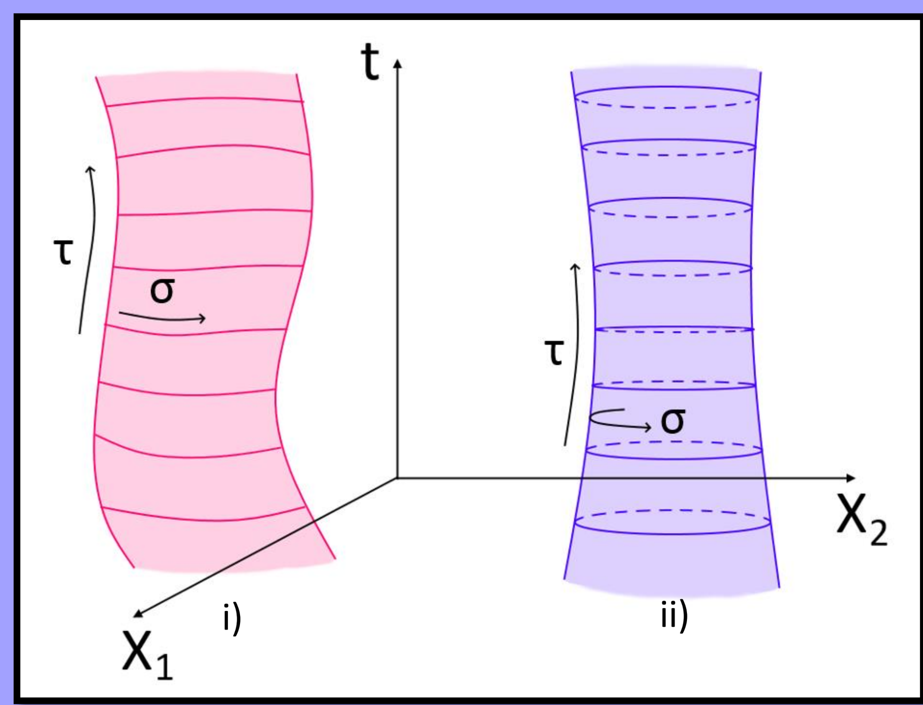


Figure 2: the worldsheet swept out by the trajectory of i) an **open** string and ii) a **closed** string. A closed string has periodic boundary conditions, an open string does not.

After **canonically quantising** the theory, we find 2 key results. Firstly, the spacetime must be **26-dimensional** ($D = 26$). Secondly, the string spectrum admits a massless spin-2 particle – the **graviton**! This truly makes string theory a quantum theory of gravity.

4. Compactification

Given $D = 26$, the first question to answer is why we only see a 4-dimensional spacetime. One explanation is called **compactification**.

The idea of compactification is to constrain the size of a dimension. Rather than having infinite length, the dimension is restricted to some **smaller range**, which may be periodic. The finite size of the compact dimension means that it may be only be visible at a certain length scale.

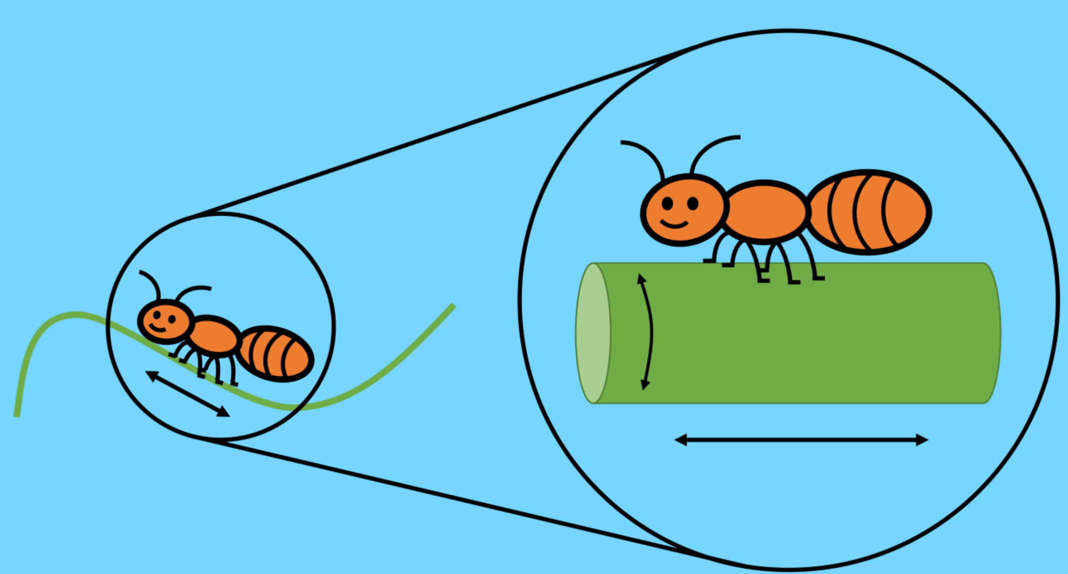
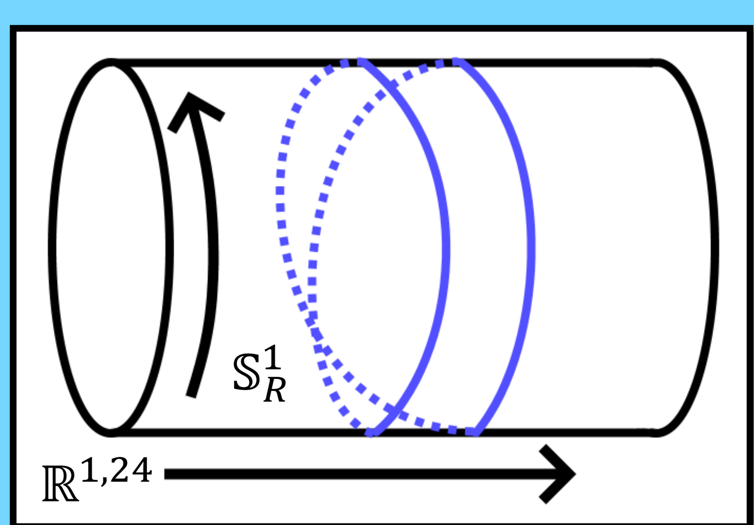


Figure 3: An ant on a hosepipe. Far away, the ant appears to move only along the **length** of the hosepipe. With a closer look, the ant may also move around the **circumference** of the hosepipe - this direction acts like a **compact dimension**.

Ultimately, 22 dimensions need to be compactified. To begin, consider the compactification of a single dimension. Let the compact dimension be a **circle** of radius R, denoted $\mathbb{S}^1_R$. The total spacetime is then denoted $\mathbb{R}^{1,24} \times \mathbb{S}^1_R$.

Figure 4: The spacetime is the product of a circle with 25-dimensional Minkowski space. The string (blue) can now wrap around the compactified direction – twice in this case. The periodicity of the circle also means **momentum is quantised**.



5. T-duality

Denote the number of times the string wraps around the compactified dimension W (the **winding number**), and the **quantised momentum** M, where $W, M \in \mathbb{Z}$ are integers. The **mass** of the string is

$$m^2 = \left(\frac{M}{R}\right)^2 + \left(\frac{WR}{l_s^2}\right)^2 + \Omega,$$

where Ω is a geometry-independent term. [1]

Consider a **second string theory** constructed identically, but now with compactified radius $\bar{R}$, winding number $\bar{W}$, and momentum $\bar{M}$. The mass is

$$\bar{m}^2 = \left(\frac{\bar{M}}{\bar{R}}\right)^2 + \left(\frac{\bar{W}\bar{R}}{l_s^2}\right)^2 + \Omega,$$

Making the **choice**

$$\bar{R} = \frac{l_s^2}{R}, \quad \bar{W} = M, \quad \bar{M} = W,$$

gives

$$\bar{m}^2 = m^2.$$

This map between string theories is known as **T-duality** [4].

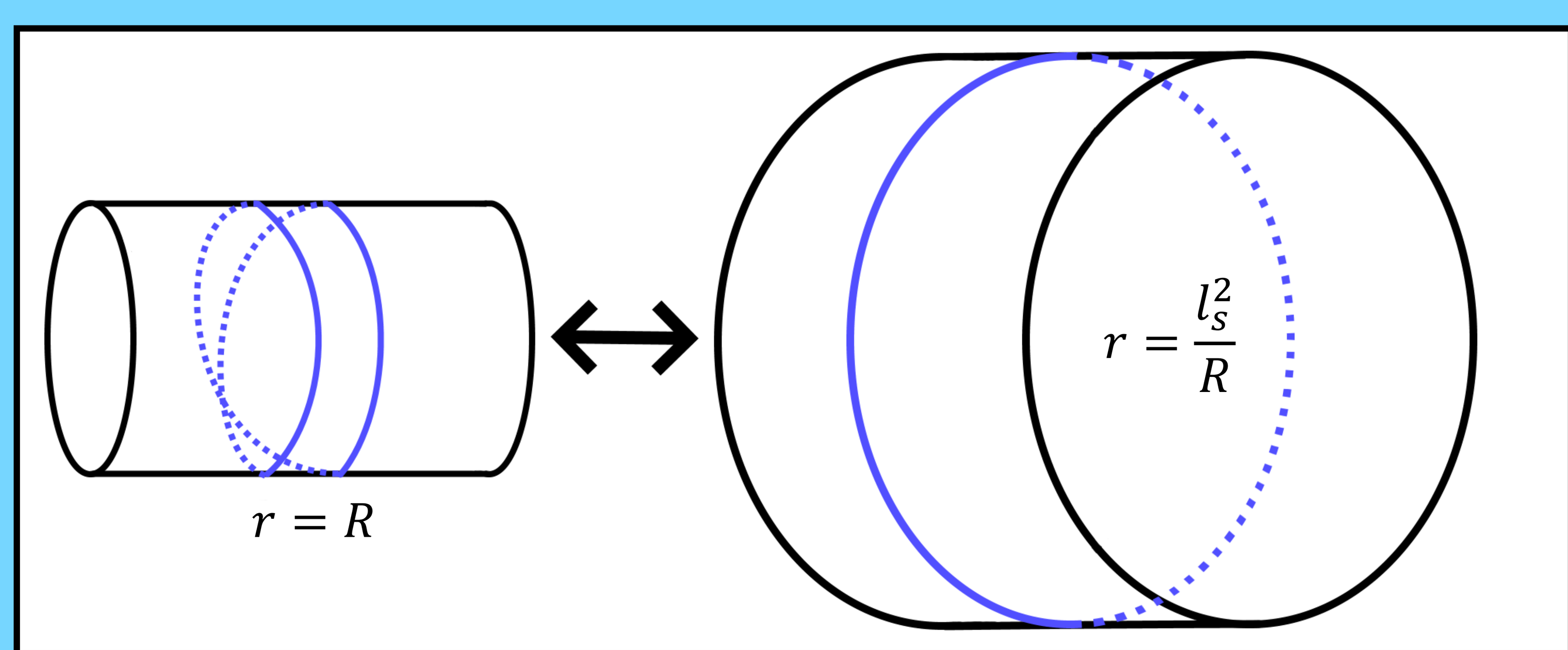


Figure 5: T-duality is a map between string theories on different compact spaces. The dual theory has a radius whose value is reciprocal to the other. The role of winding number and momentum is also exchanged between the two theories.

6. Conclusion

We have considered the **worldsheet** action and found that quantisation requires spacetime to be 26-dimensional. We analysed the string spectrum and found that it contained a **graviton**. In order to explain the extra dimensions, we studied **compactified** spacetimes; in particular, a circular dimension. This led to the key result of **T-duality**, an equivalence of string theories on different spacetimes. This result means that there is no way to distinguish string theories on circular backgrounds of reciprocal radius – that is, string theories on a large circle are **equivalent** to those on a small circle. The next steps in understanding string theory are to explore more complicated spacetime geometries.

References:

[1] K.Wray. An introduction to string theory. Berkley, 2011
[2] D. Tong. String theory. Centre for Mathematical Sciences, Cambridge, 2009.
[3] J.Polchinski. String Theory Volume 1. Cambridge University Press, 2005.
[4] J.H Schwarz. Status of superstring and m-theory. California Institute of Technology, 2008.