Imperial College London

Simulating the Braiding of Majorana Fermions with Quantum Computer

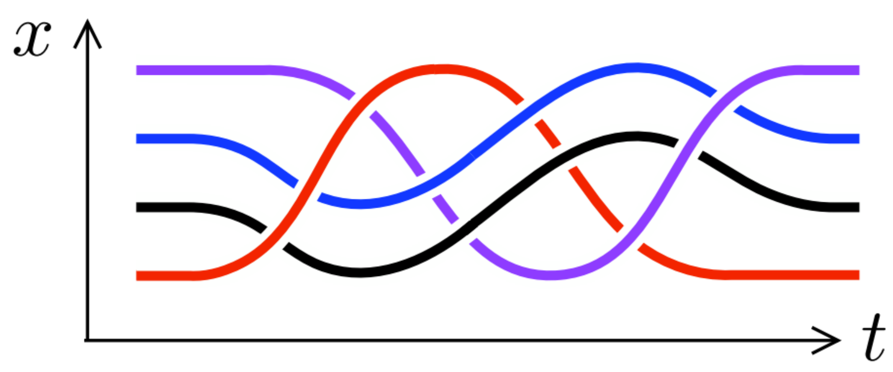
Tak Hur, Soovin Lee

Supervised by Prof. Derek Lee

Condensed Matter Physics Group

Introduction

- **Quantum computers** are yet to outperform the classical computers due to instability and noise.
- **Topological quantum computation** is a new **fault-tolerant** quantum information processing method based on **quantum braids** — formed out of **non-Abelian anyons** moving in 3D spacetime.



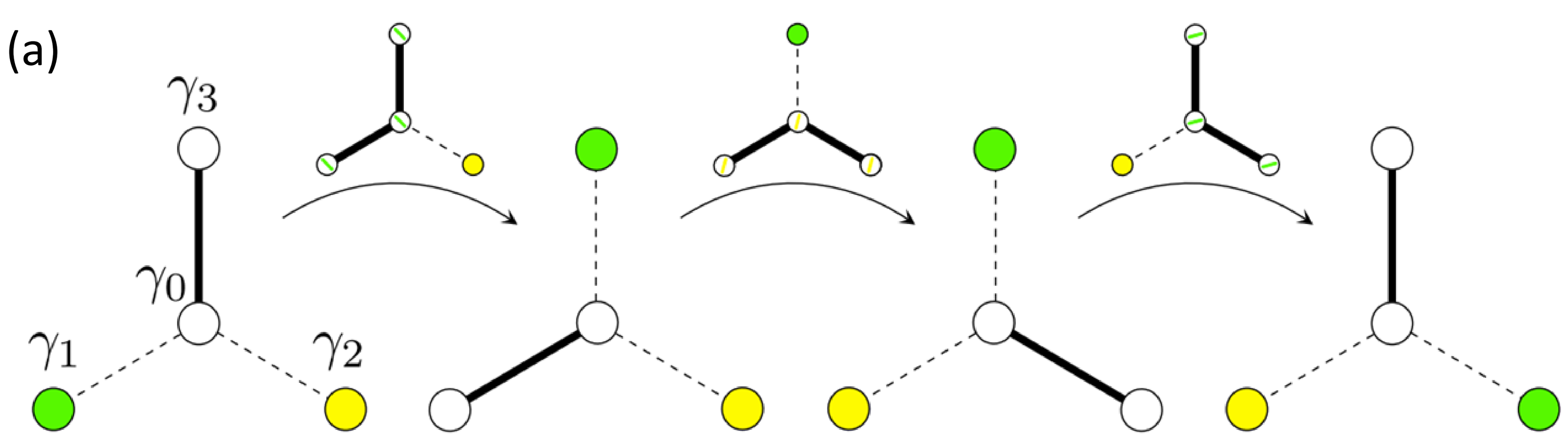
[1]

- **Majorana fermion** is a promising candidate of qubit with non-Abelian braiding statistics.

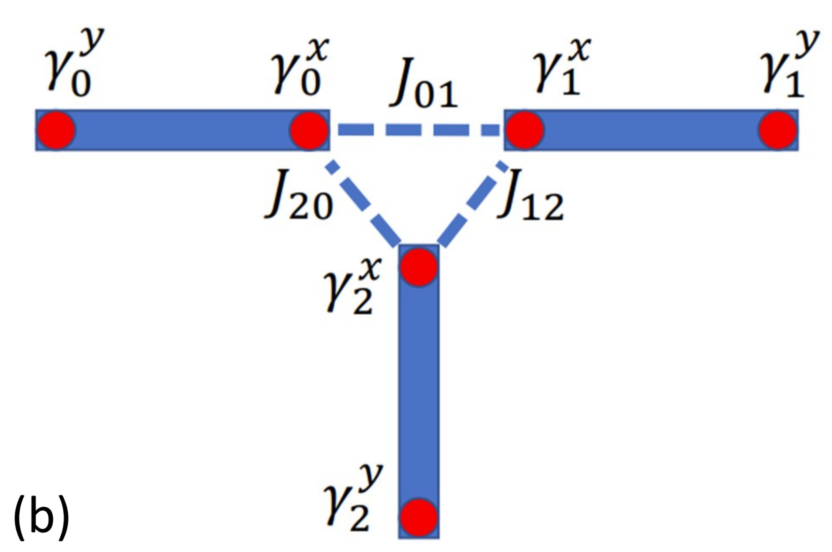
Quantum braiding

- Particles in 2D system can result in a non-trivial phase under exchange — called **anyons** with statistics **θ**.

(a)



- A **non-Abelian** braiding of a system with a degenerate set of *g* states is represented by ***g x g*** **unitary matrix** which is used as a **quantum gate**.
- Such quantum gate is **tolerant under local decoherence**.



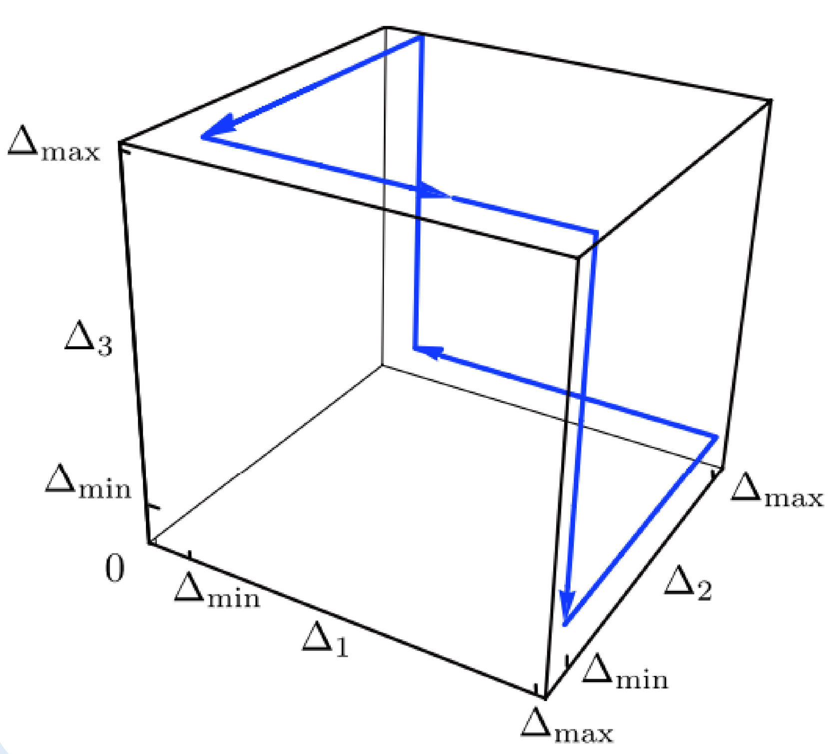
(b)

(a) Exchange of MZMs [1]. Three nanowires meet at a tri-junction, where MZM γ_0 can be coupled or decoupled.

(b) More experimentally viable schematic [2] as Majorana fermions are predicted to arise at the ends of nanowire.

Key Calculations

- **4MZM Hamiltonian** is $\sum_{k=1}^{3} \Delta_k(t) i\gamma_0\gamma_k$.
- Fermionic operators $a_1 = (\gamma_1 - i\gamma_2)/2$, $a_2 = (\gamma_0 - i\gamma_3)/2$ defines basis states $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ where 0, 1 are fermion occupation numbers.



- MZMs are braided by **tuning coupling strength** Δ_k. It forms **cubic pathway in a parameter space.**
- The ground state is **two-fold degenerate.**
- Even and odd states gains $e^{-i\frac{\pi}{4}}, e^{i\frac{\pi}{4}}$ **Berry phases.** It is **rotation in Z axis,** $U = \exp\left[-i\frac{\pi}{4}\hat{\sigma}_z\right]$.

Quantum Computation

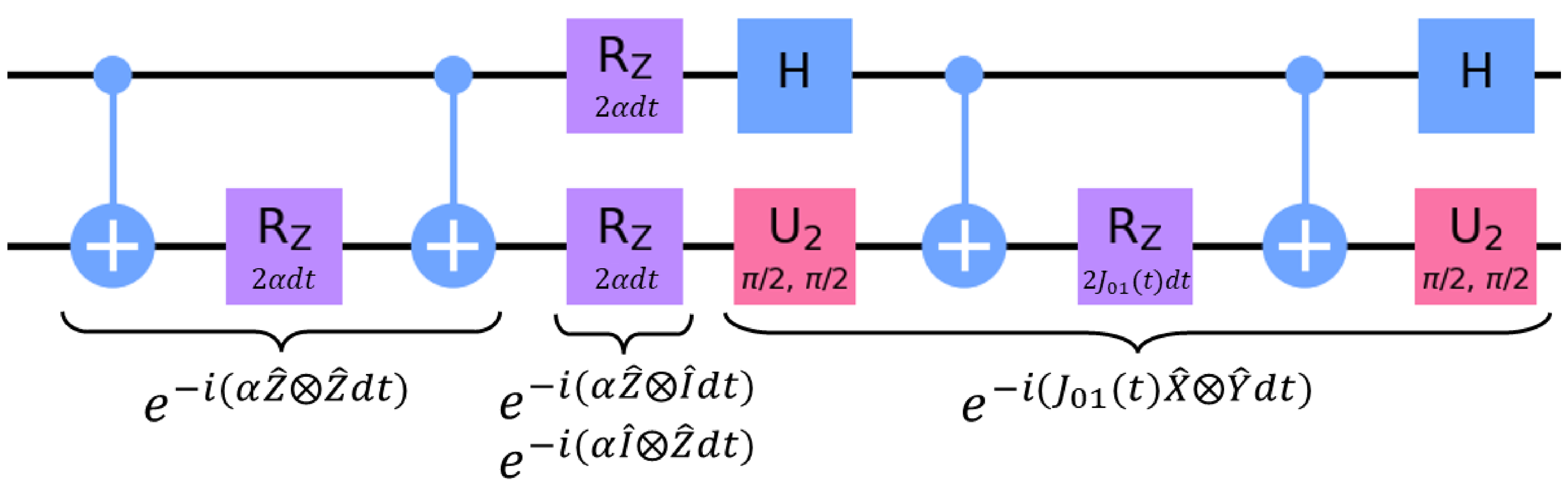
- **6MZM Hamiltonian** is:

$$H(t) = \begin{pmatrix} 3\alpha & -iJ_{12} & -iJ_{20} & -iJ_{01} \\ iJ_{12} & -\alpha & iJ_{01} & -iJ_{20} \\ iJ_{20} & -iJ_{01} & -\alpha & iJ_{12} \\ iJ_{01} & iJ_{20} & -iJ_{12} & -\alpha \end{pmatrix}$$

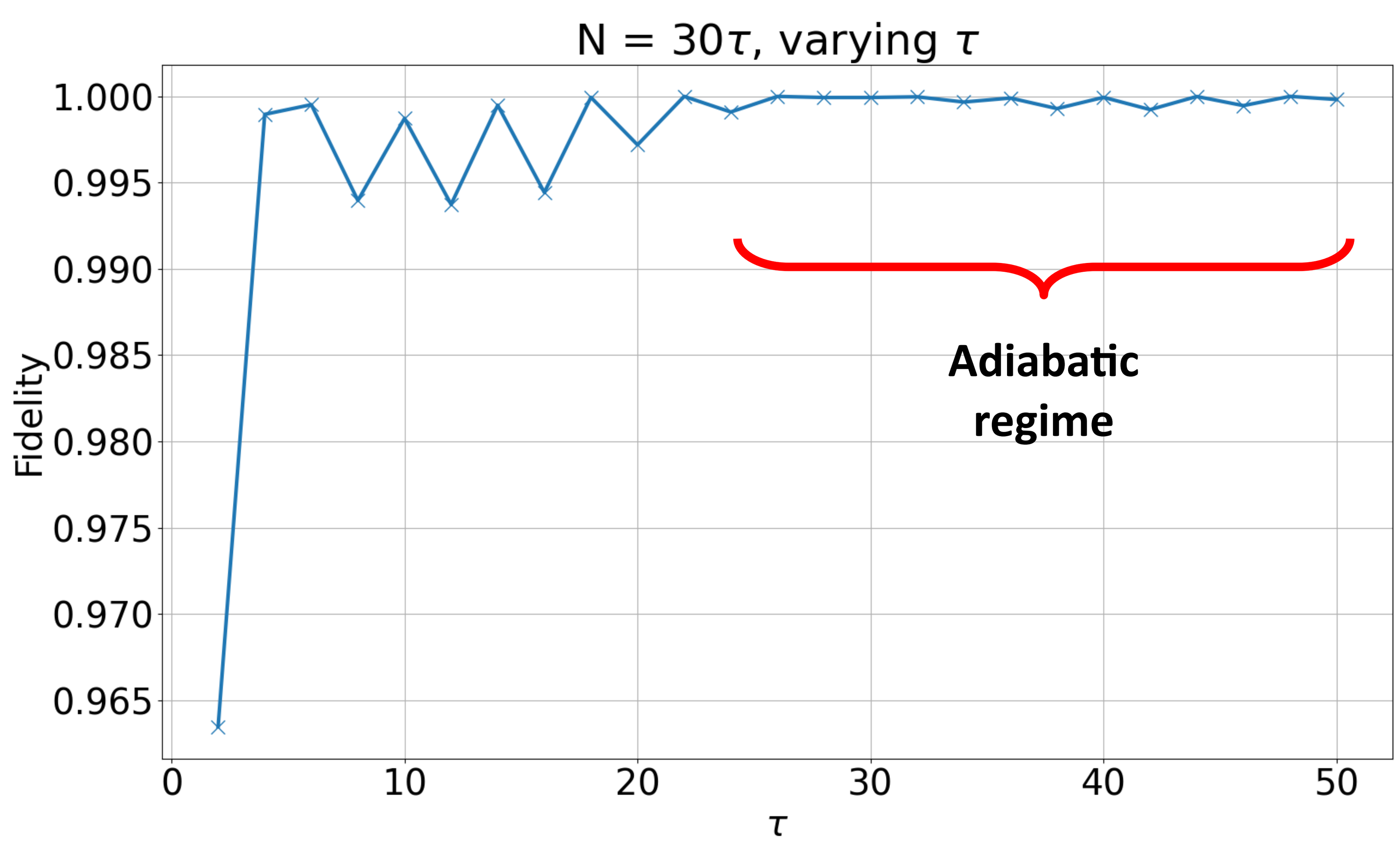
$$= \alpha(\hat{Z}\otimes\hat{Z} + \hat{Z}\otimes\hat{I} + \hat{I}\otimes\hat{Z}) + J_{01}(t)\hat{X}\otimes\hat{Y} + +J_{12}(t)\hat{Z}\otimes\hat{Y} + +J_{20}(t)\hat{Y}\otimes\hat{I}$$

- Unitary evolution is simulated by **Suzuki-Trotter decomposition.**

$$U(t, dt) = exp\left[-i\sum_n H_n(t)dt\right] = \prod_n exp[-iH_n(t)dt] + O(dt^2)$$



- The simulation is prone to two errors: 1) **trotter error** (eq) 2) mixing of the excited states from **non-adiabatic evolution**.
- Minimising the error requires a slow total evolution **(T >> 1)** but fast single-step trotter evolution **(dt << 1)**.



Conclusion

- We have **classically simulated a full braid** with the resultant phase difference consistent with the analytical solution.
- Quantum devices can efficiently probe at complex quantum dynamics.
- The full evolution requires ***O*(10³) CNOT gates**, which is far beyond what the current quantum computers can handle.
- We must balance three errors — **noisy CNOT gate error, trotterization error, and non-adiabatic regime error** — to get the best results.
- **Pulse level control** (Qiskit Pulse) can be used to implement **two qubit ZZ gates** $e^{i(Z\otimes Z)\theta}$ over the conventional CNOTs.

References

[1] Beenakker, C.W.J. (2020) SciPost Phys. Lect. Notes 15. 10.21468/SciPostPhysLectNotes.15

[2] Stenger, J.P.T (2021) Phys. Rev. Research 3, 033171. 10.1103/PhysRevResearch.3.033171