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# The Leggett-Garg Inequalities In a Spin Chain

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## Background

### Macrorealism (MR)

In the 1980s, Leggett and Garg proposed "macrorealism" as a way of clarifying our intuition about certain aspects of the macroscopic world. It is built on three assumptions [1]:

**1) Macrorealism per se (MRps):**
A system with many available macroscopically distinct states will always be in a definite one of these states.

**2) Non-Invasive Measurability (NIM):**
It is possible to determine the state of the system without disturbing its future dynamics.

**3) Induction (Ind):**
Potential future measurements on the system cannot affect its present state.

### Leggett-Garg Inequalities (LGIs)

Based on these assumptions, Leggett and Garg derived a set of inequalities called Leggett-Garg inequalities [1]. A violation of an LGI means that at least one of the assumptions of MR has failed and thus indicates that the system is exhibiting non-classical behavior.

Imagine measuring some dichotomic variable, $Q = \pm 1$, to determine pairwise correlation functions between certain times. Under MR, certain sums of these correlators are bounded – these are the Leggett-Garg inequalities.

A three-time LGI (referred to as LG3): $1 + C_{12} - C_{23} - C_{13} \geq 0$

### Fine's Theorem and the LG2 Inequalities (MR)

Fine's theorem tells us that augmenting the LGn inequalities with all the LG2 inequalities takes them from merely a necessary condition to a **necessary and sufficient condition** for macrorealism at n times. These LG2 inequalities can be expressed using the moment expansion of a particular **quasi-probability**:

$$q(s_i s_j) = \frac{1}{4}(1 + s_i\langle Q_i\rangle + s_j\langle Q_j\rangle + s_i s_j C_{ij}) \geq 0$$

where $s = \pm 1$. [2]

### Motivation

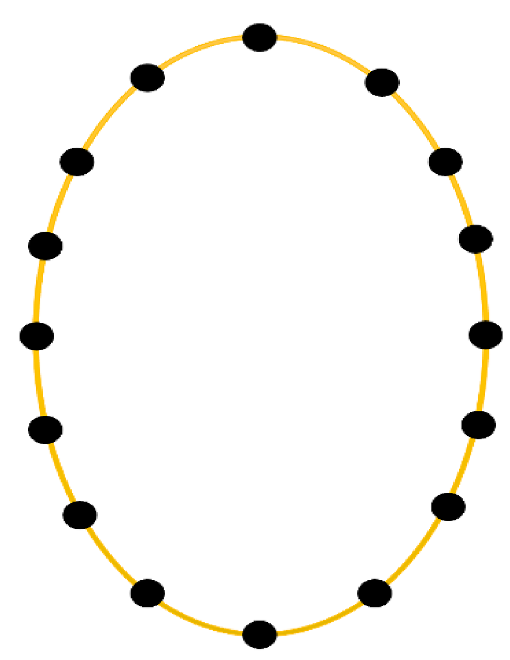
Most work so far has looked at systems with small Hilbert space. The ultimate aim of the LGIs is to explore potentially macroscopic systems and hence **we want to look at systems whose size can be scaled arbitrarily.**

## New Work

### Spin-Chain Model

We consider a 1D chain of M spin 1/2 in the absence of a magnetic field. It is subject to **periodic boundary conditions** thus giving it the topology of a loop. The Hamiltonian exchanges adjacent spins and our M first excited states occur when all but the $n$th atom are aligned (denoted $|n\rangle$). We then restrict ourselves in M-dimensional subspace consisting of only **first excited states** - effectively an M-level spin system. The energy eigenstates, $|\psi_e\rangle$, are given by the quantum Fourier transform. [3]

If we define $|n\rangle$ as a position then the free particle system emerges in the limit $M \to \infty$. This allows us to connect discrete spin systems which have been previously explored in great depth with continuous systems which have only recently been considered in the literature.



### Fine-Grained Projection Operator

Choosing a single spin site $|k\rangle$ to correspond with $Q = +1$ and the rest to $Q = -1$ is the simplest choice of dichotomic operator. Exploring this, we found the magnitude of LG violation scales as 1/M, fluctuations away from the mean decrease at long times, and the LG2 and LG3 violations occur independently.

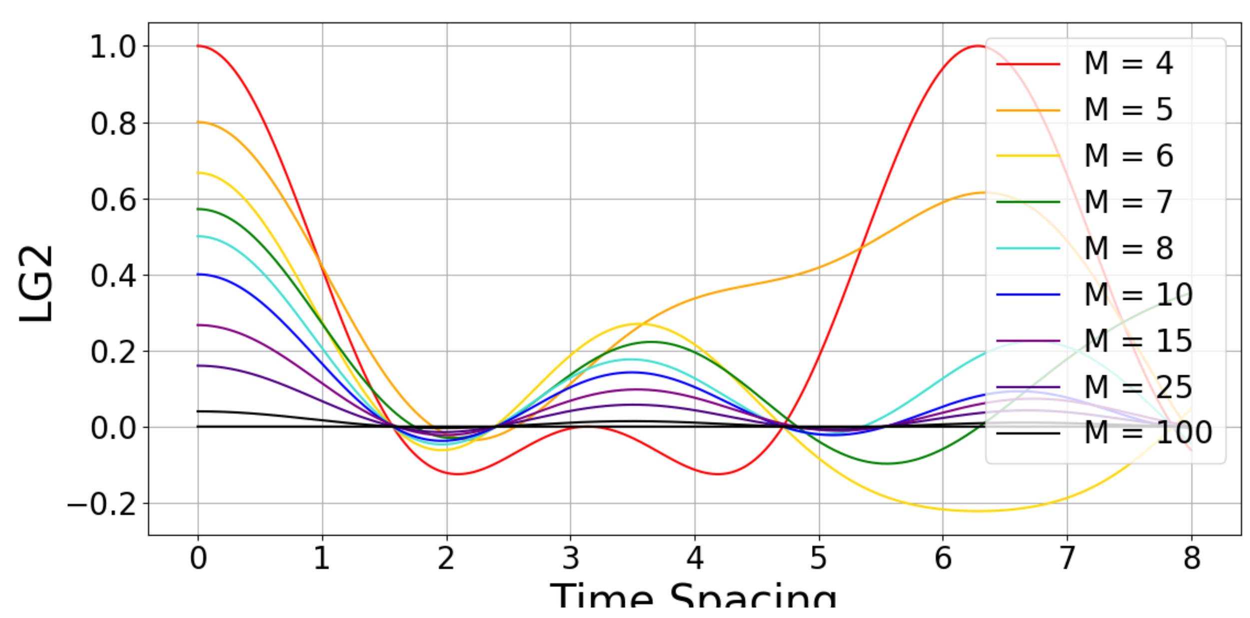



**Figure 1:** LG2 using fine-grained projection operator for varying system size M, with equal time spacing between measurements chosen.

### Course-Grained Projection Operator

Course-graining the above such that half the spin chain corresponds to $Q = +1$ and the rest to $Q = -1$ allows a connection with continuous systems, where the Heaviside is used to create a dichotomic operator. Exploring this with initial conditions that scale up with system size gives rise to M-independent behaviour and relatively large LG violations. It is important for the wavefunction to be localised near the transition point (where $Q$ changes sign) – without significant chopping of initial wavefunction, violations diminish rapidly.

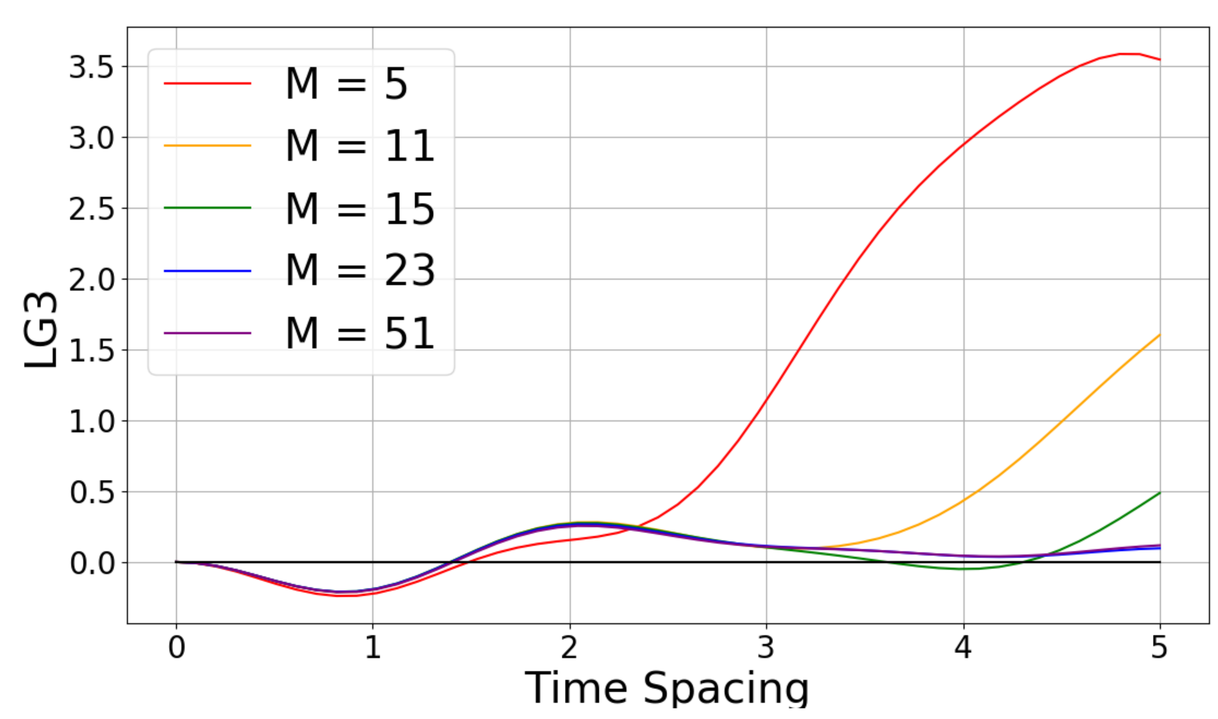



**Figure 2:** LG3 using course-grained projection operator for varying system size M. Equal time spacing between measurements chosen. Initial condition spreads over same fraction of spin chain. Violation around half of allowed maximum violation.

### Conclusion

We have seen M-independent behaviour in our spin chain for certain initial conditions and course-grainings. Since the model gives rise to the free particle system in the limit $M \to \infty$, we anticipate violations persist all the way up to this level and so our work connects simple spin systems with the continuous. It also shows the importance of spatial localisation near the transition point for large LG violations. We hope it inspires others to look at the LGIs in large Hilbert spaces and experimentally confirm LG violations in larger systems.

### References

1. A. J. Leggett, A. Garg, *Phys. Rev. Lett.* **54** 857 (1985)
2. J. J. Halliwell, *J. Phys.: Conf. Ser.* **1275** 012008 (2019)
3. T. A. Brun and J. J. Halliwell, *Phys. Rev. D.* **54** 2899 (1996)