Imperial College London

Condensed Matter Theory Group
Muhammed Furqaan Valera
Supervisor : Dimitri V. Vvedensky

Extension of The Black-Scholes Model

Background and Objectives

An Option is a contract that gives its owner the right but not the obligation to buy or sell an asset at a specified price (**the strike price**), at a particular point in time (**maturity**) [1].

Pricing Options:

Consider a scenario between a buyer and seller of a commodity, in which an option is placed:

Table 1: In both scenarios the seller is at a loss. When the strike price(Xc) is less than the market price (X) the buyer would have gained from no option in place. When Xc > X then the seller doesn't make the sale

	Seller	Buyer
Xc < X	Loses out	Exercises Option
Xc > X	Loses out	Buys from Market

SO THE BUYER DOESN'T LOSE OUT, OPTIONS ARE SOLD AT A PREMIUM.

The Black-Scholes Equation is a the most well known method for determining the fair price of an option. It is based on the assumption that the asset price returns follow a Gaussian distribution[2].

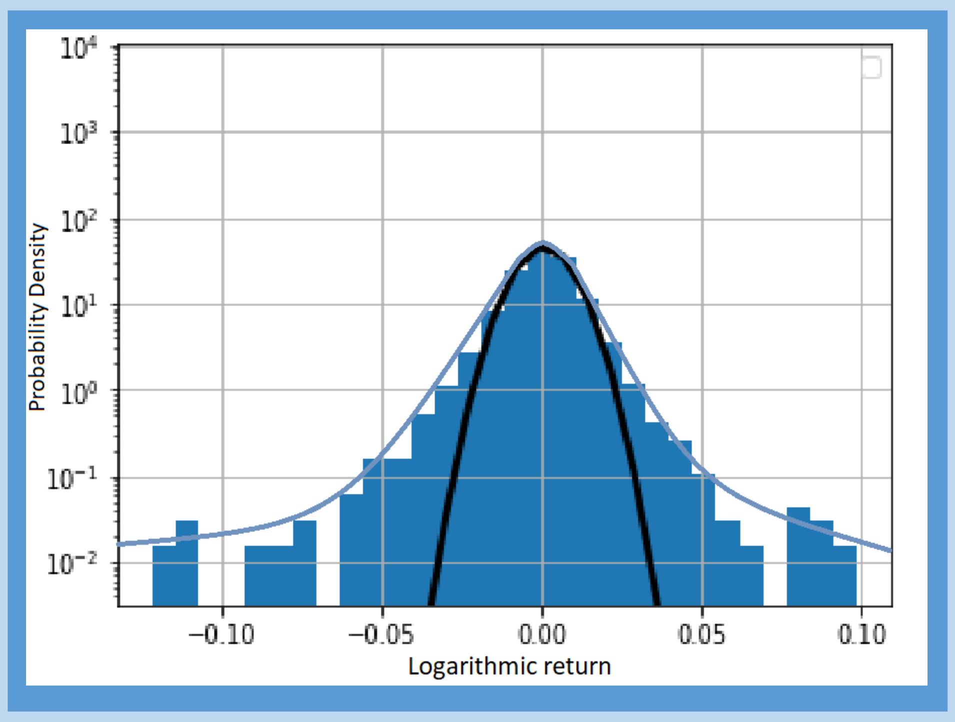

Figure 1: The distribution of the FTSE100 ,a Gaussian distribution (black), and a Stable–Lévy distribution (navy). The gaussian is not suitable as it fails to capture the extreme ends

Objectives:

1. Reformulate Black-Sholes Equation using probability theory for a general probability distribution

2. Prove that using a log–normal distribution yields a Black-Sholes results

3. Investigate the equation for a Lévy distribution

Improving The Model

1 2 3

$$\Delta W = C(x_0, x_c, T) - \theta(x(T) - x_c) + \int_0^T \phi(x,t) \frac{dx}{dt} dt$$

The wealth (ΔW) of the seller or bank is made up of

1. Gains from selling the option
2. Losses if the market price is greater than the strike price
3. Gains/losses due to variations in share prices [3]

$$\phi^*(x,t) = \int_{x_c}^{\infty} dx' \langle \frac{dx}{dt} \rangle_{(x,t)\to(x',T)} \frac{(x'-x_c)}{D(x)} P(x',T|x,t)$$

An Optimal Trading strategy is determined by minimising the risk associated with the bank, $\langle \Delta W^2 \rangle$, with respect to the number of shares being held.

When a log-normal distribution is used the Black-Scholes result is verified

$$\phi^*(x,t) = \frac{\partial}{\partial x}[C(x_0, x_c, T)]$$

Results

1

$$C(x_0, x_c, T) = x_0 e^{(\mathrm{mT})} \mathrm{N}(\mathrm{d}_1) - x_c \mathrm{N}(\mathrm{d}_2)$$

- Verification of the Black– Scholes Model from a completely general formulation, showing the validity of this approach

2

$$P(x,t|x_0,0) = \frac{1}{(\mathrm{ZT})^{\frac{1}{\mu}}} \mathrm{L}_\mu \left(\frac{x - x_0}{(\mathrm{ZT})^{\frac{1}{\mu}}} \right)$$

- For the above distribution where $L_\mu(x)$ is the corresponding Lévy distribution, one finds that:

3

$$\mathrm{C} = \frac{\mathrm{ZTC}_\mu}{\mu} \int_{x_c}^{\infty} \frac{\mathrm{d}\upsilon}{\upsilon^\mu}$$

Conclusion

A generalised **Option Pricing** formula was derived using probability theory. Using a **log-normal distribution** the model agreed with the Black–Scholes Equation. **A Stable-Lévy distribution** was used to formulate a pricing formula that doesn't ignore the "fat- tails" of the asset price returns.

References

[1] Hunt, Philip James, et al. *Financial Derivatives in Theory and Practice*. United Kingdom, Wiley, 2004.

[2] Lisa Borland, *Option Pricing Formulas Based on a Non-Gaussian Stock Price Mode,* Iris Financial Engineering and Systems; August 2002

[3] Jean-Philippe Bouchaud, Didier Sornette. The Black-Scholes option pricing problem in mathematical finance: generalization and extensions for a large class of stochastic processes. Journal de Physique I, EDP Sciences, 1994,