Quantum Simulation of Spin models Using Dressed Rydberg Atoms

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1. What is Quantum Simulation and Why do we need it?

The simulation of large quantum systems has been a challenge for decades. The computer memory required for the simulation scales exponentially with both the size of the system and time evolution.

One solution is quantum simulation – using a controllable quantum system to simulate another. There are various methods for realising quantum simulation, some popular methods include trapped ions, superconducting circuits, photons and neutral atoms.

Our project concerns the analogue simulation of anisotropic spin models using Rydberg atoms.

2. Rydberg Atoms and Dressed Rydberg Atoms: What are they?

Rydberg atoms are atoms with an electron excited to a very high principal quantum number but are still bounded by the nucleus and are not ionised. They exhibit notable properties:

- Large size $\propto n^2$
- Large electric dipole moments $\propto n^2$
- Van der Waals interaction $\propto n^{11}$
- Long lifetime $\propto n^3$, on the scale of 700 μs.[1]
- High Polarizability $\propto n^7$

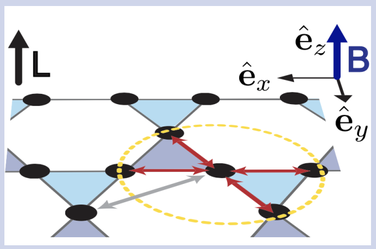


Figure showing the setup of atoms in a kagome lattice. [2]

However, the pure Rydberg state's lifetime is still not long enough to perform quantum simulation. **Thus, we can couple the Rydberg excited state and the ground state, and so form the dressed Rydberg state, which is given by**

$$|\tilde{e}\rangle = \Omega|g\rangle + (\Delta + \sqrt{\Delta^2 + \Omega^2})|e\rangle$$
$$|\tilde{g}\rangle = \Omega|g\rangle + (\Delta - \sqrt{\Delta^2 + \Omega^2})|e\rangle$$

where Ω is the Rabi frequency and Δ is the detuning.

Benefits of forming a dressed Rydberg state:

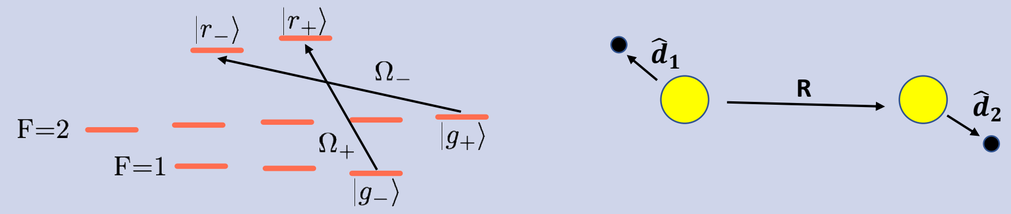
- Even longer lifetime, on the order of 100 ms to 1 s.[1]
- Finer tunability.

3.How do we use Dressed Rydberg Atoms to perform Quantum Simulation?

- Our setup: hyperfine ground states and fine Rydberg states

$$|g_-\rangle \equiv |5^2S_{1/2}, F = 1, m_F = 1\rangle$$
$$|g_+\rangle \equiv |5^2S_{1/2}, F = 2, m_F = 2\rangle$$
$$|r_{\sigma=\pm}\rangle \equiv |n^2P_{1/2}, m_j = \pm\tfrac{1}{2}\rangle \otimes |m_I = \tfrac{3}{2}\rangle$$

- Dressing by two circularly polarised lasers

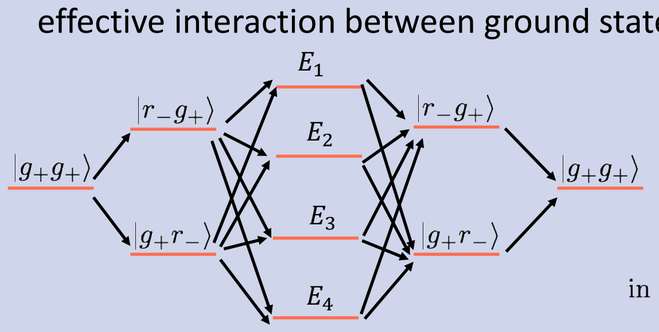


Figures showing the ground states and the Rydberg states, and the illustration of dipole-dipole interaction between two atoms separated by distance R.

- The interaction between the two atoms is mediated by the dipole-dipole interaction between the Rydberg states. The full Hamiltonian is

$$H = \sum_{i=1}^{2}\left[H_{Atom}^{(i)} + H_{Laser}^{(i)}\right] + H_{VDW}$$

- Treating the laser interaction as a fourth-order perturbation to find the effective interaction between ground states:



$$\tilde{H} = \begin{pmatrix} \tilde{V}_{00} & \tilde{V}_{01} & \tilde{V}_{02} & \tilde{V}_{03} \\ \tilde{V}_{10} & \tilde{V}_{11} & \tilde{V}_{12} & \tilde{V}_{13} \\ \tilde{V}_{20} & \tilde{V}_{21} & \tilde{V}_{22} & \tilde{V}_{23} \\ \tilde{V}_{30} & \tilde{V}_{31} & \tilde{V}_{32} & \tilde{V}_{33} \end{pmatrix}$$

in the basis $\{|g_+g_+\rangle, |g_-g_+\rangle, |g_+g_-\rangle, |g_-g_-\rangle\}$

Diagrams showing the fourth-order term in the fourth order perturbation and the resulting effective Hamiltonian.

- The effective interaction Hamiltonian is shown in $\widetilde{H}$. Note that this is a general expression for all the interactions, including **anisotropic** interactions.

4. Our results and outlook

- We then map the spin systems to the Rydberg atom systems in the following procedure:

$$|g_+\rangle \rightarrow |\uparrow\rangle \qquad |g_-\rangle \rightarrow |\downarrow\rangle$$

- For instance, the Pauli z matrix would be mapped as:

$$S_z = (|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) \rightarrow (|g_+\rangle\langle g_+| - |g_-\rangle\langle g_-|)$$

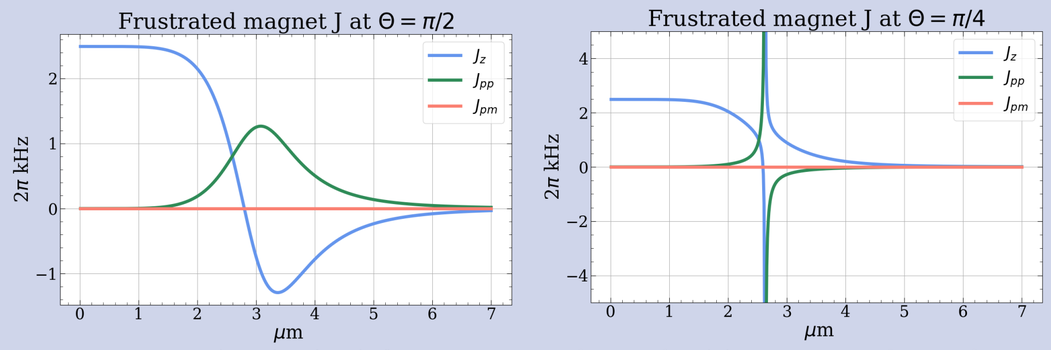
- We first simulate a well-known Hamiltonian, the frustrated magnetic, to test our computational model, where the Hamiltonian is given by[2]:

$$H = \sum_{i<j}\left[J_z(\mathbf{r}_{ij}) S_z^i S_z^j + J_{\parallel}(\mathbf{r}_{ij}) S_z^i + \frac{1}{2}\left(J_{+-}(\mathbf{r}_{ij}) S_+^i S_-^j + J_{++}(\mathbf{r}_{ij}) S_+^i S_+^j + \text{H.c.}\right)\right]$$

- Thus, we have the mapping to our effective interactions

$$J_z(\mathbf{r}_{ij}) = \frac{1}{4}\left[\tilde{V}_{33}(\mathbf{r}_{ij}) - 2\tilde{V}_{22}(\mathbf{r}_{ij}) + \tilde{V}_{00}(\mathbf{r}_{ij})\right]$$

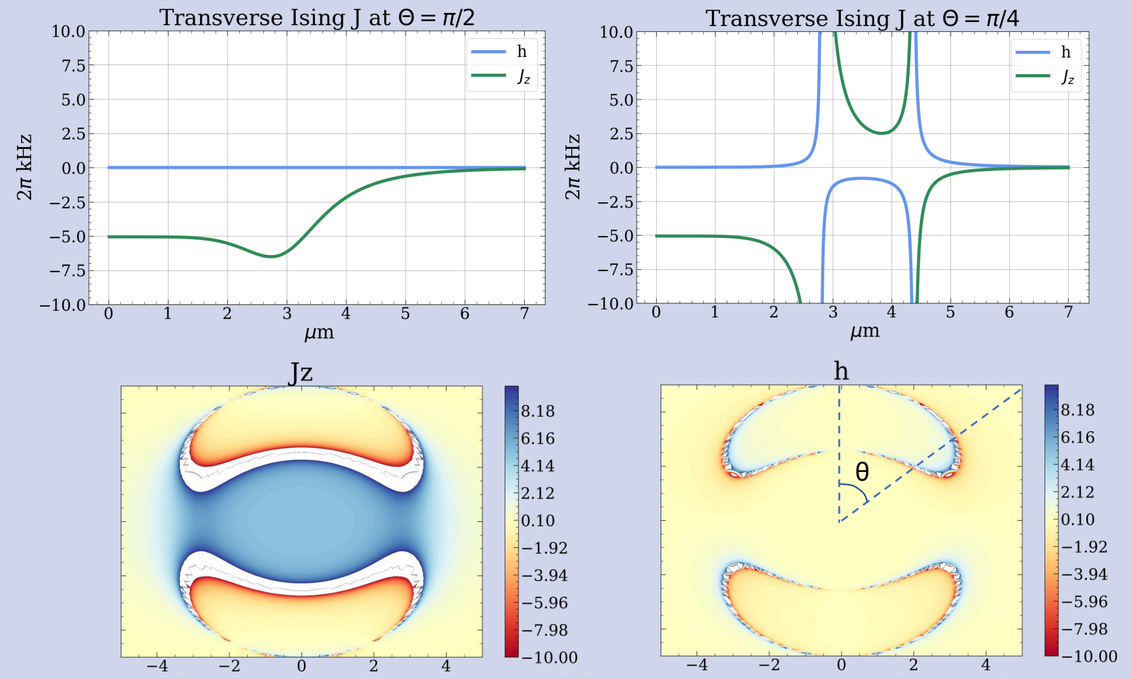
$$J_{+-}(\mathbf{r}_{ij}) = 2\tilde{V}_{12}(\mathbf{r}_{ij}), \text{ and } J_{++}(\mathbf{r}_{ij}) = 2\tilde{V}_{03}(\mathbf{r}_{ij})$$



Figures plotted with Rabi frequencies 10, 2.5 [2π MHz] and detuning 50, -50 [2π MHz].

- We mainly consider the transverse Ising model, which has a simpler form, which makes it easier to study certain properties. Its Hamiltonian is given:

$$H = \sum_{i<j} J_z(r_{ij}) S_z^i S_z^j + \sum_i h_i^x S_x^i$$



Figures plotted with Rabi frequencies 10, 5 [2π MHz] and detuning 40, -60 [2π MHz].

- By manipulating the detuning and laser frequency, we can get the above figures. Note that in the transverse Ising θ = π/2 figure, we can construct a potential that looks like a step function using the J_z term.
- By optimizing the angle and detuning, we can get any non-trivial potentials between the spins. And we can use that to simulate various exotic condensed matter dynamics, for example interacting domain walls.

5.Conclusion

- We demonstrate the possibility of performing quantum simulation for spin systems using Rydberg atoms due to their useful properties.
- We construct a method to theoretically calculate the anisotropic Van-der-Waals interactions between Rydberg atoms.
- Furthermore, we show that by choosing specific detuning and laser frequencies, we can generate certain interesting and promising potentials, such as the step-function-like potential, and apply it to simulate other condensed matter dynamics.

6.Reference

- [1]:Nils Henkel. Rydberg-dressed bose-einstein condensates. Dresden, 2013.
- [2]:Alexander W. Glaetzle, Marcello Dalmonte, Rejish Nath, Christian Gross, Immanuel Bloch, and Peter Zoller. Designing frustrated quantum magnets with laser-dressed rydberg atoms. Physical Review Letters, 114, 2015.