The Defenestration of Locality

Throwing Intuition Out of the Window

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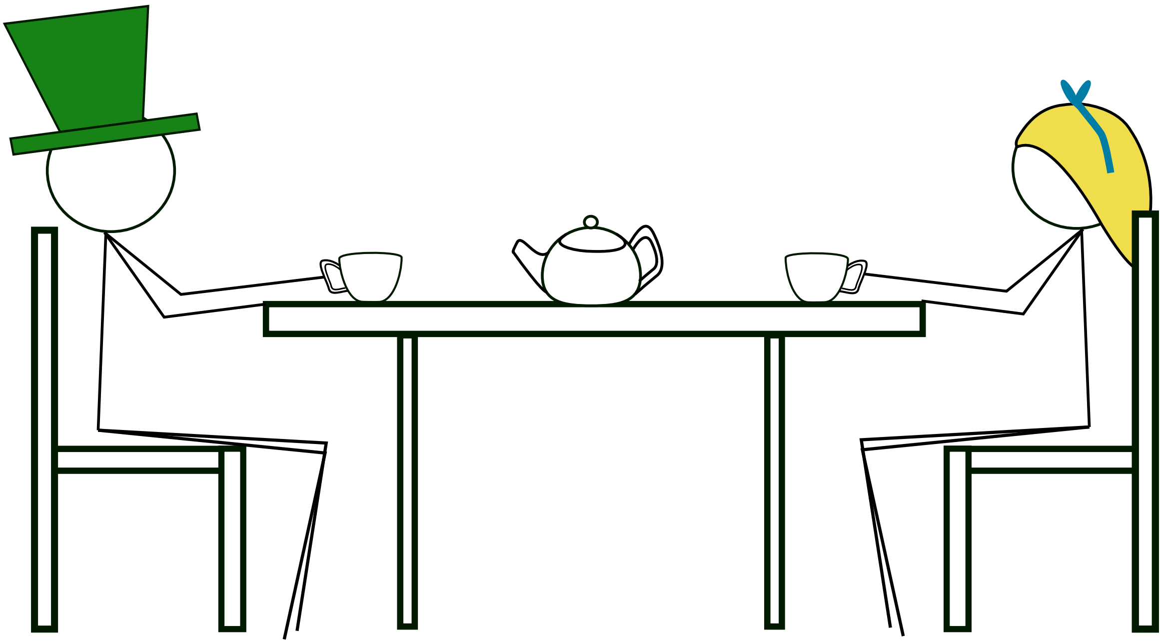

Introduction

Two friends, Alice and Bob, are sat around a table for their bi-monthly 'physics tea party'. During these occasions, the two relish the opportunity to raise and discuss confounding problems – preferably with profound philosophical implications – over a good pot of earl grey, and various assortments of cake and biscuits.

Today, Bob wears a notably perplexed expression.

"My dear fellow," remarks Alice "you've hardly been your talkative self and you've barely touched your tea. What on earth is the matter?"

"My apologies," Bob replies, "but there's a problem bothering me that I just cannot ignore."

"Well, that is why we are here," Alice responds. "What is this problem?"

Bob thus begins to divulge: **"Well, picture – if you will – a stationary particle. This decays into two smaller particles, which I send to opposite ends of the universe. By simple conservation, these particles should have equal and opposite momenta. So far so good?"**

"Indeed."

"Well, each particle could be in a superposition of any number of momentum states. But if we measure the momentum of one, then we can immediately know that of the other particle via conservation."

Alice nods her head in agreement.

"But finding a particle in a given momentum state is a matter of probability. If I measure the momentum of one particle, the exact outcome should be random. So how can the other particle be in the correct momentum state the instant I measure the first? How can it instantaneously know this random outcome from so far away?"

Bob looks back to Alice. She too now wears the same look of perplexion.

Now, if you didn't immediately understand Bob then don't worry. Hopefully, all shall be made clear.

The problem that Bob has just raised is quite similar to a famous one in quantum mechanics. It was brought to light in 1935 by physicists Nathan Rosen, Boris Podolsky and Albert Einstein (EPR) [1] and has since been titled the "EPR paradox" after its authors. Its purpose was to show that quantum mechanics, as was understood at the time, was incomplete.

So, let's get into unpacking this problem.

Quantum Phenomena: Briefly

To start, Bob makes use of the terms "state" and "superposition" when describing particles – but what do these mean?

In basic terms, a state is way of describing a system with specific physical properties. For example, a particle with momentum p_n might be associated with a momentum state $|\phi_n\rangle$. A particle does not necessarily have to exist in just one state but can exist in multiple states at once – in a superposition:

$$|\psi\rangle = \sum_n c_n |\phi_n\rangle \tag{1}$$

Here $|\psi\rangle$ is the wave function of the particle. You can think of it giving information on its "total state".

Of course, this is rather formal. A more approachable[1] example might be to consider something like Schrödinger's cat – which is in a superposition of the states: alive and dead.

$$|Cat\rangle = \frac{1}{\sqrt{2}}|Alive\rangle + \frac{1}{\sqrt{2}}|Dead\rangle \tag{2}$$

The cat is said to be in both states at once until it is observed. When it is observed, its wave function is said to "collapse" to a single state – either alive or dead. For a particle in a superposition of momentum states, it collapses to a single state $|\phi_n\rangle$ when its momentum is measured.

The magnitude squared of the coefficient for each momentum state (in this case: $|c_n|^2$) gives the probability that a particle is found in the state $|\phi_n\rangle$ when it is measured. In the case of the cat, it has a probability of $(\frac{1}{\sqrt{2}})^2$ for being found alive, and for being found dead.

However, in Bob's thought experiment, there is something more going on. Bob describes measuring one particle as exactly determining the state of both particles in the system. In doing so, he has indirectly referred to the idea of entanglement. Interestingly, EPR do not explicitly refer to the idea either since the term was coined by Schrodinger a few months after they wrote their paper [2].

You will no doubt have heard the term entanglement before, at the very least because it is one of the more common buzzwords in physics media. In short, for two particles to be entangled means that their states are intrinsically linked, so that knowledge of one gives direct knowledge about the other [3, p. 175]. For entangled particles 1 and 2 with momentum states $|\phi_{1,n}\rangle$ & $|\phi_{2,n}\rangle$ respectively, the mathematical representation of this might look something like the following:

$$|\psi_{entangled}\rangle = \sum_n b_n |\phi_{1,n}\rangle |\phi_{2,n}\rangle \tag{3}$$

If particle 1 was measured, and found to be in momentum state $|\phi_{1,k}\rangle$, then the two-particle system could be inferred to be in the state $|\phi_{1,k}\rangle|\phi_{2,k}\rangle$. If the momentum of particle 2 were to be measured, we could say with 100% certainty that it would be in the state $|\phi_{2,k}\rangle$.

It is this concept which gives the EPR paradox, and Bob's problem their strangeness. As we will see, this is because it appears to conflict with a relatively intuitive idea called "locality".

What is Locality?

Exactly how locality is defined seems to vary slightly depending on where you look, but there are two key aspects that are particularly relevant. The first is that an object should only respond to forces and fields in its immediate surroundings [4]. For an example, an electron in an electric field should experience a force based on the strength of the field at its position, and not some other location.

The second is that for an event at one point to be able to influence an event at another, the time between events must be large enough to allow for light to travel between the two [5]. This ensures special relativity is adhered to – specifically that no information is travelling faster than light.

At a glance, it seems more than reasonable that locality should hold. However, the entangled particles referred to by both Bob and EPR appear to behave non-locally. Measuring the momentum of one determines the momentum state of the other, regardless of where each particle is located. This would imply that the second particle is responding to measurement at the position of the first – violating the first idea we established.

[1] Unless you are a cat

Furthermore, if at the instant we measure the momentum of one particle, we know the state of the other, would this not suggest some faster-than-light interaction between the two? Have we broken relativity?

You can see why someone like Einstein might believe quantum mechanics to be incomplete. After all, relativity was kind of his thing...

So, What *Exactly* Did EPR Suggest?

The original paper by EPR roots itself in the idea that the wavefunction is not a complete description of the two particles – based on an idea named "the criterion of reality":

> *"If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." [1]*

This may seem a rather overwhelming statement, but bear with me. What it suggests is that if something can be known for certain about a system/particle/etc without interacting with it, then it should be predetermined by something [2] [7].

EPR also assumed that if the particles were spatially separated, they could not interact during measurement [8]. This is a direct analogue to Bob putting his particles at opposite ends of the universe, and an example of their belief that locality should hold.

You may be able to see where this is going. EPR's logic was that if the state of one of particle can be determined without interacting with it, then it must be pre-decided in some way shape or form. When the entangled pair of particles are measured, we are not changing the system in any way; we are simply measuring something that was always there [9]. Thus, nothing happens faster than light, there is no "spooky action at a distance" [3] and we're back to obeying locality as we always did. Right...?

Back to the Tea Party

As it turns out, Alice is having much the same thought process as EPR did.

"In your problem, when you make your measurements, could the results be pre-determined? Could there be some hidden variable, influencing what we measure? It could be something we just don't yet understand or even something inherently unknowable."

As keen, meticulous and diligent scientists, Alice and Bob want to verify this through experimentation...

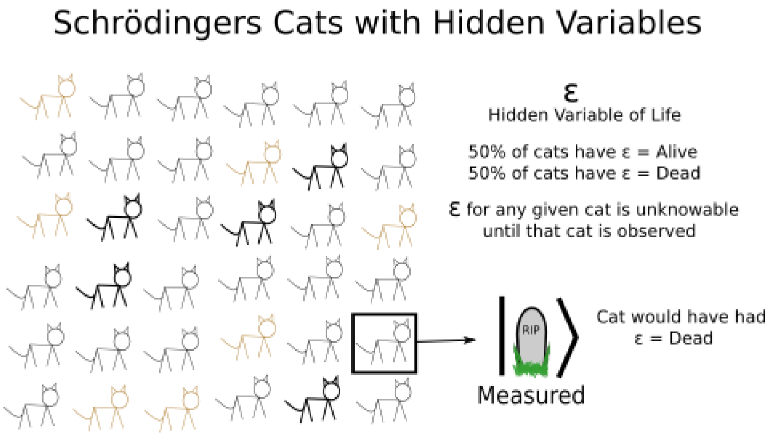

Figure 1: An example where a hidden variable is applied to Schrödinger's Cat. The Cats have a variable ϵ which determines whether it is Alive or Dead when observed. However, the observer cannot know ϵ for a cat until it is observed.

Hidden Variables?

For Alice and Bob to test their hypothesis of a 'hidden variable', they will ultimately end up at 'Bell's Theorem". In 1964, John Stewart Bell published a rather famous paper: "On The Einstein Podolsky Rosen Paradox" [11]. The basic premise of this was to determine whether measurements and behaviours of quantum phenomena made sense if there were hidden variables controlling them. The basic outcome: no.

Bell's methods for showing this involved the use of the first iterations of rather famous expressions known as "Bell Inequalities". These inequalities, when found to be violated, showed that hidden variables in quantum mechanics were not possible.

The mathematics involved gets rather complicated, but a good demonstration of the kind of logic used is still possible without going too far down this rabbit-hole. To do this, however, a new thought experiment is required...

Alice and Bob will use a form of the EPR experiment proposed by the physicist David Bohm [12], in a way described by David Mermin [13].

To test the hidden variable idea, they will measure the spin of an entangled pair of "spin-1/2" particles:

[2] This relates to another idea known as realism [6]
[3] An actual quote from Einstein [10]

A and B. In the same way that their particles earlier had a combined total momentum of zero, these particles will have zero total spin. In technical terms, this makes them anti-correlated.

An Interlude on Spin

You may have heard of spin before now, but may not have an entirely firm grasp of the concept. Well, fear not; the exacts of what spin is are not so important. Instead, it is what happens when we measure it that is key.

Spin is a form of intrinsic angular momentum, and therefore is a vector that points in a specific direction. When a particle's spin is measured along any given axis, one of two possible results can be found. The *full magnitude* of the particle's spin will either point in the same direction as the axis (AKA "spin-up"), or in the opposite direction (AKA "spin-down") [14, p. 138].

So, what does this look like in a pair of entangled particles? What happens if we measure the spin of each particle along the same axis? Since our particles are anti-correlated, with 0 total spin, measuring the spins of each particle will always result in one up and one down measurement.

$$A : |\uparrow\rangle \;\; B : |\downarrow\rangle \;\; or \; A|\downarrow\rangle \;\; B : |\uparrow\rangle$$

If one particle is measured along an axis, and the other is in the opposite direction, then the measurements will always be either both up or both down.

$$A : |\uparrow\rangle \;\; B : |\uparrow\rangle \;\; or \; A|\downarrow\rangle \;\; B : |\downarrow\rangle$$

This is exactly the same result as before. Imagine Alice and Bob are standing on opposite poles of the Earth. Up for Alice will be down for Bob and vice a versa.

But what happens when the spins of the particles are measured along completely different axes? For example, if the spin of particle B is measured along an axis at an angle θ to the axis used to measure A? Quantum mechanics predicts that B will exist in a superposition of the states:

$$|B\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow_\theta\rangle + \sin\left(\frac{\theta}{2}\right)|\downarrow_\theta\rangle \quad if \;\; |A\rangle \;\; is \;\; |\downarrow\rangle \tag{4}$$

$$|B\rangle = -\sin\left(\frac{\theta}{2}\right)|\uparrow_\theta\rangle + \cos\left(\frac{\theta}{2}\right)|\downarrow_\theta\rangle \quad if \;\; |A\rangle \;\; is \;\; |\uparrow\rangle \tag{5}$$

as given by [14, p. 213].

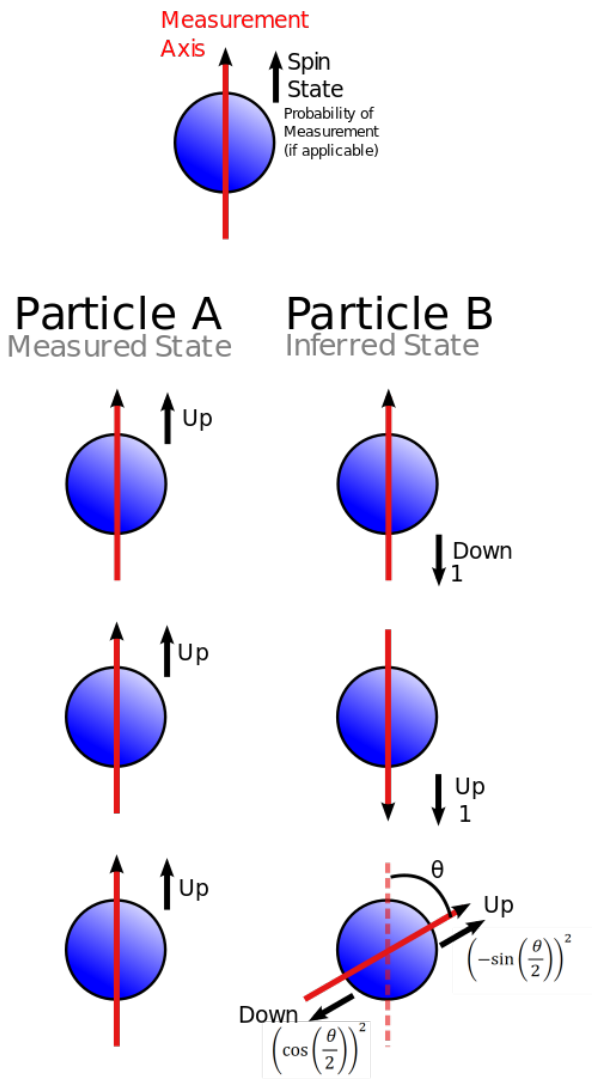

Figure 2: Expected spin states of particle B when A is measured to be in a "spin-up" state. The spin state of B along the axis used to measure A is known for certain. Along other axes, it is in a superposition of states.

The Big Experiment

The Method

Alice and Bob have acquired a machine that emits the entangled particles they require, and shoots them to opposite ends of the universe. At each end they have a machine able to measure the spin of the particles along one of three different axes. The directions of these axes are evenly spaced – separated by angles of 120°. When a pair of entangled particles arrive, each machine selects at random an axis along which they will measure their particle's spin. The machines then simultaneously make these measurements. If a machine finds the particle to be in a "spin-up" state along the axis it has chosen, it flashes a green light. If it finds "spin-down", a red light flashes instead.

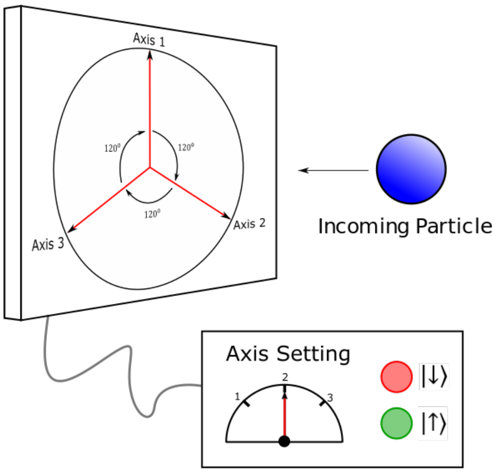

Figure 3: An example of one of the machines used to measure the spins of particles A and B in Alice and Bob's experiment

Once they have collected an infinitely large amount of data, Alice and Bob ask a question: Could this data have been produced if the measurements were pre-determined?

To answer this question, they look at the number of times both machines record the same spin state. More simply put, they note the fraction of the time that the machines' lights flash the same colour.

The Theory

What should Alice and Bob expect to see?

To start, if A is measured along a given axis, there is a $\frac{1}{3}$ Chance that B will be measured along the same axis, and a $\frac{2}{3}$ chance that B will be measured along a different one.

So, what happens if A and B are measured along the same axis? Since we know these particles are anti-correlated, they must have opposite spin states. This means that in this case, the lights on the machines measuring A and B can never flash the same colour.

For the lights to flash the same colour, the machines measuring A and B must therefore use different axes. As a result, these will be separated by an angle $\theta = 120°$. Supposing different axes are chosen, then the probability of the lights flashing the same colour is given as:

$$P(A\ is\ |{\downarrow}\rangle) \times P(B\ is\ |{\downarrow}_{120°}\rangle\ if\ |A\rangle\ is\ |{\downarrow}\rangle) + P(A\ is\ |{\uparrow}\rangle) \times P(B\ is\ |{\uparrow}_{120°}\rangle\ if\ |A\rangle\ is\ |{\uparrow}\rangle)$$

For the probabilities regarding the state of B, these end up being equal to $\sin^2(\frac{120°}{2})$ (from equations 5 & 6). We can therefore take the above expression, and rewrite it like so:

$$\sin^2\left(\frac{120°}{2}\right) \times (\ P(A\ is\ |{\downarrow}\rangle) + P(A\ is\ |{\uparrow}\rangle)\ )$$

Of course, A can only be up or down, so:

$$P(A\ is\ |{\downarrow}\rangle) + P(A\ is\ |{\uparrow}\rangle) = 1$$

Putting all this together, when A and B are measured along different axes, $\frac{3}{4}$ of measurements will result in the lights flashing the same colour.

Accounting for the fact, that A and B are only measured along different axes in $\frac{2}{3}$ of measurements, we wind up with $\frac{1}{2}$ of all pairs of particles causing the lights in the machines to flash the same colour.

So, is this the same if the states of the particles are decided by some hidden variable? If the spin states of the particles that Alice and Bob will observe at their machines are all predetermined, then we can imagine measuring them to be like drawing results from a pre-written list. For a given instance of particle A, it might have a pre-determined response to being measured along each axis as follows:

$$A: \quad 0° : |{\uparrow}\rangle\,, 120° : |{\downarrow}\rangle\,, 240° : |{\uparrow}\rangle$$

Since A and B must have opposite spins along each axis, this instance of B would have the pre-determined responses:

$$B: \quad 0° : |{\downarrow}\rangle\,, 120° : |{\uparrow}\rangle\,, 240° : |{\downarrow}\rangle$$

From this, finding the fraction of tests where both machines flash the same colour would simply be a case of running through the different permutations of A and B's responses, and considering the colours the lights will flash when different combinations of axes are chosen. As it turns out, the fraction we get from this method must be at least $\frac{5}{9}$ [13].

Upon reaching this same result, Alice and Bob agree that $\frac{5}{9}$ is not equal to $\frac{1}{2}$.

Hidden variables are not an option

So... What Now?

Of course, to you and I, this is still but a thought experiment – and is only useful if it can be backed up by experimental data. Thankfully for Alice and Bob, this seems to be the case. A well-known example is a test performed by Alain Aspect, Philippe Grangier, and Gérard Roger in 1981 [15] which used the polarisations of pairs of entangled photons to show violation of Bell Inequalities (something that was briefly touched on earlier). Since then, further tests have been carried out to account for various "loopholes" that may allow local theories to seep through the cracks [16].

But if this is all true, and locality is very much out the window, then you may be wondering what this means for relativity. After all, it seems we have particles interacting at faster-than-light speeds. Could we make a superluminal transmitter? For example, we might have Alice transmitting a message by setting the state of the entangled particles by measuring A along a specific axis. Bob might receive this message by measuring B on the other side of the universe. The existence of a "no-communication theorem" [17] should probably be a good indicator. Once again, the mathematics are rather complicated, but the general gist is that it wouldn't be possible for Bob to tell the difference between Alice's message and random data.

So where does that leave our good friends Alice and Bob? The truth is, they don't really know. They have learnt that at the fundamental level the future is seemingly unknowable, unpredictable...

"**I suppose there's always fatalism,**" says Bob

And thus, the metaphysical musings continue.

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