Investigating Single Pion Production from Neutrino-Nucleon Interactions Using the MK Model

HEP Research Group
Balaji Harihar, Nicholas Mok
Supervisors: Prof Yoshi Uchida, Dr Minoo Kabirnezhad
HEPH-Uchida-2 Modelling of Inelastic Neutrino Scattering

Introduction

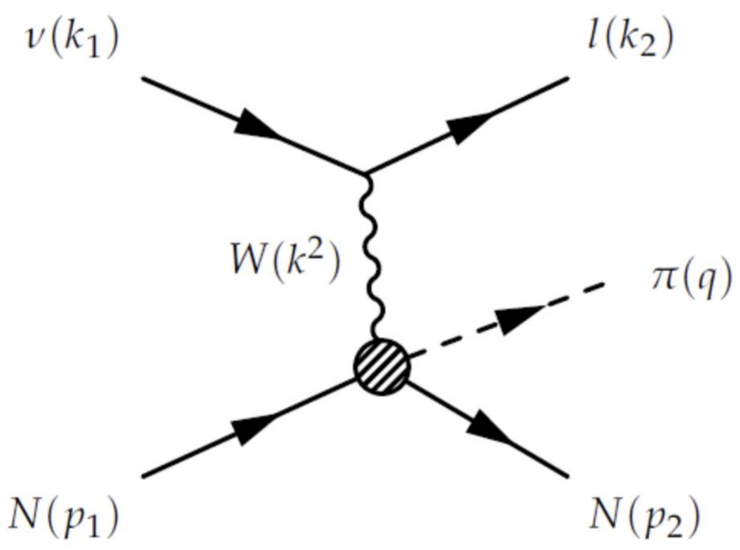
- Neutrinos are not massless particles as stated in the Standard Model.
- Understanding the interaction cross section of neutrinos is essential to neutrino research.
- Single pion production (SPP) represents a large fraction of the total cross section for neutrino oscillation experiments.
- Neutrino cross section measurement gives invaluable information about the axial properties and transitional form-factors as the nucleon axial form-factor is still poorly known.

MK Model

- Rein and Sehgal model used in neutrino simulation generators such as NEUT and GENIE.
- MK model expands on that by including non-resonant interactions and lepton mass effects. [1, 2]
- The model provides a full kinematic description of the final state particles, which are listed below.

- For charged current (CC) interactions:
- $\nu_l + p \to l^- p\pi^+$, $\bar{\nu}_l + n \to l^+ n\pi^-$,
- $\nu_l + n \to l^- p\pi^0$, $\bar{\nu}_l + p \to l^+ n\pi^0$,
- $\nu_l + n \to l^- n\pi^+$, $\bar{\nu}_l + p \to l^+ p\pi^-$

- For neutral current (NC) interactions:
- $\nu + p \to \nu p\pi^0$, $\bar{\nu} + p \to \bar{\nu} p\pi^0$,
- $\nu + p \to \nu n\pi^+$, $\bar{\nu} + p \to \bar{\nu} n\pi^+$,
- $\nu + n \to \nu n\pi^0$, $\bar{\nu} + n \to \bar{\nu} n\pi^0$,
- $\nu + n \to \nu p\pi^-$, $\bar{\nu} + n \to \bar{\nu} p\pi^-$



Single pion production [1]

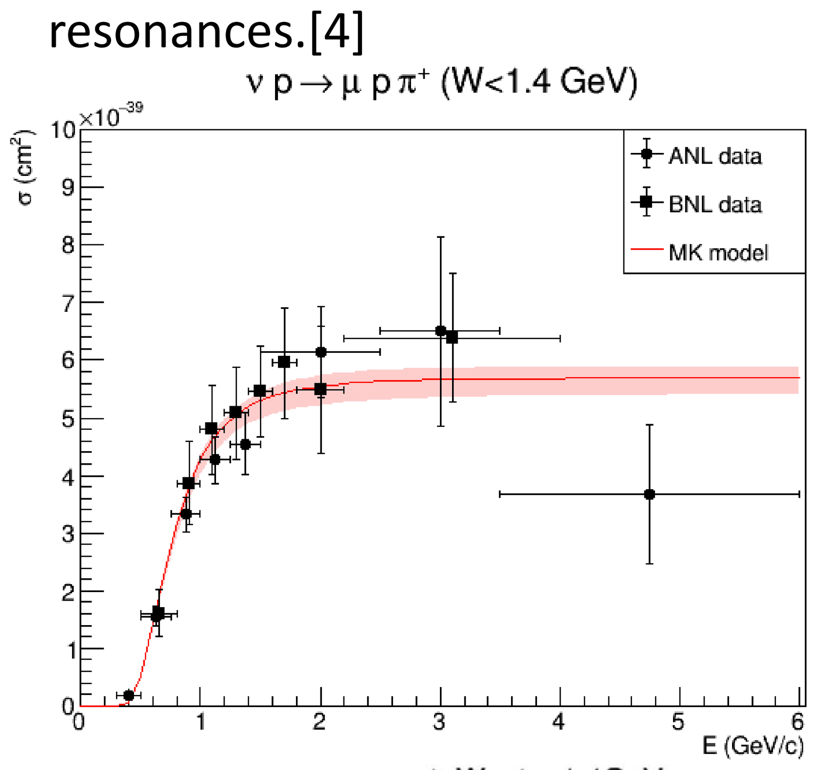
In general:
$\nu(k_1) + N(p_1) \to l(k_2)N(p_2)\pi(q)$

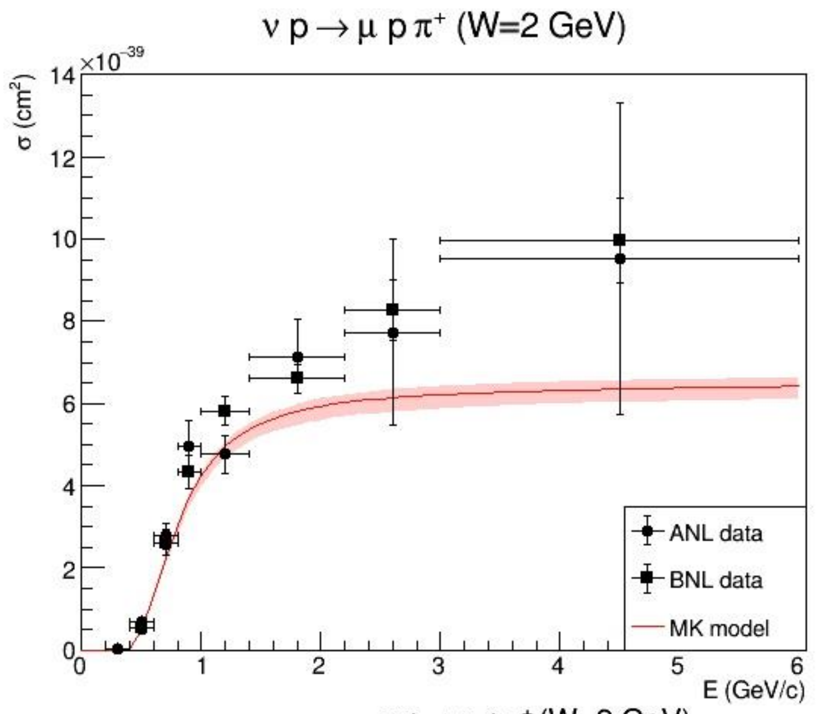
Find parameters and validate model

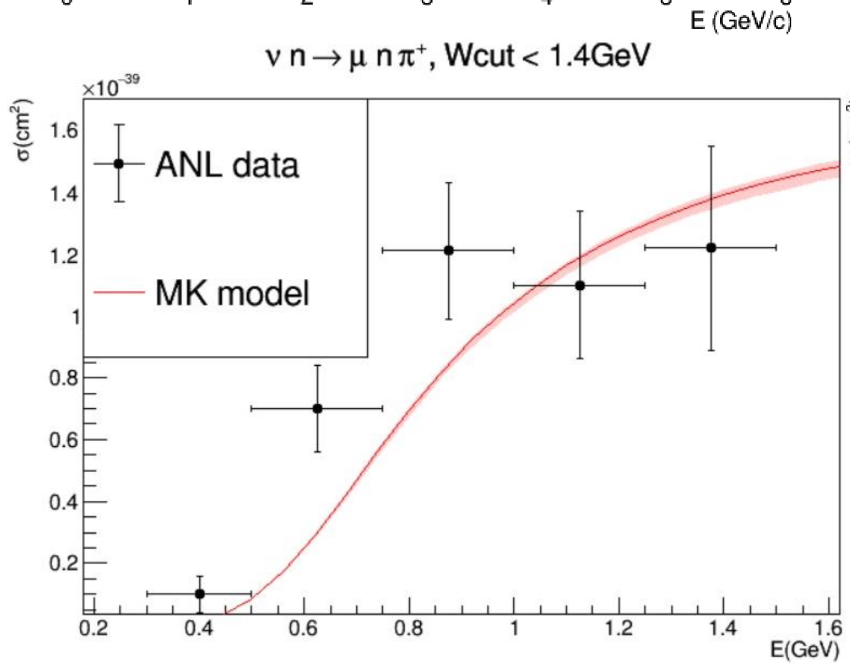
X^2 function with difference between expected value from MK model and ANL data → Minimise X^2 using ANL data to find best fit parameters → Use best fit parameters for MK model and plot against ANL and BNL data

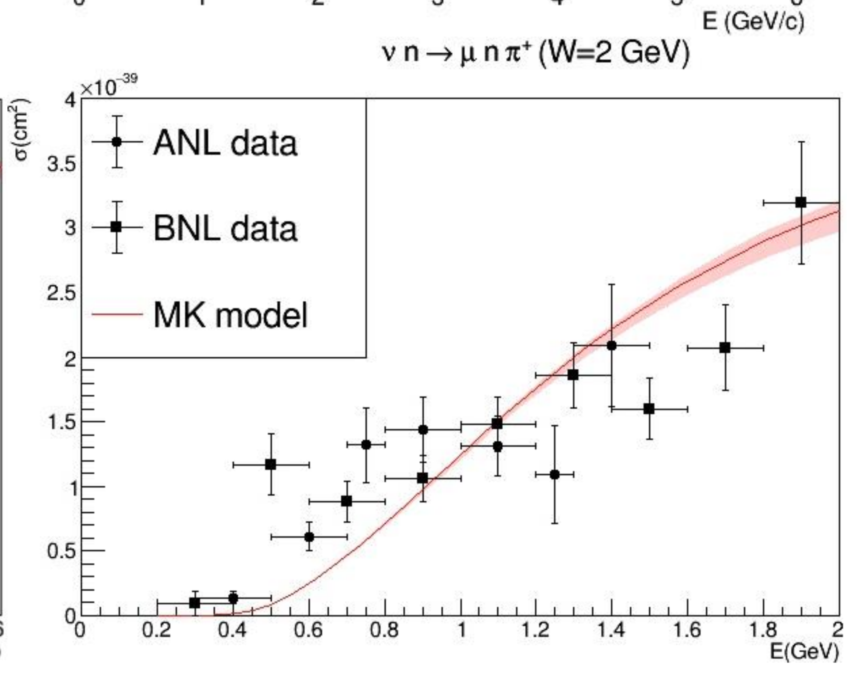
Original MK Model Results

- Plotted graphs of integrated cross section σ against neutrino energy E for limited hadron invariant mass ($W < 1.4$ GeV and $W < 2$ GeV) with 1σ error band predicted by the MK model.
- Data constraints $W < 1.4$ GeV neglects contribution from heavier resonances.[4]









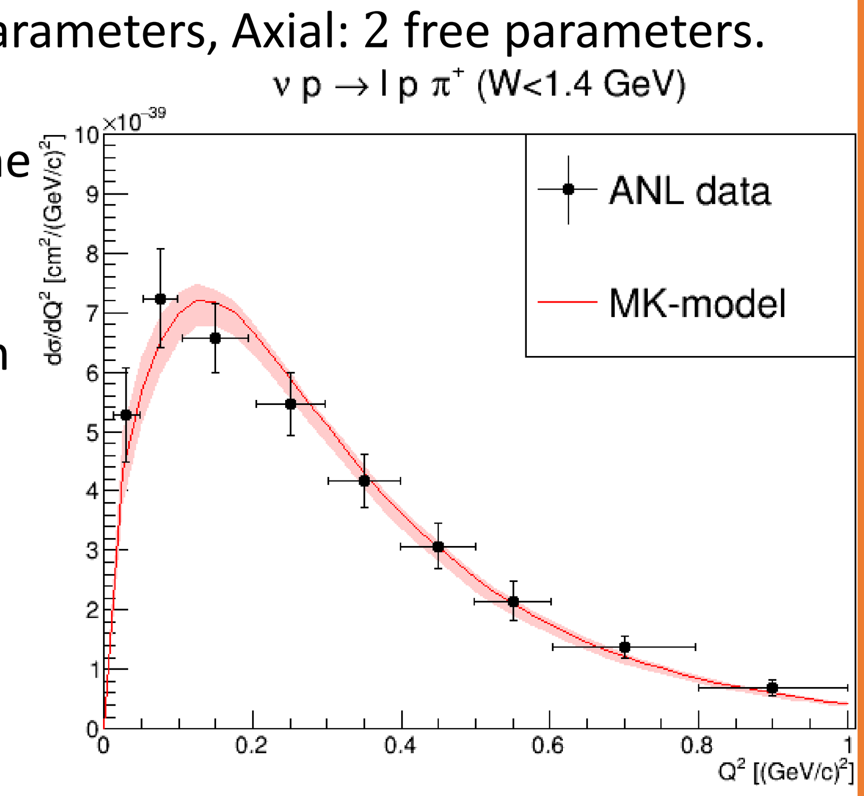
- MK model agrees much better with lab data in both high and low energy regimes.

Objectives

- Testing the MK model against observed results from Argonne National Laboratory (ANL) and Brookhaven National Laboratory (BNL).
- Using different minimisation implementations to find the best parameters for the original MK model.
- Comparing minimisers to find the best one for our purpose.
- Using electron and pion scattering data to fit parameters to the Vector part and the Axial part of the new MK model respectively.

Minimisers

- Original MK model → Vector: 0 free parameters, Axial: 2 free parameters.
- Minimising X^2 value to find best fit values for $C_5^A(0)$ and M_A, which are the free parameters of the form-factor for the axial current. [3]
- Using ANL Q^2 differential cross section data.
- Minimisation libraries MINUIT2 and GNU Scientific Library in ROOT.
- The following table shows the best fit parameters and X^2 value of the minimiser used at using 9 data points.



Minimiser	Min Method	M_A	$C_5^A(0)$	X^2	Time (s)
MINUIT2	Migrad	0.871039	0.882856	4.54834	202.067
	Simplex	0.870358	0.883293	4.54842	271.797
	Scan	0.98	0.98	28.7156	429.295
GSL	conjugateFR	0.871098	0.882819	4.54834	1955.88
	conjugatePR	0.871098	0.882819	4.54834	1704.29
	BFGS	0.871098	0.882819	4.54834	1709.57
	BFGS2	0.871098	0.88282	4.54834	1093.33
	SteepestDescent	0.871096	0.882822	4.54834	4872.31
	SimAn	0.871453	0.88253	4.54836	68916.4

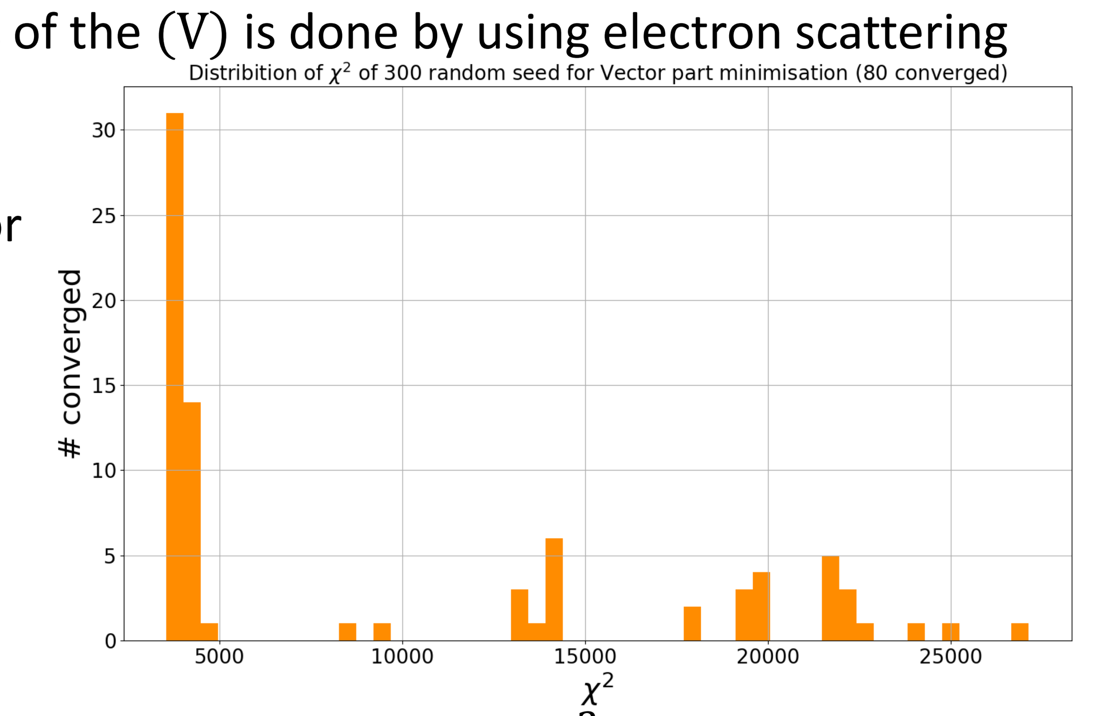
- The correlation matrix between the two parameters was calculated and the parameters were found to be anti-correlated, (−0.8765).

Vector & Axial Current (VC & AC)

- Lack of quantity and quality of neutrino data → We cannot verify the model.
- Weak current consists of the Vector (V) and the Axial (A) current. Due to the parity violation, it has a V − A structure.
- We can use abundant electron and pion scattering data by splitting the model into a Vector and an Axial part → New MK model.
- Conservation of vector current (CVC) and Partial conservation of axial current (PCAC) constrains the nucleon form-factors.

- $k^\mu J_\mu^V = 0$, is the CVC equation and is an exact conservation law. [5]
- $\partial_\mu A^\mu = -f_\pi m_\pi^2 \varphi$, where f_π is the pion decay constant, m_π is the pion mass.
- The symmetry is exact when $m_\pi \to 0$.

- Finding the 23 parameters of the (V) is done by using electron scattering data.
- ~2000 data points used.
- 300 random seeds used for initial starting point for parameters. 80 converged using Migrad.
- X^2 values concentrated at around 3600, 4200 and higher values signify multiple local minima.



- Simplex and SimAn were also used to minimise the X^2 but fail to converge. Other gradient methods are computationally infeasible and are not used.

Conclusion & Further Exploration

- Validating MK model by evaluating the parameters from the Vector and Axial parts, make comparisons with NEUT and GENIE model predictions.
- Improving the axial current by using more form-factors to each resonances → Will have more free parameters to fit.
- Newer MK model uses PCAC relation and pion scattering data to constrain parameters for the Axial part.
- Using Markov chain Monte Carlo methods to minimise the parameters of the Vector part and investigate performance gains.

References
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- [3] - Graczyk, K.M., Kielczewska, D. and Sobczyk, J.T., 2009. $ C_5^ A $ form factor from ANL experiment. *arXiv preprint arXiv:0907.1886.*
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- [5] - Alevizos, T., Celikel, A. and Dombey, N., 1977. PCAC analysis of weak pion production in the first resonance region. *Journal of Physics G: Nuclear Physics*, *3*(9), p.1179.