Imperial College London

Branes and Blackholes in Supergravity[1]

Ziwen Arno Liu, Yuelin Shen
Theoretical Physics Group, Department of Physics, Imperial College London
Supervisor: Professor Kellogg Stelle

Introduction

- ***Supergravity***: semi-classical field theory containing gravity; the low energy effective theory of superstring and M-theory.
- ***p-branes***: generalisation of particles in p-dimension (a p=0 brane is a particle; a p=1 brane is a string). p-branes are solutions to the field equations of supergravity.
- ***World-Volume & Transverse Space***: a p-brane occupies a p+1 spacetime known as the world-volume; the transverse space is orthogonal to the world-volume, shown in Fig.1.
- ***D-branes***: fundamental strings are attached to D-branes, shown in Fig.2. p-branes in Anti de-Sitter Space (AdS) is the low energy manifestation of the D-branes in SU supersymmetric Yang-Mills. This interplay between the two is the primary motivation of supergravity. [2]
- ***Aim of the Project***: understand the supergravity and its p-brane solutions which include the derivation, phenomenology and kinematics.

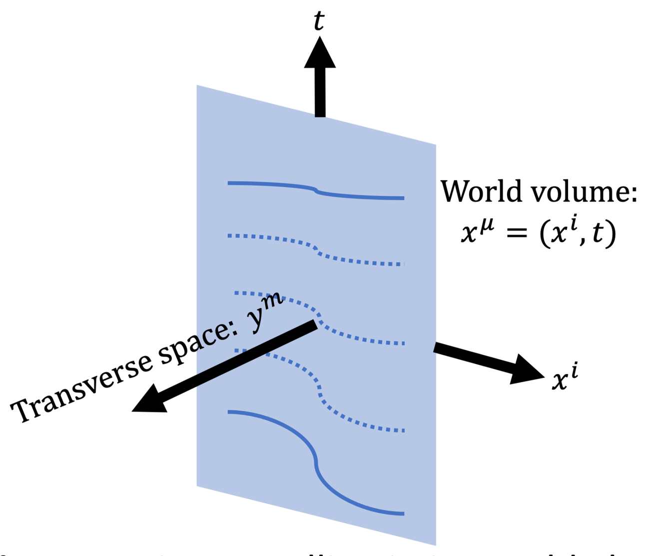

Fig.1: a string travelling in its world-sheet (2-D world-volume).

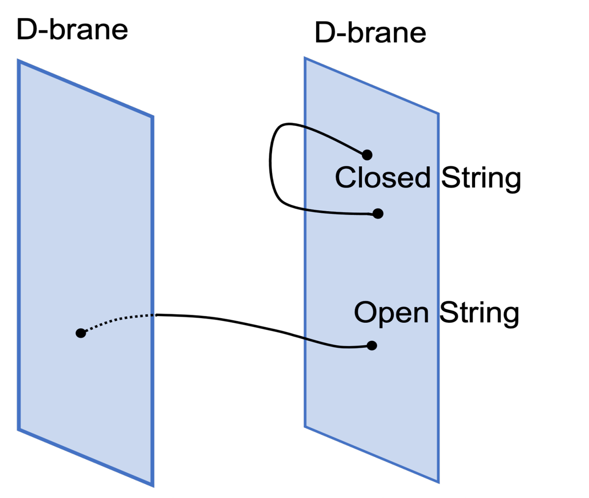

Fig.2: p-brane only contains closed strings; open strings can only attach to D-branes, but the two share a similar geometry.

Formalism

- ***General Relativity***: supergravity inherits some formalism in GR. The metric $g_{\mu\nu}$ in GR characterises the spacetime geometry by defining the interval $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Other properties of spacetime such as connection and curvature are derived from the metric.
- ***Form Fields***: a k-form, $F_{[k]}$, is an antisymmetric volume element in k-dimension with some orientation, analogous to a co-vector. A form field (differential k-form) assigns a k-form onto each point of the manifold. The wedge product between differential forms, given by the wedge $\wedge$, produces another differential form. $\overline{F_{[D-k]}} = \star F_{[k]}$ is the orthogonal to $F_{[k]}$ in a space with D maximum dimensions.
- ***Weyl Transformation***: local rescaling of the metric tensor, $g_{ab} \to \lambda^2 g_{ab} = \hat{g}_{ab}$. AdS is the combination of spherical and conformal symmetry, which our ansatz satisfies.

Supergravity Model

- ***Action Composition***: gravity, a Majorana spinor ψ_μ, a vielbein e^a_μ and an antisymmetric form field $A_{[3]}$ on the supermanifold. [3]
- ***Fermionic Sector***: the spinors exist in the tangent space, which is connected to the curved manifold via a vielbein, $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$.
- ***Bosonic Sector***: the action containing only gravity and $A_{[3]}$,

$$I_{11} = \int d^{11}x\left\{\sqrt{-g}\left(R - \frac{1}{48}F^2_{[4]}\right)\right\} - \frac{1}{6}\int F_{[4]} \wedge F_{[4]} \wedge A_{[3]}. (1)$$

- ***Single Charged Action***: dimensional reduction is used to recreate the Type-IIA string theory action in D=10, during which a scalar ϕ is introduced to the action.

$$I = \int d^D x\sqrt{-g}\left[R - \frac{1}{2}\nabla_M\phi\nabla^M\phi - \frac{1}{2n!}e^{\sigma\phi}F^2_{[n]}\right]. (2)$$

  The $F \wedge F \wedge A$ term in (1) vanishes for our symmetric ansatz. Setting $\sigma = 0$ can recreate the $F \wedge F$ term in (1).
- ***Equation of Motion***: EoM of (2) is a differential equation system containing ϕ, R, F.

Symmetric Ansatz

- ***Chosen Symmetry*** : the ansatz preserves $Poincaré_{(d)}$ in world-volume and $SO_{(D-d)}$ in the transverse space, hence the metric becomes,

$$ds^2 = e^{2A(r)}dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B(r)}dy^m dy^n \delta_{mn},$$

  where $r = \sqrt{y^m y_m}$; $\mu = 0,1,\ldots,p$ is world-volume indice; $m = p+1,\ldots,D+1$ is the transverse space indice.
- ***Solving Procedure***: R is obtained from the metric; F is obtained from its antisymmetry and r dependence. With R and F determined, we can solve for the $A(r)$, $B(r)$ and $\phi(r)$ in the EoM.
- ***Full Metric***:

$$ds^2 = H^{\frac{-4\tilde{d}}{\Delta(D-2)}}dx^\mu dx^\nu \eta_{\mu\nu} + H^{\frac{4d}{\Delta(D-2)}}dy^m dy^m, (3)$$

  where $d = p+1$ is the dimension of world-volume, $\tilde{d} = D - d - 2$, and $H(r) = 1 + \frac{k}{r^{\tilde{d}}}$ is the harmonic solution of the spherical Laplacian.

Electric/Magnetic p-branes

- ***Duality***: directly from (1), we can infer a 2-form and a 5-form supercharges due to the duality between $F_{[4]}$ and $\overline{F_{[7]}} = \star F_{[4]}$. $F_{[4]}$ comes from a $A_{[3]}$ that couples to a d=3 world-volume, which implies a 2-brane (similar with $\overline{F_{[7]}}$ and the 5-form).
- ***Dyonic Branes***: EoM yields two solutions due to the duality, the elementary/electric 2-brane and the solitonic/magnetic 5-brane. The metric for each brane is obtained by identifying the corresponding dimensionality, d, in (3).
- ***Horizons & Singularity***: for the electric case, after applying the coordinate transformation, $r^6 = \tilde{r}^6 - k$, one can identify a horizon at $\tilde{r}^6 = k$. The causal structure of both branes is shown in Carter-Penrose diagram in Fig.3; the geometry of the electric metric is illustrated in Fig.4.
- ***BPS Bound***: based on supersymmetry, $M \geq Q$, which is known as the BPS bound. Generally, brane metric yields two horizons, but our ansatz both satisfy the equality, $M = Q$, in which case the two horizons coincide. [1]
- ***Dimensional Reduction***: ϕ is introduced by Kaluza-Klein dimensional reduction, which eliminates the gauge field's dependence on one spatial dimension. KK reduction used in our ansatz it is a consistent truncation that doesn't change the solution to EoM. New families of brane-like ansatz will emerge from further dimensional reduction.

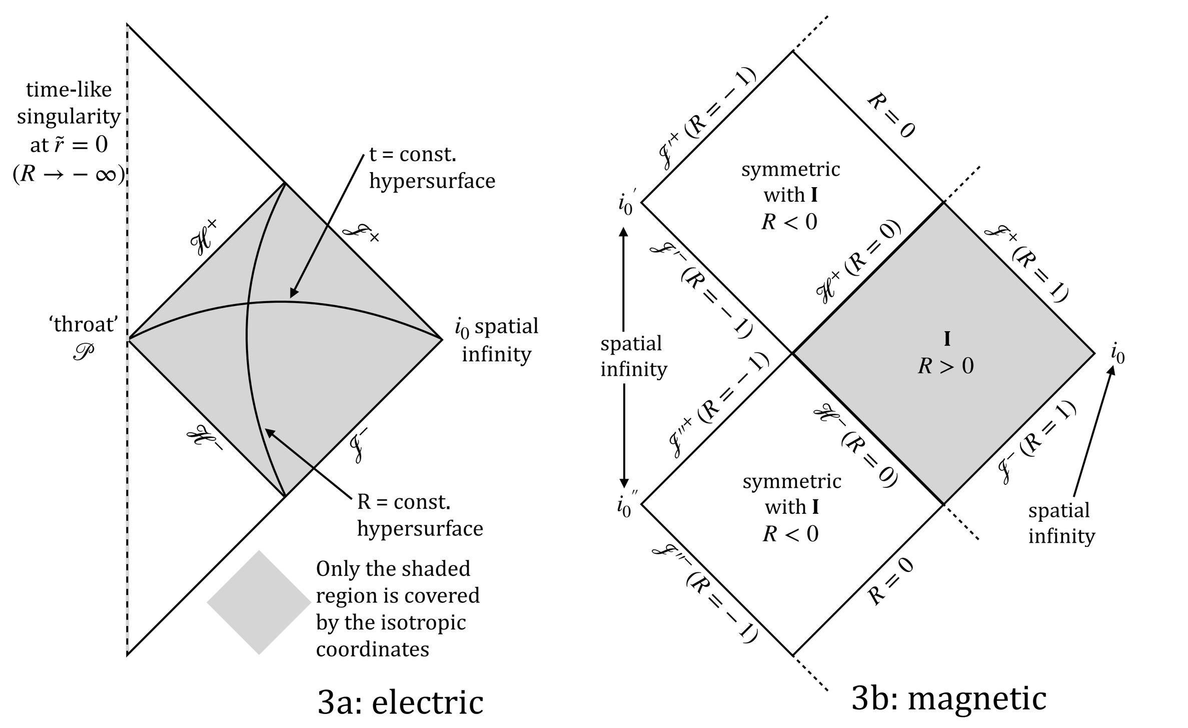

Fig.3: another transformation is applied here $\tilde{r} = k^{1/6}(1 - R^3)^{-1/6}$

3a) Electric brane: $\mathcal{H}^-$ and $\mathcal{H}^+$ are the two horizons, which has the same R, so they coincide; $\mathcal{J}^-$ and $\mathcal{J}^+$ are regions of flat space.

3b) Magnetic brane: the horizons now appear at $R = 0$; no singularity, instead there is a symmetry from $R \to -R$; once crossing the horizon, one can enter another identical space. [1]

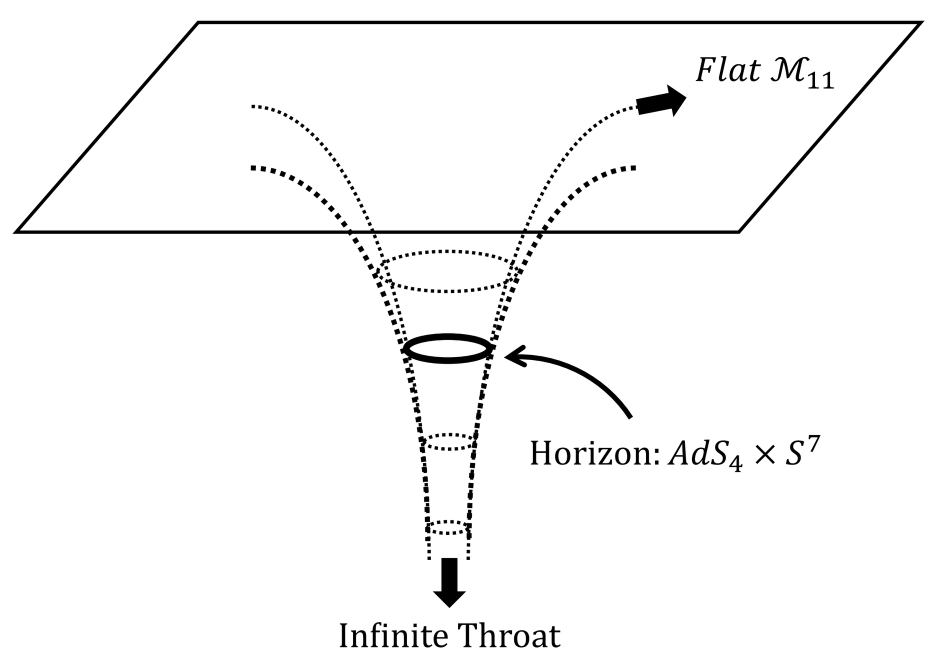

Fig.4: geometry described by the metric of the electric brane. Exterior of the horizon tends to D=11 flat space; on the horizon, the spacetime is $AdS_4 \times S^7$; down the infinity throat, one meets the singularity.

Brane Kinematics

- ***Branic orbits***: if two BPS branes are parallel in the world-volume, they can be dimensionally reduced to blackhole liked objects in the transverse space. By having one heavy brane as the background, we have investigated the orbital motion of the other probe brane, which is analogous to the orbit around a charged black-hole shown in Fig.5.
- ***Branic Motion***: if two parallel branes are stationary (∂y^0), the potential vanishes, because the form field repulsion offsets the gravitational attraction. In the second order (∂y^2), the probe brane will experience a flat metric from the heavier brane. Higher ordered motion will be investigated later in the project.

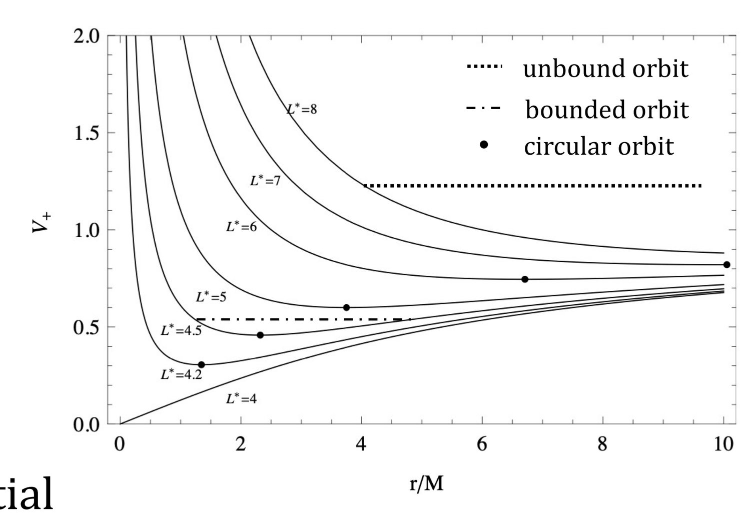

Fig.5: V^+ is the effective potential of a charged blackhole in natural units. V^+ depends on the charge, mass and angular momentum, L, of the probe. [4]

Reference

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[3] Polchinski, J. (1997). *TASI LECTURES ON D-BRANES*.
[4] Pugliese, D., Quevedo, H. and Ruffini, R. (2011). Motion of charged test particles in Reissner-Nordström spacetime. *Physical Review D*, 83(10).