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Bosonic String Theory, T-Duality and Extended Geometry

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INTRODUCTION

In the late 1960s, string theory arose as one of the candidates for unifying all four fundamental forces in the premise of identifying particles as different vibrational modes of a string. Our motivation is to understand the string dynamics both classically and quantum mechanically, which involves studies in ***symmetries*** and ***quantisation***. One of the outstanding results is the physical promotion of the existence of ***extra*** spacetime dimensions. Furthermore, we shall see how ***T-duality*** provides the equivalence of string dynamics with compactified spacetimes.

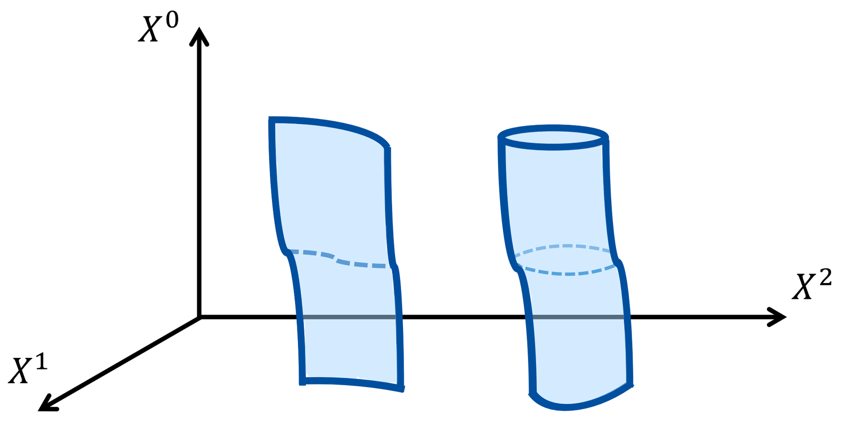

Figure 1. A sketch of the worldsheets of an open string and a closed string.

CLASSICAL RELATIVISTIC STRING

$$S_p = -\frac{T}{2}\int d^2\sigma\sqrt{-h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

There are several symmetries which the ***Polyakov*** action enjoys:
- ***Poincaré*** transformation invariance (global)
- ***Reparameterization*** invariance (local)
- ***Weyl*** symmetry (local)

Upon that, we can fix a gauge such that the field metric $h_{\alpha\beta}$ is ***flat***:

$$h_{\alpha\beta} = \begin{pmatrix} h_{00} & h_{01} \\ h_{10} & h_{11}\end{pmatrix} \xrightarrow{\text{Diffeomorphisms}} \begin{pmatrix} h_{00} & 0 \\ 0 & -h_{00}\end{pmatrix} \xrightarrow{\text{Weyl}} \eta_{\alpha\beta}$$

There exists a ***residual*** symmetry that a certain diffeomorphism ε on the field metric can be undone by a Weyl rescaling Λ^{-1}, this is a ***conformal*** symmetry.

Equation of motion from varying $X \to X + \delta X$ and $h^{\alpha\beta} \to h^{\alpha\beta} + \delta h^{\alpha\beta}$:[1]

$$\partial_\alpha \partial^\alpha X^\mu = 0 \qquad\qquad T^{\alpha\beta} = 0$$

where $T^{\alpha\beta}$ transforms into ***constraints*** for the solution of X^μ.[1]

For a closed string, the solution in its expanded Fourier modes is:

$$X^\mu = x^\mu + 2\alpha' p^\mu \tau + \frac{i}{2}\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}(\tilde{\alpha}_n^\mu e^{-2in\sigma} + \alpha_n^\mu e^{2in\sigma})e^{-2in\tau}$$

By imposing two constraints from the stress-energy tensor, it yields:

$$L_n = \frac{1}{2}\sum_m \alpha_{n-m}\cdot\alpha_m = 0, \qquad \tilde{L}_n = \frac{1}{2}\sum_m \tilde{\alpha}_{n-m}\cdot\tilde{\alpha}_m = 0$$

which are Noether charges that generate the conformal symmetry.

OLD COVARIANT QUANTISATION

Just as in the ordinary ***canonical quantisation***, we define creation/annihilation operators acting on a vacuum state $|0;p\rangle$ to obtain excited states, e.g., for open strings

- Level 1: $\alpha_{-1}^\mu|0;p\rangle$
- Level 2: $\alpha_{-1}^\mu\alpha_{-1}^\nu|0;p\rangle$ *or* $\alpha_{-2}^\mu|0;p\rangle$

However, this Hilbert space is ***non-physical*** due to the existance of negative-probability states (also called ***ghost states***), e.g.

$$\text{Prob}(\alpha_{-1}^0|0;p\rangle) = \langle 0;p|\alpha_1^0\alpha_{-1}^0|0;p\rangle = -\langle 0;p|0;p\rangle$$

This can be solved by imposing the constraints:

$$L_{n>0}|\phi\rangle = \tilde{L}_{n>0}|\phi\rangle = 0, \qquad L_0|\phi\rangle = \tilde{L}_0|\phi\rangle = a|\phi\rangle$$

which are gorverned by the ***Virasoro algebra:***

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{D}{12}n(n^2-1)\delta_{n+m,0}$$

Varying the parameters a and D gives a ***physical*** Hilbert space, and it happens to be $\boldsymbol{a = 1}$ and $\boldsymbol{D = 26}$, it restricts the spacetime ***dimensions*** exactly to **26**.

STRING SPECTRUM

$$M_{\text{Open}}^2 \propto N - a, \qquad M_{\text{Closed}}^2 \propto \tilde{N} + N - 2a$$

Open String
$N = 0 \to$ Tachyon
$N = 1 \to$ Photon

Closed String
$N = \tilde{N} = 0 \to$ Tachyon
$N = \tilde{N} = 1 \to$ Graviton, B-field, Dilaton

COMPACTIFICATION & T-DUALITY

However, we only feel 4 spacetime dimensions. A natural attempt is to let these extra 22 dimensions to be small (and periodic) such that it is only noticeable to strings. For simplicity, we can consider the spacetime $\mathbb{R}^{24,1}\times S_R^1$:

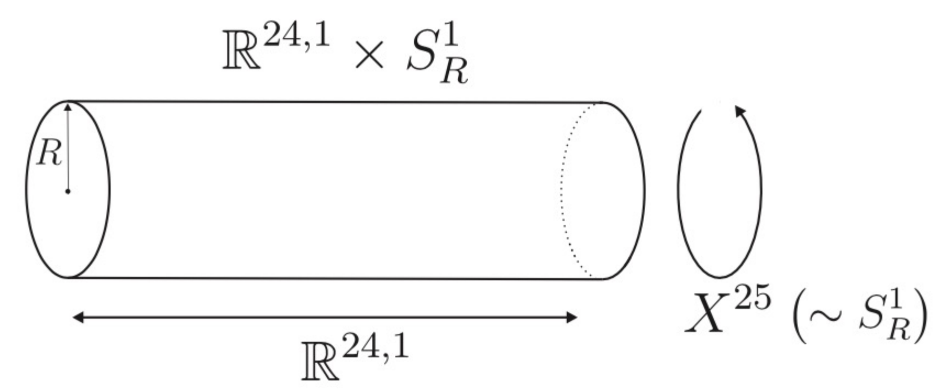

Figure 2. An analog of a compactified spatial dimension described by the coordinate X^{25}, it is topologically a circle of radius R.[2]

The string gains a new intrinsic property (***winding number***) as it can wrap over the compactified dimension. The momentum associated to X^{25} is quantised. These two ***quantum numbers*** changes the string spectrum:

$$M_{\text{Closed}}^2 = \frac{2}{\alpha'}(\tilde{N} + N - 2) + \left(\frac{n}{R}\right)^2 + \left(\frac{wR}{\alpha'}\right)^2$$

Now, consider another spacetime $\mathbb{R}^{24,1}\times S_r^1$ with the string spectrum:

$$M_{\text{Closed}}^2 = \frac{2}{\alpha'}(\tilde{N} + N - 2) + \left(\frac{m}{r}\right)^2 + \left(\frac{\omega r}{\alpha'}\right)^2$$

They are equivalent under the transformation:

$$R \to \alpha'/r, \qquad n \to \omega, \qquad w \to m$$

- The string sees no difference between the two backgrounds, one with radius R and the other with radius r.
- This is a symmetry, known as ***T-duality***, from which two seemly different spacetime backgrounds are equivalent.
- In general, a curved spacetime is described by the massless closed string states, which are the metric $G_{\mu\nu}$ and the B-field $B_{\mu\nu}$ (ignoring the dilaton).
- There is a set of rules, called the ***Buscher rules***, that allows us to find a T-dual background spacetime, described by the metric $\tilde{G}_{\mu\nu}$ and the B-field $\tilde{B}_{\mu\nu}$.

$T^3 + H$-flux → Twisted Torus → T-fold

Figure 3. An example of applying Buscher rules starting from the background $T^3 + H$-flux.

IMPLICATION

Although string theory lacks experimental evidence and the full picture is yet to be revealed, it still offers a framework in which people can start to attempt the puzzles contained in quantum gravity. Particularly, we have just seen different geometries arose from the T-duality, such as non-geometric backgrounds, which provides new perspectives of the nature of gravity.

References

[1] Thomas Mohaupt (2007/2008). Liverpool Lectures on String Theory, The Free Bosonic String Summary Notes.
[2] Kevin Wray. An Introduction to String Theory, revised May 6, 2011