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Planet 9 Detection with High-stability Atomic Clocks in Space

What is an Atomic Clock?

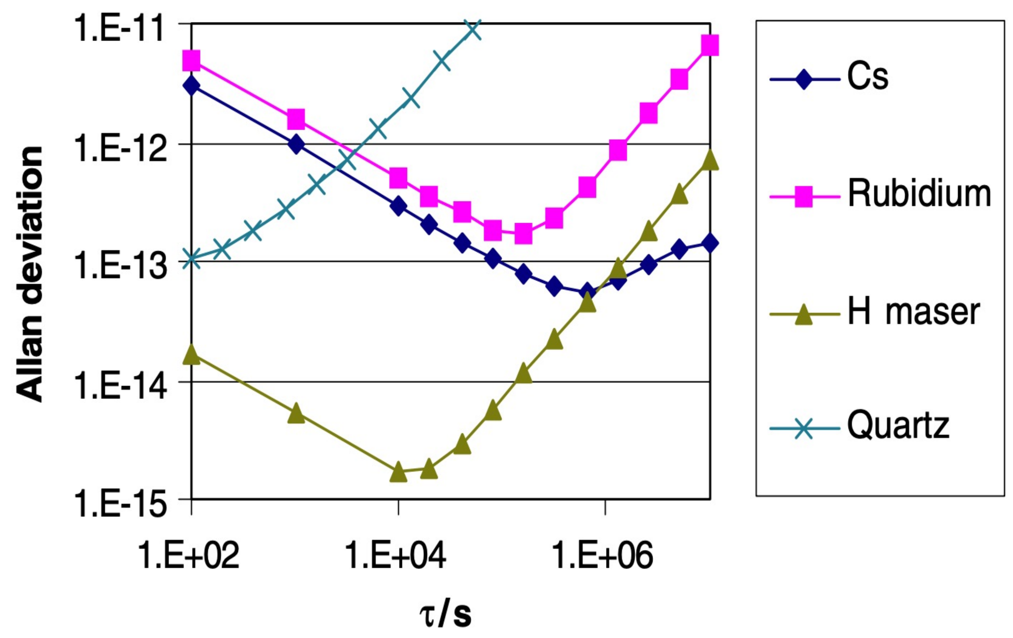

Fig.1. Allan deviation of different types of oscillators as a function of the measurement time τ [1].

- **Atomic Clocks** measure time based on atomic transitions between energy levels, they are more stable over long time period than quartz oscillators.
- **Frequency Stability** reflects the oscillators' ability in resisting frequency fluctuations. The measurement "Allan Deviation" is often used in the time domain.
- **Optical Lattice Clocks** improve the stability by trapping multiple coupled sources in a laser field. The latest record was set by a Sr optical clock in 2022, having a minimum frequency stability of 7.6x 10^{-21}[2].

What is Planet 9?

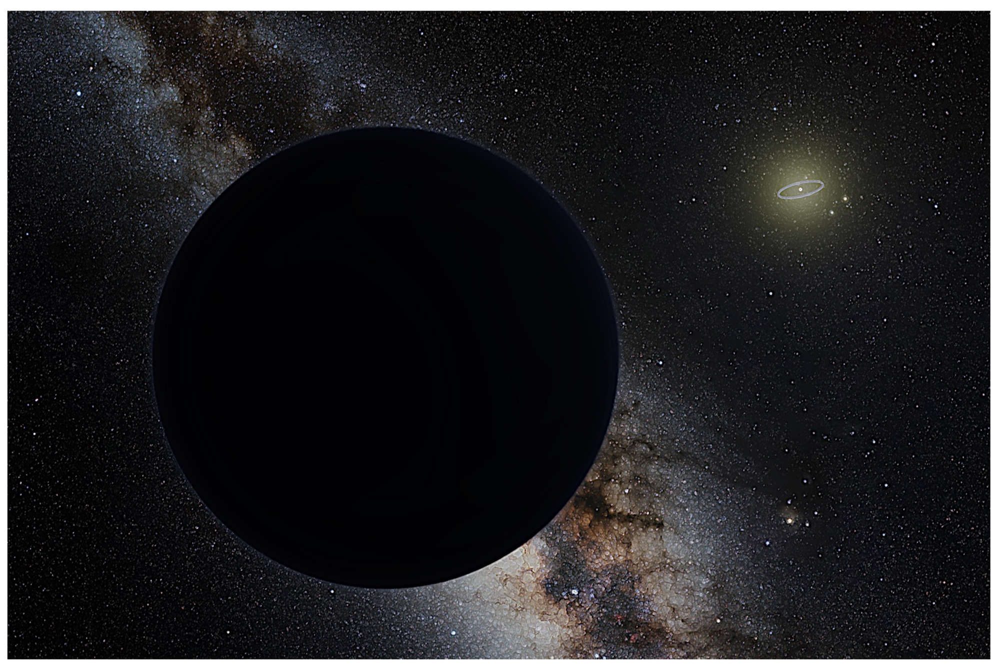

Fig.2. An illustration of the hypothetical Planet 9[3].

- The orbital motion of clustered small bodies in the Kuiper belt have led astronomers to believe there to be another planetary sized body lying beyond Pluto.
- Based on the observed gravitational influence, Planet 9 is believed to weigh in at 10 Earth Masses, 6×10^{25} kg and lie 700 AU away from the Sun[4].
- Planet 9 remains elusive to astronomers as the far distance causes the intensity of light reflected by its surface to be greatly reduced. Furthermore, even with modern day powerful telescopes scanning potential areas of the Solar System for Plant 9 is like "looking through a straw" .

Aim and Approach

- We propose the idea of placing high-stability atomic clocks at the Earth-Sun Lagrange Points to detect redshift signals from the solar system objects.

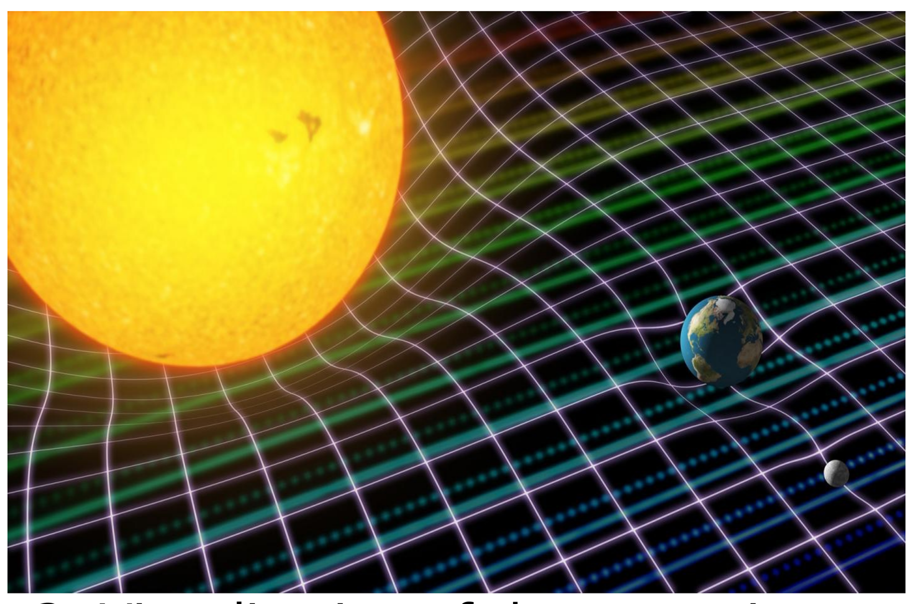

Fig.3. Visualisation of the spacetime metric effects due to massive bodies[5].

- We also add realistic oscillator noise to our simulation, allowing us to determine the level of stability required to detect planet 9 in the outer solar system.

I. The Lagrange Points

- The **Restricted Three Body Problem** reveals the 5 equilibrium points surrounding the orbit of 3 bodies of mass, with the restriction that the third mass must be much smaller than the others.
- These Lagrange Points are crucial to the research as they allow for clocks to be placed at points in space without requiring large amounts of fuel to keep them in a stable orbit.
- The resulting equation is chaotic, making it impossible to solve for the points analytically.

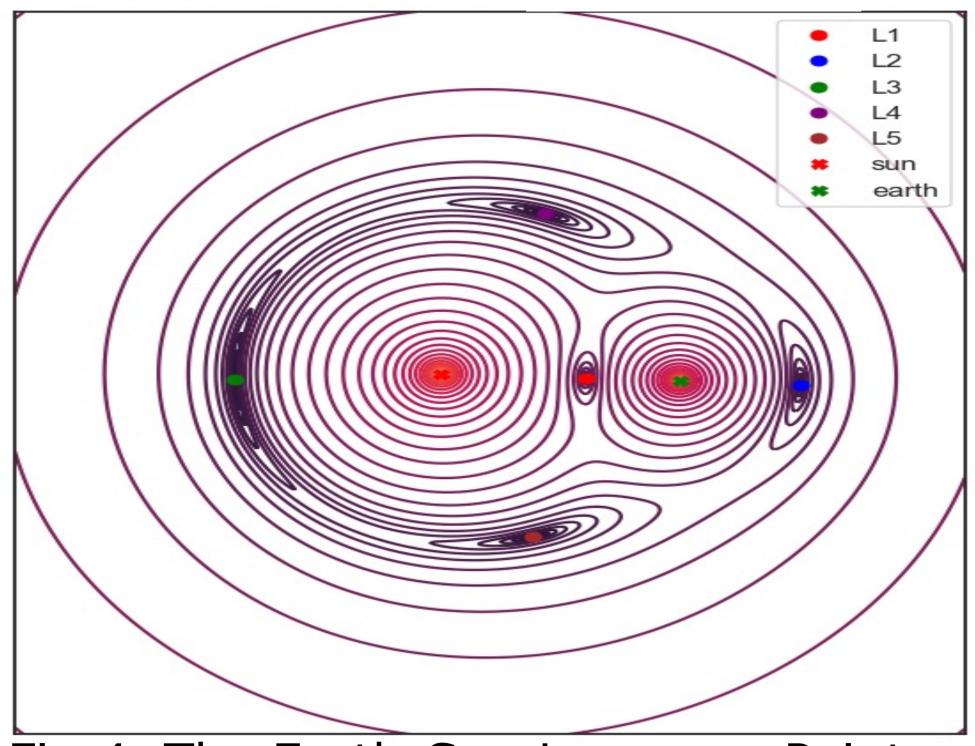

Fig.4. The Earth-Sun Lagrange Points contour plot

II. Redshift Signals Based on the Schwarzschild Metric

The spacetime metric serves as a tool for measuring the distance between events. In particular, the Schwarzschild metric describes the gravitational field outside a static, spherical symmetrical mass. Assuming that all solar system bodies are stationary, we treat each body as an individual Schwarzschild metric and calculate the redshift contributions from each one. This given by

$$\frac{\Delta\nu}{\nu} = -\frac{GM}{rc^2} - \frac{v^2}{2c^2} + \frac{r^2}{2c^2}\left(\omega^2 + \Omega^2 \sin^2\theta\right),$$

where the spherical coordinates (r, θ, ϕ) are used, and (v, ω, Ω) are the corresponding velocity components, G, M are the gravitational constant and the mass of the body.

Results - can we find Planet 9?

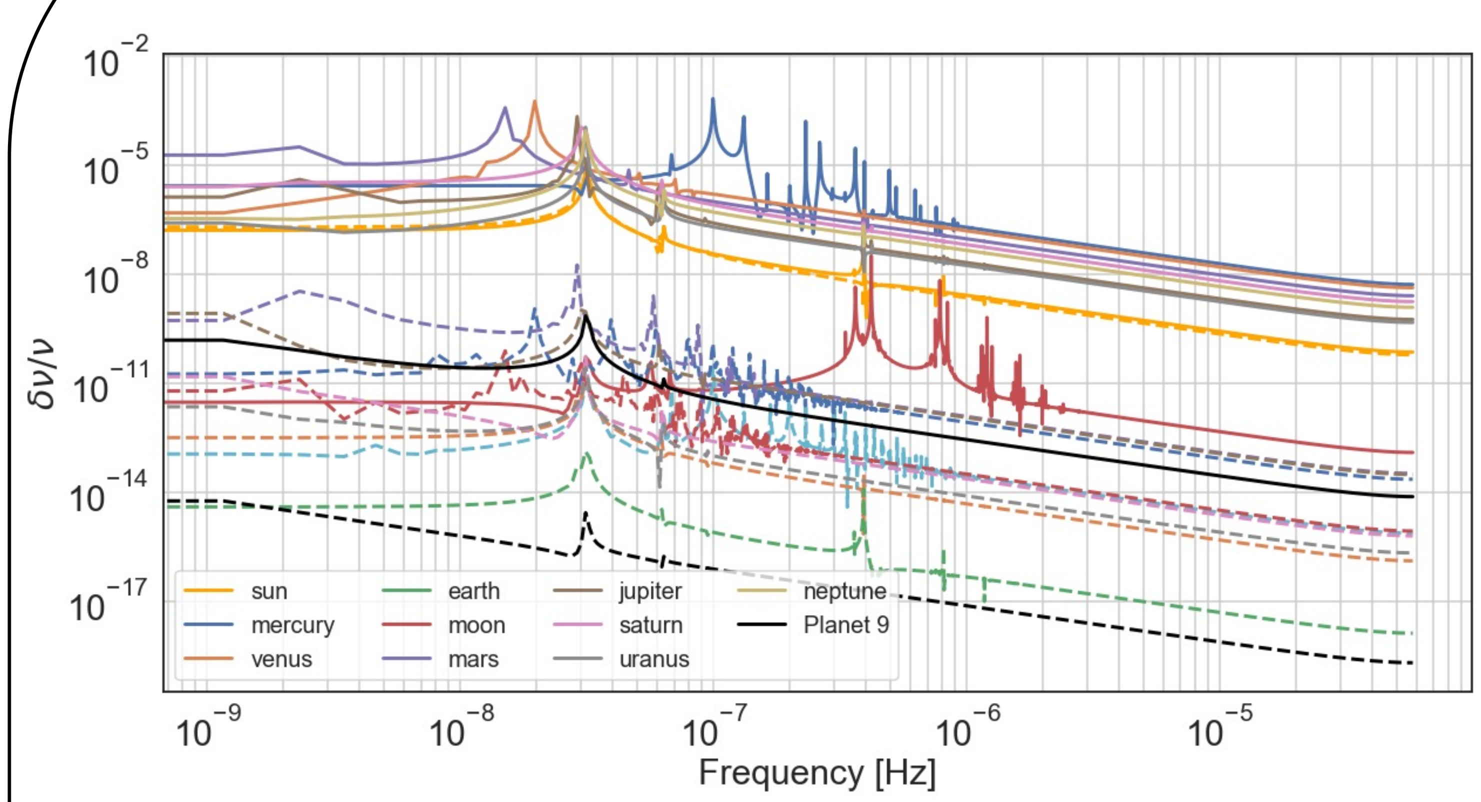

Fig.5. The Fourier magnitude of the redshift signal $\delta\nu/\nu$ of each Solar System object, with the dashed and solid curves indicating the potential and velocity contributions to the redshift, respectively. The positions and velocities were extracted from NASA's JPL ephemeris, an open-source data log of the motions of the Planets from 1549 to 2650. By Fourier Transforming the signal, we can identify the frequencies in the redshift caused by gravitational influences of other masses.

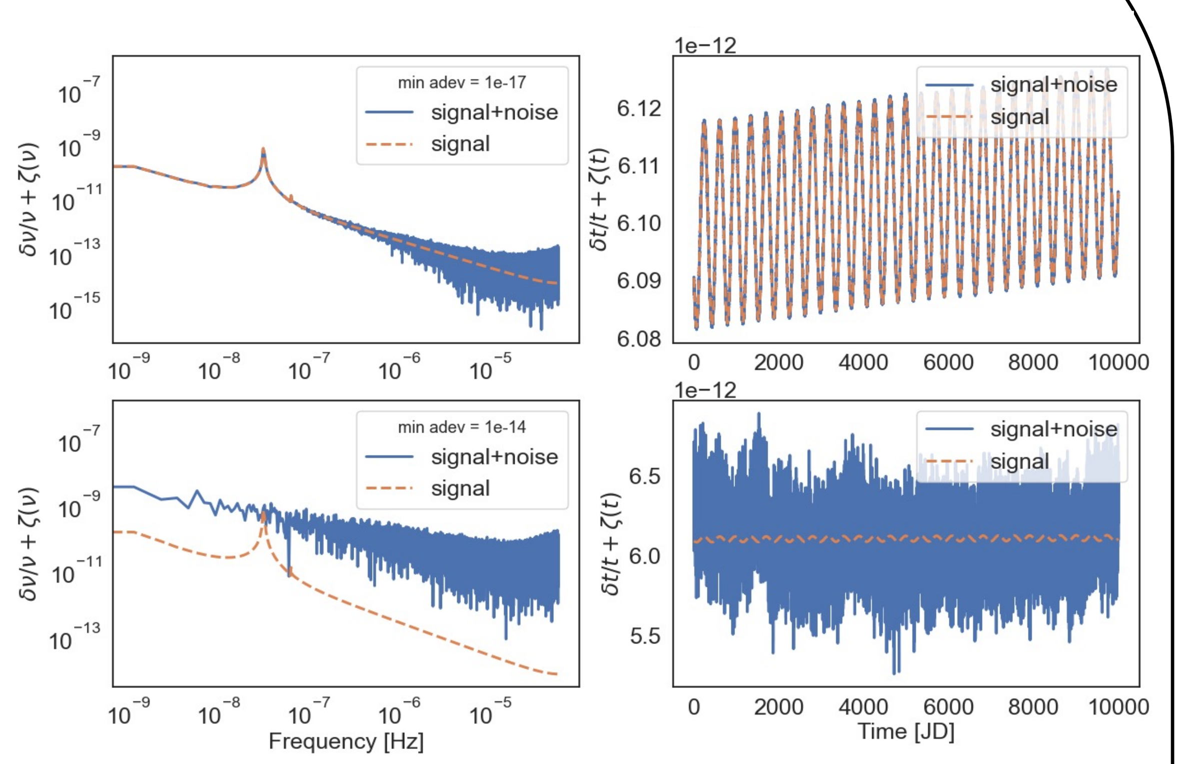

Fig.6. Planet 9 redshift signal added with noise ζ at levels of 10^{-17} and 10^{-14} respectively. As the clean signal line lies above the noisy signal curve, our research suggests that state-of-the-art atomic clocks have high enough frequency stability to measure perturbations on the space time metric caused by Planet 9.

Implications

- In conclusion, our simulations have shown that, theoretically, given the current state-of-art of atomic clocks, we may be able to detect massive bodies in the outer solar system solely due to the metric effects.
- The clock models we built can be applied to further research, such as performing sophisticated Fisher Matrix Analysis to determine a desirable signal-to-noise ratio with various sets of Planet 9 parameters.

References

[1]P. Tavella, "Statistical and mathematical tools for atomic clocks", *Metrologia*, vol. 45, no. 6, pp. S183-S192, 2008.

[2]T. Bothwell et al., "Resolving the gravitational redshift across a millimetre-scale atomic sample", *Nature*, vol. 602, no. 7897, pp. 420-424, 2022.

[3] Artist's impression of Planet Nine eclipsing the central Milky Way, *https://en.wikipedia.org/wiki/Planet_Nine*

[4] M. J. Holman and M. J. Payne. Observational Constraints on Planet Nine: Cassini Range Observations. Astronomical Journal, 152(4):94, October 2016.

[5] Artistic Representation of the Sun, the Earth and the Moon with the Space-Time Curvature of Einstein, Instituto de Astrofísica de Canarias (IAC), *https://www.eurekalert.org/multimedia/660646*