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Horizon Molecules In Causal Set Theory

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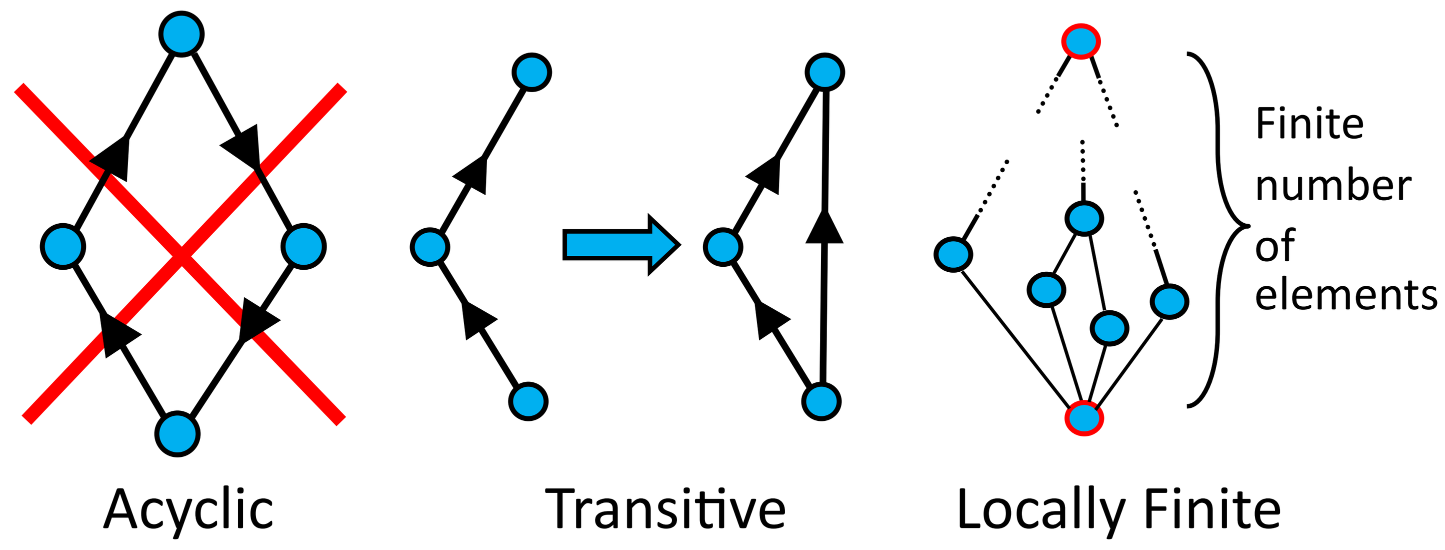
What is Causal Set Theory

Causal Set Theory (CST) is a radical approach to Quantum Gravity postulating that spacetime is discrete at the most fundamental scales. This idea has a wealth of supporting evidence such as curvature singularities in GR and the breakdown of all theories at the Planck Scale. CST states that our continuum spacetime is an approximation to a **causal set** (causet)**,** which can be thought of as the discrete 'atoms of spacetime' with relations between them representing the causal order between these elements. It may be represented as a Hasse diagram (a type of digraph whose arrows always point upward) with the following properties:



Black Hole Horizon Entropy

Once an object falls into a black hole, it is no longer accessible to the outside universe. This suggests that the black hole itself must have an associated entropy to not violate the Second Law. In 1973, Hawking and Bekenstein [1] showed that the entropy is proportional to the area of the event horizon in Planck Area units L_P^2:

$$S_{BH} = \frac{A}{4L_P^2}$$

One heuristic to recover this law is to subdivide the horizon into N Planck area sized "plaquettes" which are each assigned a state 0 or 1. The total number of microstates Ω for a black hole with area $A = NL_P^2$ is therefore $\Omega = 2^N$, corresponding to the **Boltzmannian entropy**:

$$S = k\, log(\Omega) \propto \frac{A}{{L_P}^2}$$

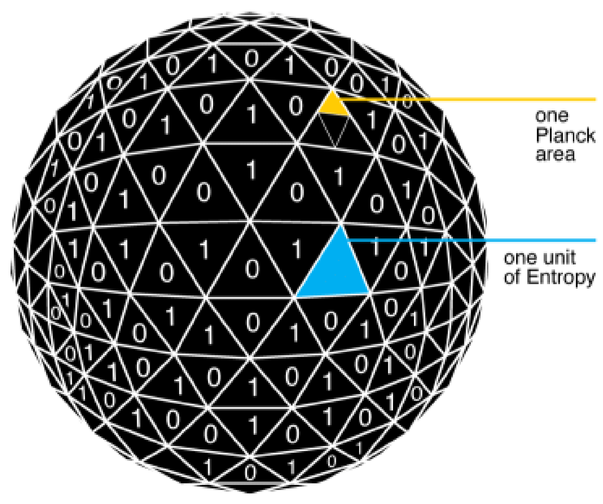


Fig. 1: A diagram of the black hole event horizon tiled with plaquettes [2]. One unit of entropy is attributed to 4 plaquettes due to the 1/4 factor.

The Proposal

The analogue of the plaquettes in CST are **Horizon Molecules:** sub-causets localised to the intersection of the horizon $\mathscr{H}$ and a spacelike hypersurface Σ. We build on past work by [3] and [4], and propose alternative definitions for these objects, as well as the states that they occupy. We focus on the **B-Molecules** and a particular definition of their states:

A B-Molecule in state n is a connected sub causal set with n+1 elements. It contains at least one element is in C^-_- that is **maximal** within $J^-(\Sigma)$. These maximal elements are related to at least one element in C^+_+ that is **minimal** within $J^+(\Sigma)$.

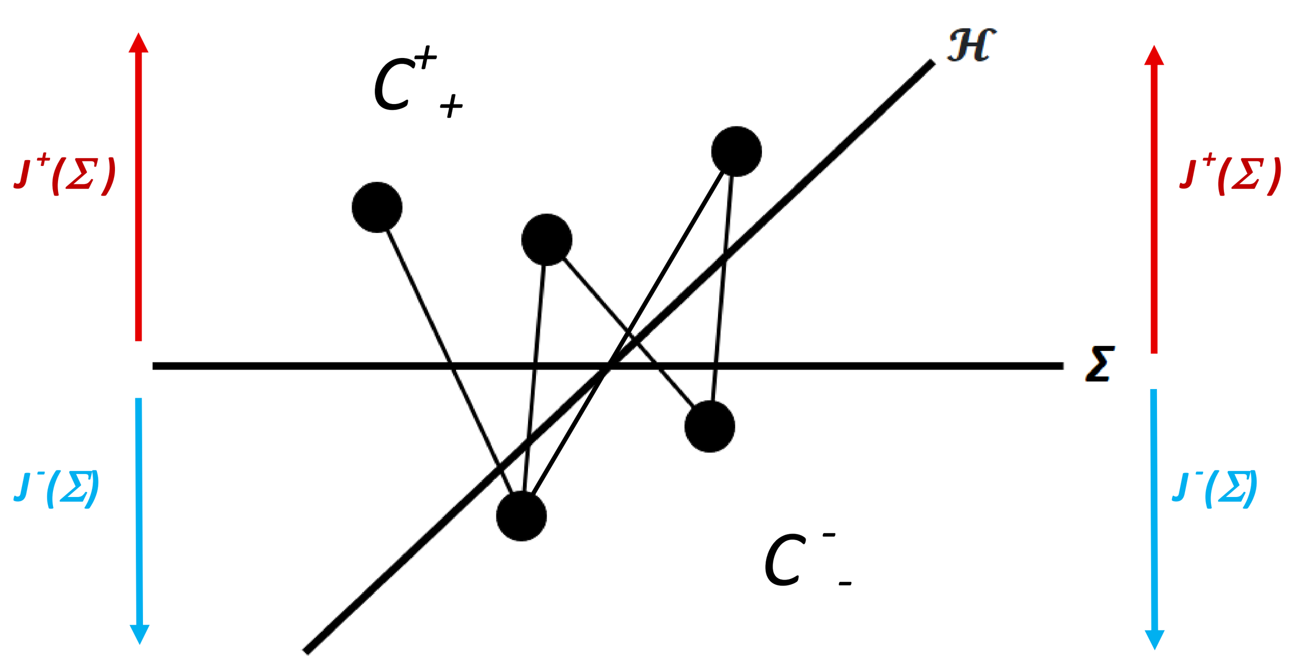


Fig. 2: A diagram of a B-Molecule in state 4 centred on a horizon in 2D. The horizon at a particular time is just a point, given by the intersection of H and Σ.

- Minimal / Maximal : having no elements to its causal past / future
- $J^{+/-}(\Sigma)$: Causal future/past of Σ

Aims

1. Generate horizon molecule simulations in any dimension
2. Verify that the number of molecules scales linearly with the horizon area in 2, 3 & 4D
3. Investigate the distribution of states for these molecules in 2, 3 & 4D
4. Develop a suitable state-counting procedure to calculate the horizon entropy

Our Results

2D & 3D

Scaling: In 1+1 dimensions, we proved analytically there is only ever one molecule, which will trivially scale with the horizon area which in 2D is a single point. Furthermore, the number N of horizon molecules for a simulation of a given horizon area also scales correctly in 2+1 dimensions.

State-Counting: The state distributions are well behaved in 2 and 3D, both decaying exponentially and do not change for increasingly large simulations. Here we define the horizon microstate as the occupancies **n** = (n_1, n_2, …) , where n_k is the number of molecules in the k'th state. Then, the **Gibbs Entropy**:

$$S = \sum p_j ln(p_j)$$

scales with N, where p_j is the probability of being in the j'th microstate.

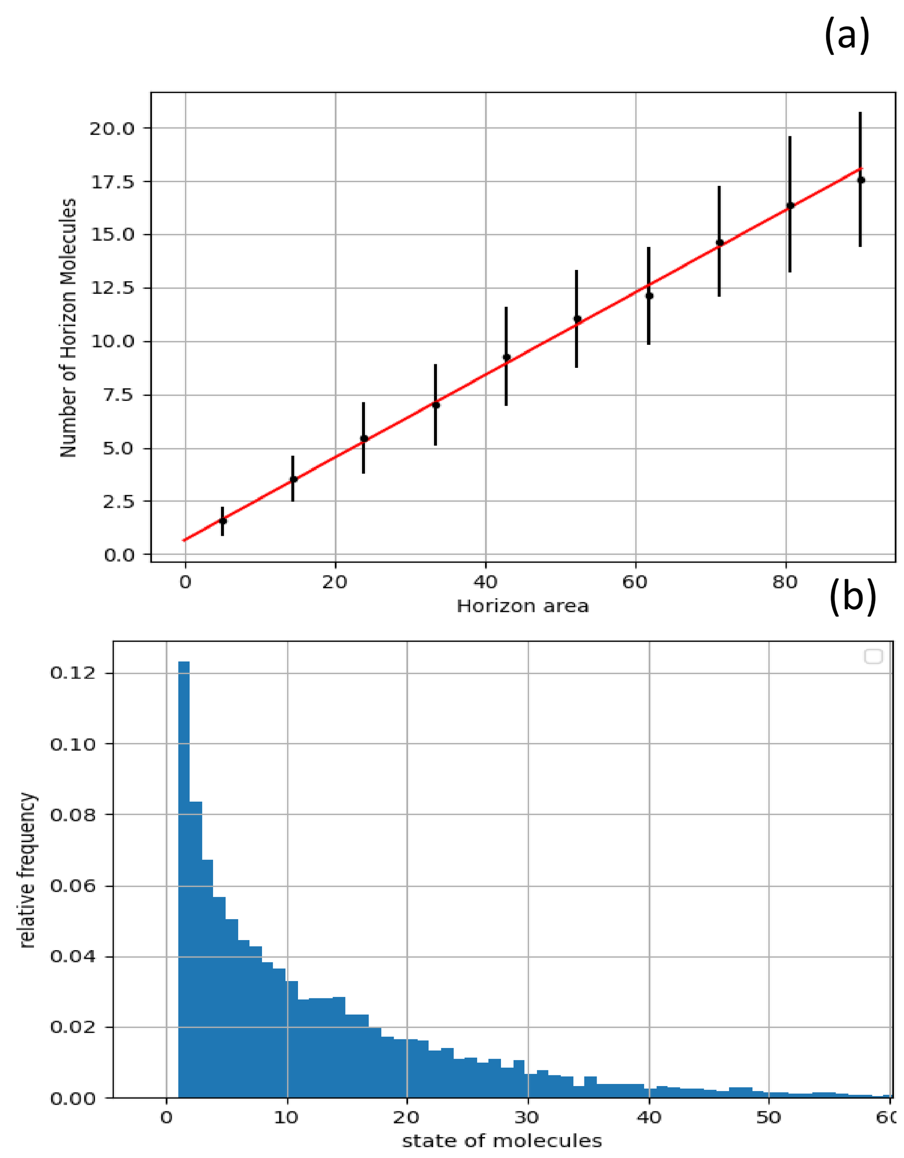


Fig. 3: (a) -The relationship between horizon area (in the 2+1 case this is a length) and number of molecules. The error bars are represent the standard deviation of these results over 100 simulations.
(b) - the state distribution of an individual molecule

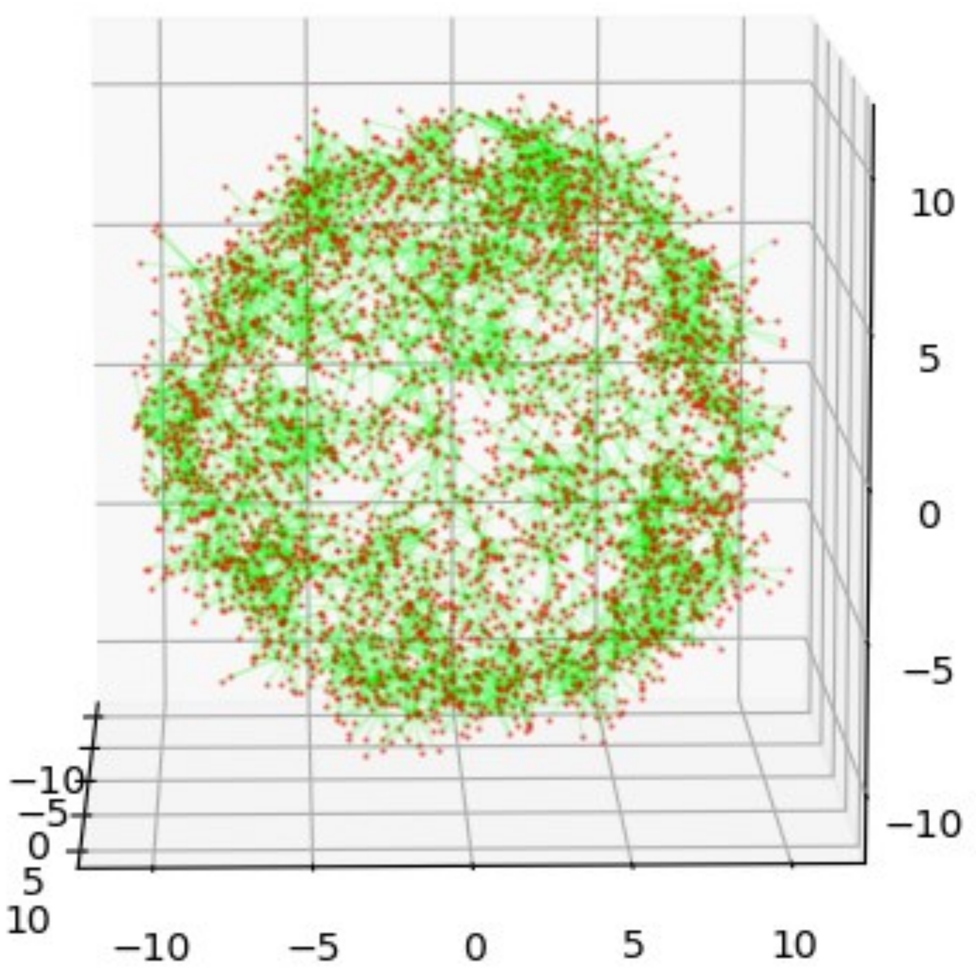


Fig. 4 – The spatial coordinates of a Black Hole Horizon Causet in 4D. The red points are the causet elements and the green lines represent the links between elements.

4D

In 4D, we find these molecules now connect up completely into one large causal set that spans the whole horizon, which we call the **horizon causet**. The prior state-counting procedure is no longer valid, however we can define a microstate to be every unique horizon causet that could exist for a given horizon area. This definition works better within the Boltzmannian interpretation of entropy like the plaquette heuristic.

Further Research

1. Study alternative molecule definitions with fewer states
2. Investigate the correlation between neighbouring molecules
3. Investigate the homology of the horizon causet

References

1. J. D. Bekenstein, '*Black Holes and Entropy*', April 1973, pp. 2333-2346 Phys. Rev. D, 7(8)
2. Bekenstein-Hawking Entropy, [online], Available: http://www.scholarpedia.org/article/Bekenstein-Hawking_entropy
3. D. Dou, R. D. Sorkin, '*Black-Hole Entropy as Causal Links*', Jan 2003, pp. 279-296 Foundations of Physics, 3(2)
4. C. Barton et al. Sep 2019 '*Horizon Molecules in Causal Set Theory*'