From One Electron to a Dead-living Cat

Imperial College London

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Yianni, Theodoros; Peng, Yuhang; Supervisor: Professor Jonathan Halliwell[1]
[1]Theoretical Physics Group, Department of Physics, Imperial College London

Introduction

In Quantum Mechanics, measuring a quantum system would change the wavefunction and could affect the outcome of future measurements on the system. The Leggett-Garg inequalities (LGIs) test this 'quantumness', by taking successive measurements of a system. We are interested in whether macroscopic coherence, quantum superpositions of macroscopically distinct states, is possible [1]. If a system behaves classically, it would satisfy the assumptions of Macrorealism (MR): the system is always in a definite state and can be measured non-invasively. These tests could rule out some classical hidden variable models by showing that a system violates the assumptions of MR.

Currently, testing on truly macroscopic systems is very hard, but studying simplified systems could give us insight to larger systems. This project aims to shed light on a possible route to an experimental demonstration of macroscopic coherence. Such an experiment would rule out superselection theories.[2] LG tests could also be applied to quantum computers to test for qualities of qubits [3]. Applying LGIs on large (macroscopic) systems could test for whether there's a size limit for systems that could exhibit quantum superpositions.

Leggett-Garg Inequalities

Imagine making many copies of a system and taking measurements of a dichotomic variable Q at any one or two of three different points in time for each copy. The averaged measurement outcomes of a macrorealistic system should satisfy the LGIs. The family of LGIs consists of many inequalities but here we mainly focus on two types of LGIs:

- Two-time LG inequalities (LG2s):

$$1 + \langle Q_1 \rangle + \langle Q_2 \rangle + C_{12} \geq 0 \ (1)$$

- Three-time LG inequalities (LG3s):

$$1 + C_{21} + C_{32} + C_{31} \geq 0 (2)$$

Where Q_i is a measurement of Q at time t_i, and the correlators $C_{ij} = \langle Q_i Q_j \rangle$. The other LG2s and LG3s are obtained by replacing two of the + signs with – signs. The notation will be explained below.

- Sequential Measurements of a dichotomic variable at two times:

Probability of two-time outcome= $p(s_1,s_2)$	Q_1 outcome=s_1	Q_2 outcome=s_2	Correlation= $(s_1 s_2)$
p(+,+)	+1	+1	+1
p(+,-)	+1	-1	-1
p(-,+)	-1	+1	-1
p(-,-)	-1	-1	+1

When a system obeying MR is initialised, we expect it would have pre-determined measurement outcomes according to the above table, and we can see that the rows individually satisfy the LG2s.

Methods

- Find examples of initial states of quantum systems where successive measurement at specific time intervals show large quantum effects;
- Focusing on states that can be easily fabricated in a lab, e.g., energy eigenstates or coherent states;
- Looking for violations that can either be calculated analytically or require a small amount of numerical simulation, usually by varying the time intervals and parameters of the initial states;
- Search for initial states and operators that have macroscopic analogues, e.g., QHO coherent states can represent classical particle trajectories in a harmonic potential. A simple choice of Q is Q=+1 in some region of space and -1 elsewhere, which can be measured in quantum and macroscopic systems.

Models Used

We investigate how to find LGI violations in the following model systems: the spin chain and quantum harmonic oscillator. The spin chain can be extended, and the QHO shares many properties with its classical counterparts.

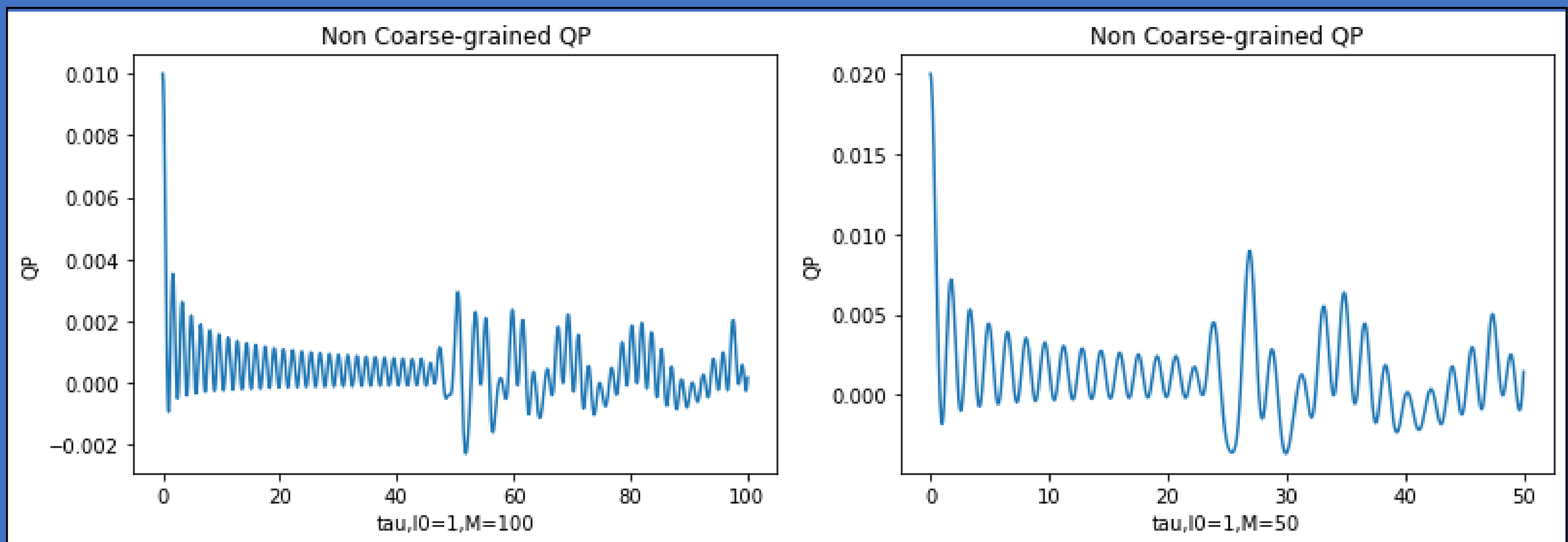
- Spin chain Hamiltonian:

$$\hat{H} = \sum_{n=1}^{M} |n\rangle\langle n+1| + |n+1\rangle\langle n|$$

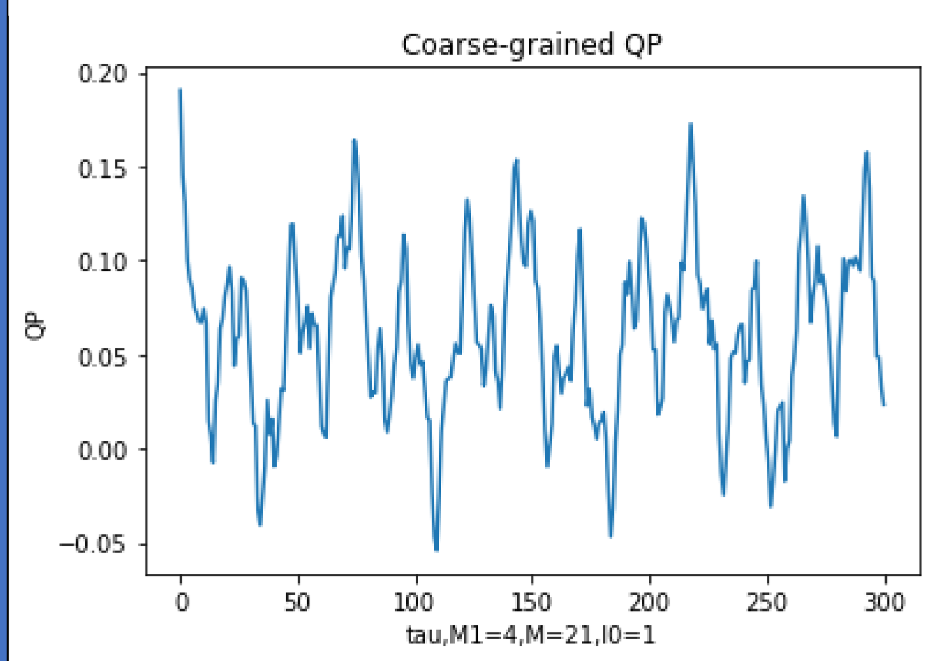
The system is a chain of interacting spin and can be considered a tensor product of many two-level spin systems. The n basis state here represents the state at which the nth spin site is spin is up, and the rest of the chain is spin down.

- The QHO Hamiltonian :

$$\hat{H} = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{1}{2}\hat{x}^2$$



- Figure 1. Simulation of spin chain model, at different sizes. The quasi-probability (QP) is a quarter of the LHS of (1), and the x-axis is the time interval. Projector and initial state: a spin basis state and a single energy eigenstate. Left: System size=100, violation up to -0.002983; Right: system size M=50, violation up to -0.008411 out of a possible -0.125.



- Figure 2. Simulation of spin chain model of size 21 using coarse-grained position measurements. A violation of-0.06575 was found. A decent violation of -0.03417 was also found when the system size is arbitrarily large.

Results and Discussion

- Using parity and sign operators, large violations in the QHO and spin chain are easy to find;
- The spin chain system has a guaranteed violation of about 25% of the theoretical bound with a single Bloch state projector;
- Single state projectors onto arbitrary $|a\rangle$ and $|b\rangle$ can have maximal violations if the initial state is a superposition of $|a\rangle$ and $|b\rangle$;
- Violations from a projection onto a single spin chain position state scale with the inverse of system size; see Fig 1. This is because the energy eigenstates are spread in position;
- Certain coarse-grained projectors on the spin chain, i.e., projectors onto a range of states, can have significant violations that persist in the limit of large system size. If a spin chain with many spin sites can be considered macroscopic, this could demonstrate macroscopic coherence; see Fig 2.
- The spin chain model could be tested on a quantum computer

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