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Is Bach Chaotic?

Interpreting Bach Chorale Works Through Information Theory

Aim: Analyse the information content of written music through mutual information and entropy, investigate the role of vocal parts and their interplay, arrive at a quantitative description of musicality and monotony in music.

Mutual Information and Entropy

Mutual Information – Gives the expected amount of information gained about the value of X from the prior knowledge of Y, or vice-versa:

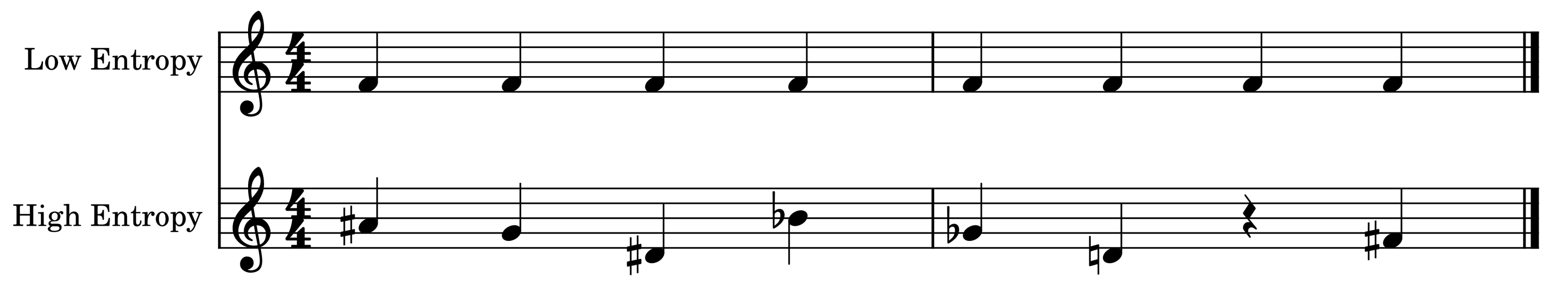
$$\mathbb{I}(X,Y) = \sum_{n=1}^{N}\sum_{m=1}^{M} \mathbb{P}(X = a_n \cap Y = b_m) log_2 \left[\frac{\mathbb{P}(X = a_n \cap Y = b_m)}{\mathbb{P}(X = a_n)\mathbb{P}(Y = b_m)}\right] bits$$

Entropy – Quantifies the (non-redundant) information content of X. It also corresponds to reflexive mutual information, i.e. when $X = Y$:

$$\mathbb{I}(X,X) = \mathbb{H}(X) = \sum_{n=1}^{N} \mathbb{P}(X = a_n) log_2 \left[\frac{1}{\mathbb{P}(X = a_n)}\right] bits$$

In music, entropy relates to the melodic content of a part and mutual information describes the counterpoint and harmony in a rudimentary way.

Entropy in Music



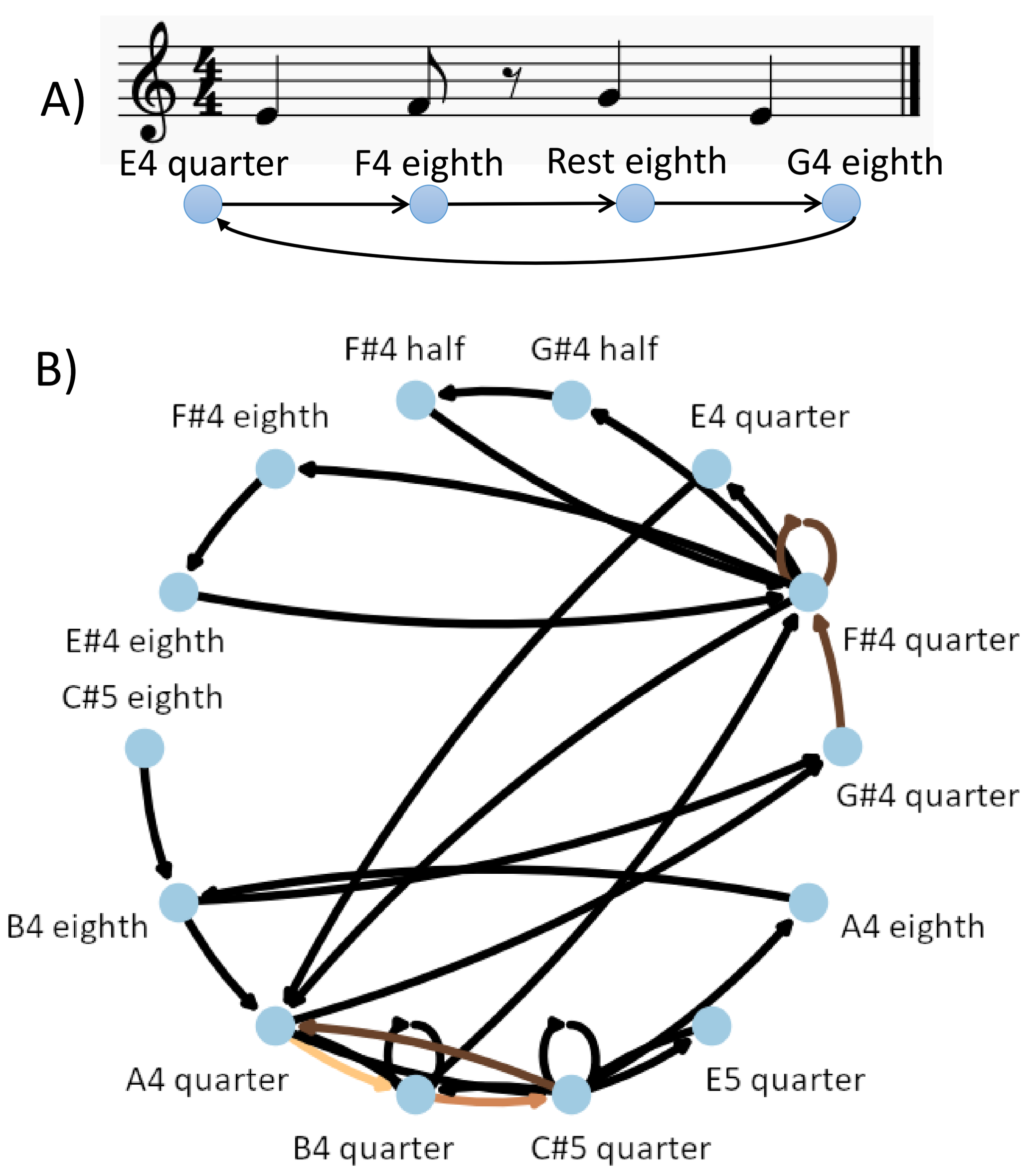
Here, the top line is entirely redundant, and has 0 entropy. The bottom line is entirely unique, and has 3 bits of entropy from 2^3 unique notes.

In general, composition treads a fine line between having recognisable patterns and introducing new musical ideas, which is why entropy is a useful metric.

Informational entropy is used similarly in physics: high entropy implies greater chaos (lots of unique parts), low entropy implies order.

Uncertainties

It is impossible to understand the significance of the information measures we observe without an estimation of how practical differences manifest.



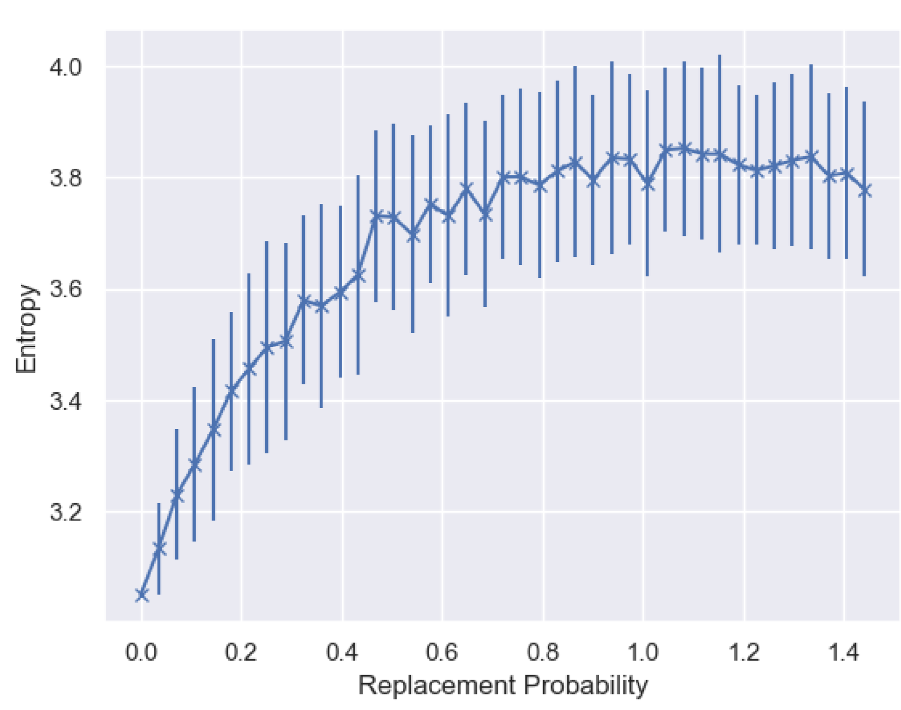
A) The line of music as a network - edges show the note movements. ('E4 quarter' is the note E in the 4th octave that lasts for a single beat – a quarter of a four beat bar)
B) This is applied to Bach chorale BWV 66.6

Random Walks

By treating the music as a network of notes and durations, as seen on the left, it is possible to generate a baseline random dataset by following a path on the edges, much like exploring roads in a town.

With a sample of 1000 such 'walks' on a network of around 300 Bach chorales, the uncertainty in the entropy due to randomness was found to be 0.3 bits.

Bootstrapping (Re-sampling with replacement)

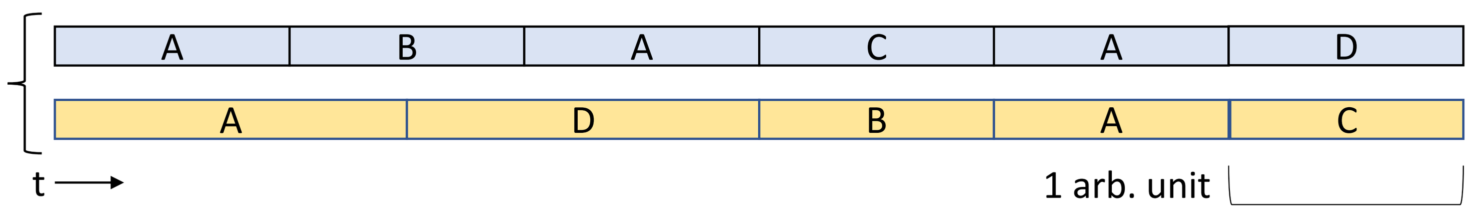


Bootstrapping allows us to understand how our measurements would change under small perturbations in our data sample.

Here, the perturbation consists of randomly replacing individual notes according to a replacement probability.

The graph shows how, as the replacement probability increases, patterns in the music vanish and the sequence of notes becomes more chaotic. The uncertainty remains on the order of 0.1 bits.

Mutual Information Measurements



By treating the music as a series of simultaneous notes, it is possible to find the relevant conditional probabilities for measuring mutual information by measuring how long notes coincide for, as in the visualisation.

	A	B	C	D
A	2.5	0.5	0	0
B	0	0	1	0
C	0	0	0	1
D	0.5	0.5	0	0

Musically, mutual information implies a harmonic pattern between two parts. However, direction of influence and the specific musical effect are not captured. For finer detail, there are further metrics to be used. [2]

Results

	S	A	T	B
S	2.6	1.2	1.2	1.8
A	1.2	2.7	1.2	1.7
T	1.2	1.2	2.7	1.5
B	1.8	1.7	1.5	3.4

Table 1. Mutual Information Matrix for BWV 66.6

	S	A	T	B
S	2.7 ± 0.3	1.1 ± 0.3	1.1 ± 0.2	1.3 ± 0.2
A	1.1 ± 0.3	2.7 ± 0.3	1.1 ± 0.3	1.3 ± 0.2
T	1.1 ± 0.2	1.1 ± 0.3	2.8 ± 0.3	1.3 ± 0.2
B	1.3 ± 0.2	1.3 ± 0.2	1.3 ± 0.2	3.4 ± 0.3

Table 2. Mutual Information Matrix for the corpus of 371 Bach Chorales

- Bach's Bass parts have the greatest entropy (larger variation of note choice) and the greatest mutual information with the other voices, (most similar harmonic interplay).
- The other three voice parts are treated quite equally by Bach.

This can be attributed to a contemporary use of figured bass, in which the harmony is defined from the bass line being written first.

References

1. MacKay DJC. Information theory, inference, and learning algorithms. Cambridge University Press; 2003. 628 p.
2. Scagliarini T, Marinazzo D, Guo Y, Stramaglia S, Rosas FE. Quantifying high-order interdependencies on individual patterns via the local O-information: theory and applications to music analysis. 2021 Aug 26; Available from: http://arxiv.org/abs/2108.11625