Spatial Networks

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1. Introduction

- Spatial network: nodes and edges embedded in a metric space (often geometric distance).
- A node can represent anything, and the edge signifies the interaction and connection between them.
- Given a set of data, different network models give different results

2. Research Question

- How do different network models compare against each other?
- How does coarse graining affect the models?
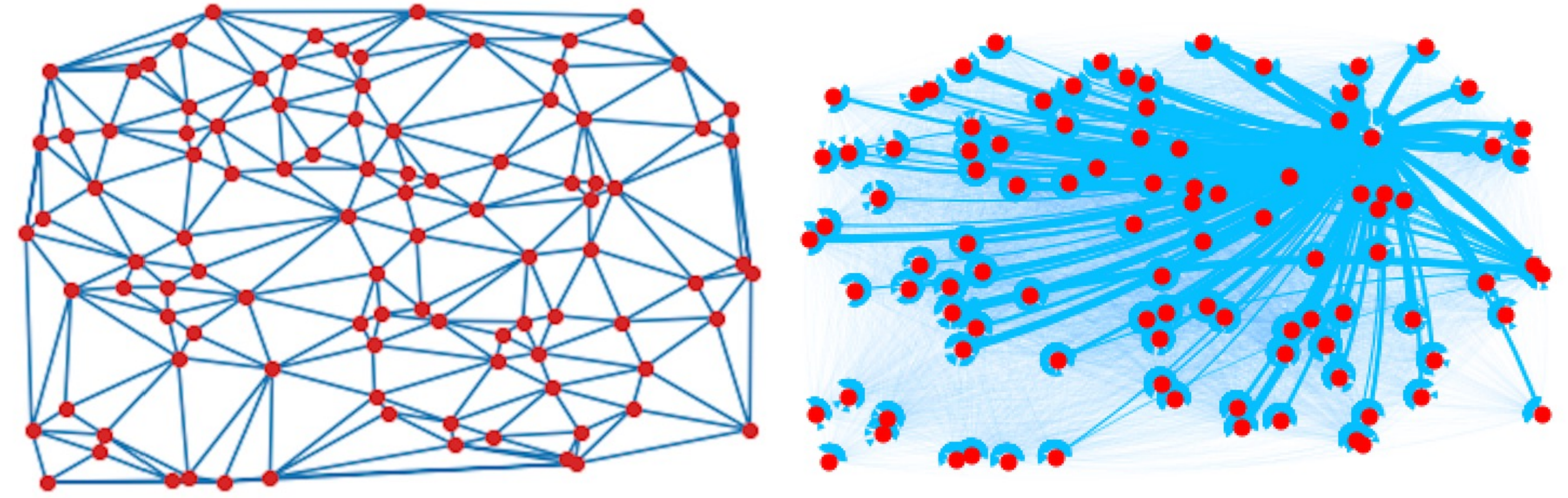- How important are the errors in the data of the nodes?

Fig 1. A comparison between a network of 100 nodes formed by Delaunay Triangulation (left) and a simple gravity model (right).
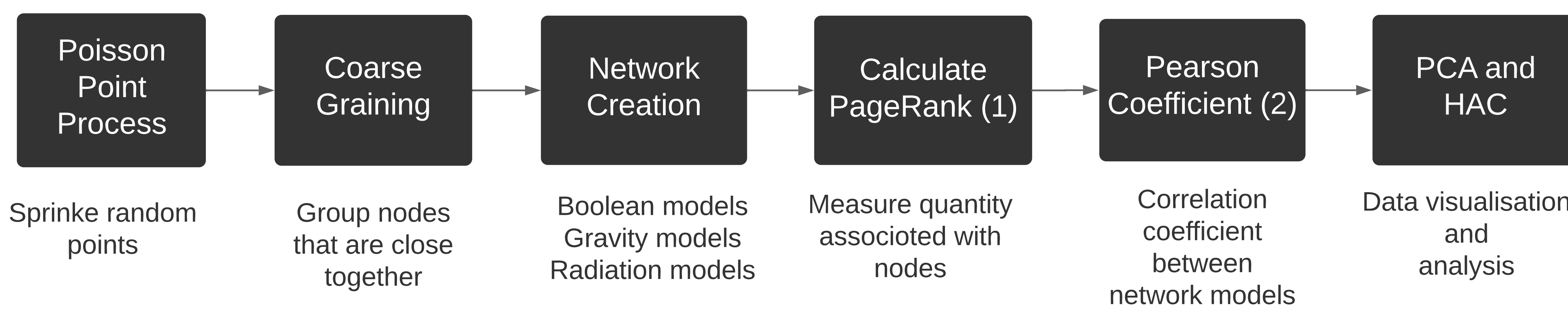
3. Method

Poisson Point Process — Sprinke random points

Coarse Graining — Group nodes that are close together

Network Creation — Boolean models, Gravity models, Radiation models

Calculate PageRank (1) — Measure quantity associoted with nodes

Pearson Coefficient (2) — Correlation coefficient between network models

PCA and HAC — Data visualisation and analysis

Fig 2. Illustrative example of PageRank. (1)

4. Results

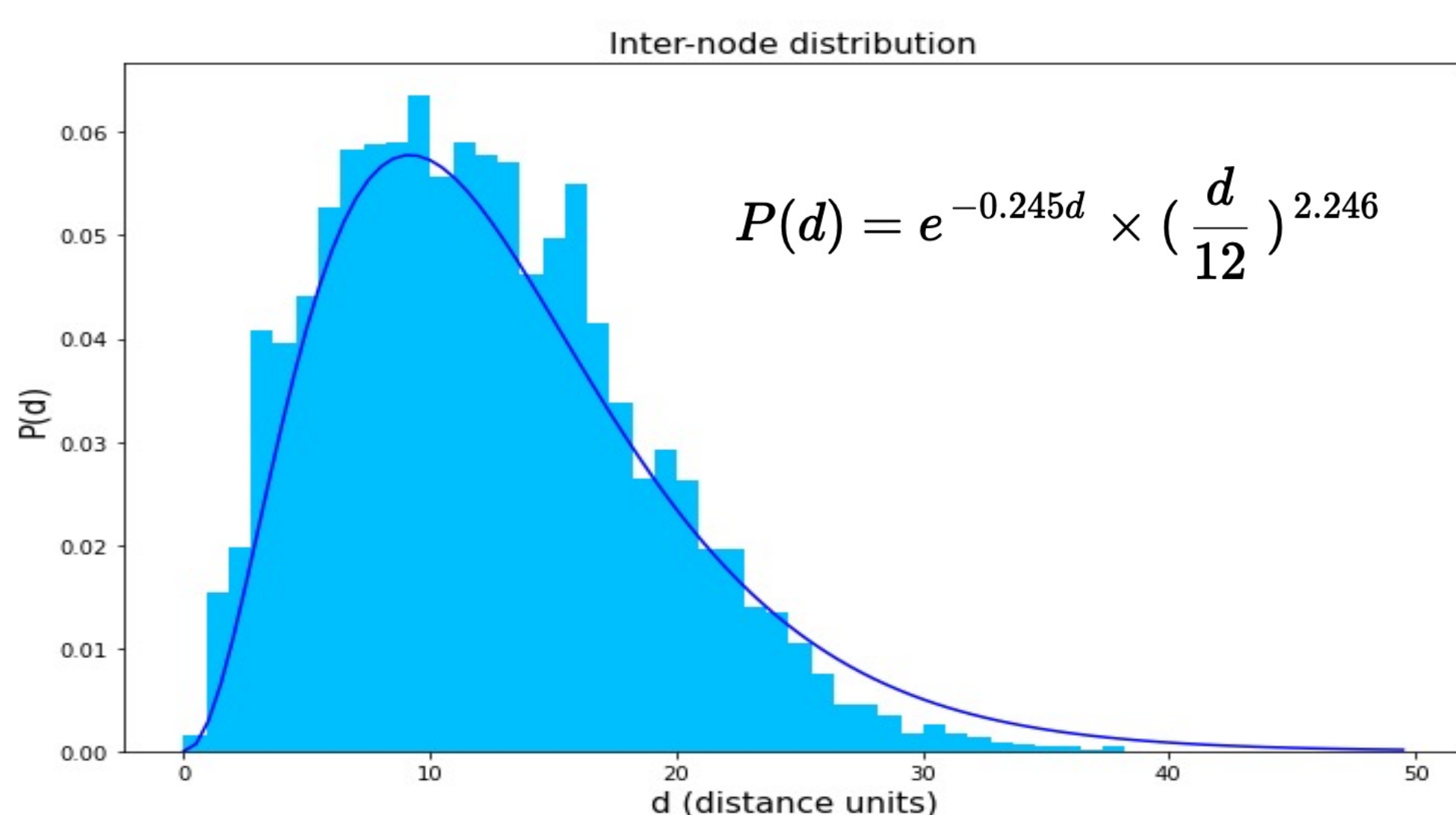$$P(d) = e^{-0.245d} \times \left(\frac{d}{12}\right)^{2.246}$$

Fig 3. Geographical distance distribution of any two nodes among 100 nodes, d representing inter-node distance. The distribution follow that as observed by W.Zhang (2020) (3)
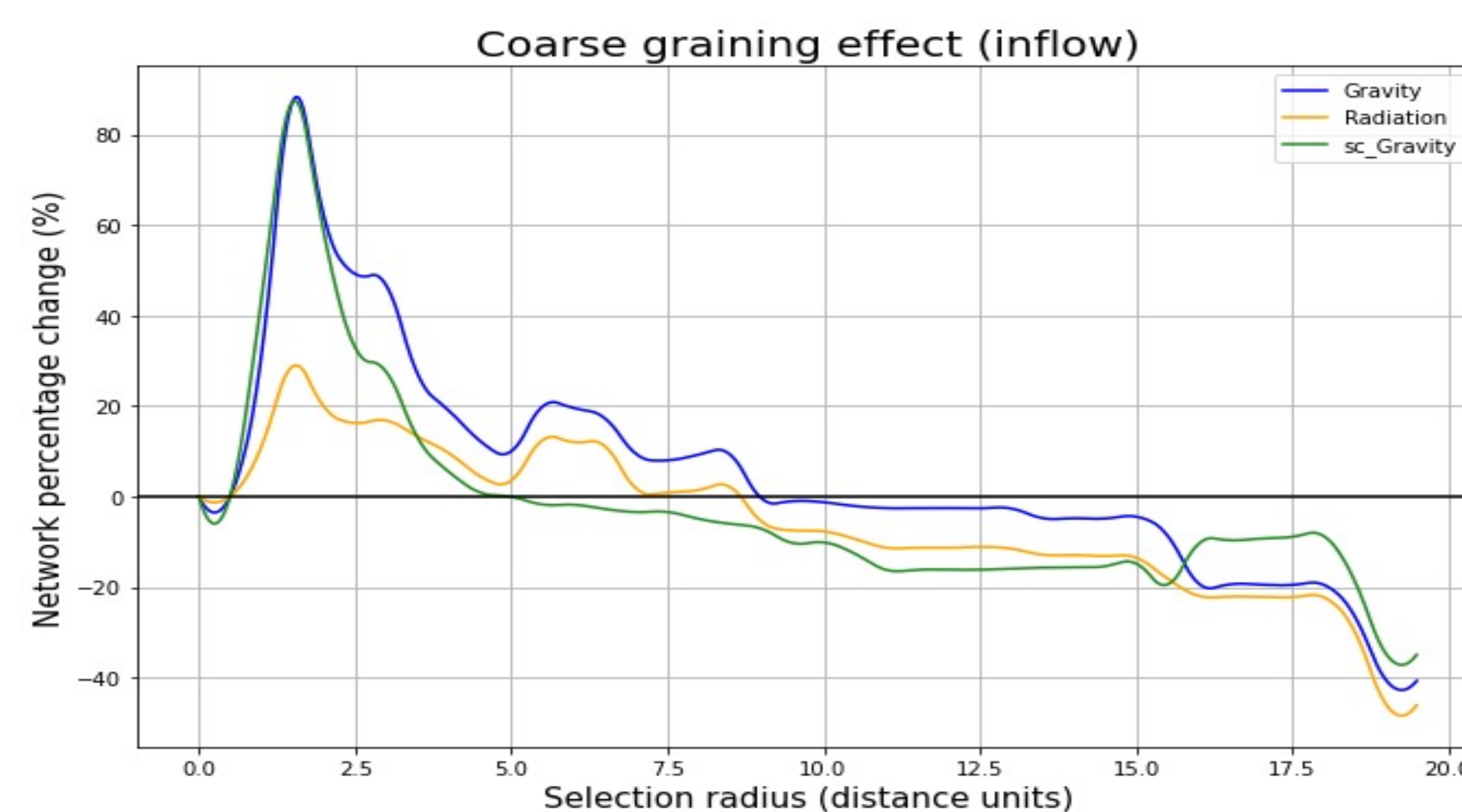

Fig 4. The selection radius indicates the scale of coarse graining. The graph shows the change in total inflow of the network at different coarse graining distance.
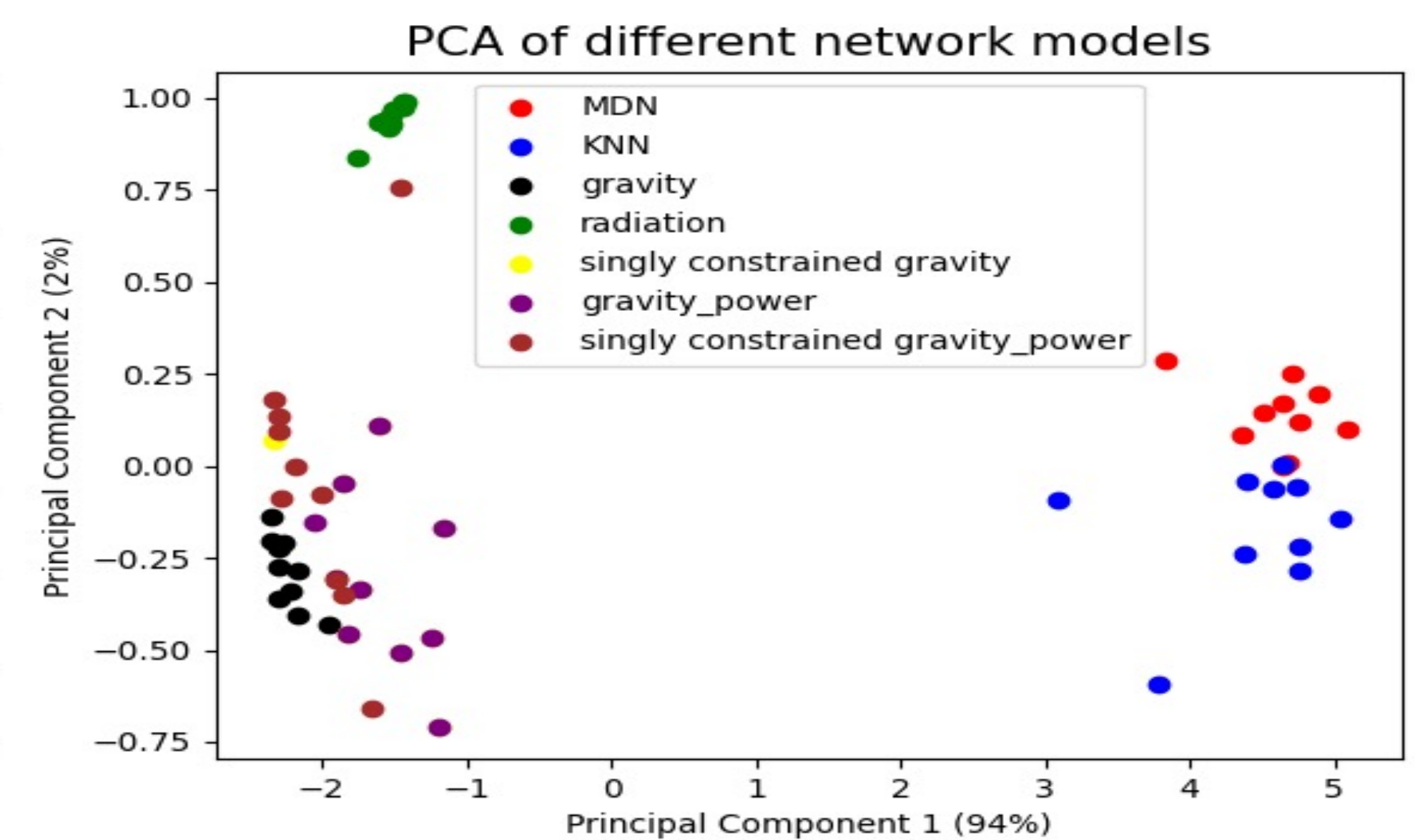

Fig 5. The positions of the nodes were randomly varied to produce ten networks and the similarity between the models are visualized using PCA. With the X axis holding much more weight, horizontal separation signifies a drastic difference in behavior.

5. Conclusion

- Two separate groups of models (Boolean models comparing to ntose with a flow).
- Boolean models are more sensitive to error in the information Tf the nodes and should be used carefully if there is a large uncertainty in the data.
- Distance dependent models (Gravity models) are heavily affected at short coarse graining distances. At long distances, more complex models (which have more constraints) are more heavily affected.
- There is a distance scale at which the effect of coarse graining is nullified for different models.

References

1. S. Brin, L.Page. (1998). "The anatomy of a large-scale hypertextual Web search engine"
2. A. Bravais. (1844). "Analyse mathématique sur les probabilités des erreurs de situation d'un point"
3. W.Zhang. et.al (2020) "Chinese cities' air quality pattern and correlation"