Casimir's Phantom Hand

CID [redacted]

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Casimir's Phantom Hand

Picture this: in front of you are two mirrors, perfectly parallel, with their reflective sides facing each other. The surfaces of these two plates are close together, so close you can hardly make out their separation. These plates are surrounded by a perfect vacuum: no dust, no human, and no machine touch the plates. Perfect, undisturbed silence.

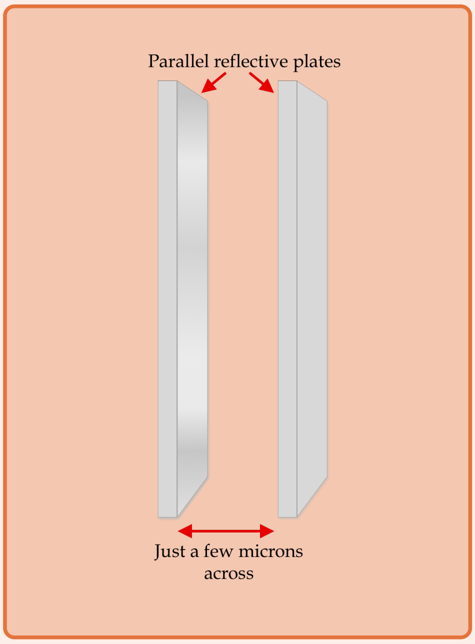


Figure 1: The set-up as described above. The plates are suspended in a vacuum, perfecty parallel to one another. The plates are conductors and reflective on their adjacent sides.

If I were to ask you whether anything would happen to the plates, whether they move or vibrate or spontaneously combust, you would look at me and say, "Well of course not! Newton's first law says objects at rest should remain at rest in the absence of external forces, which is precisely the case here."

What if I told you these plates would, in fact, be pushed together? Would you believe me? Your common sense surely warns you against it.

How about if I gave you a hint and said this was a quantum effect? Ah, I see now your ears have perked up.

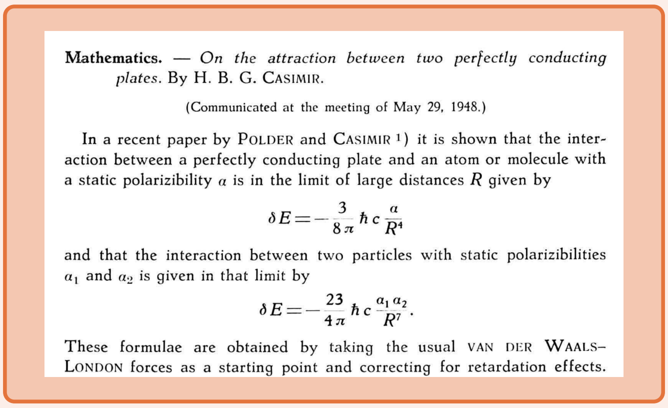
At first glance this has two explanations, both equally implausible: either Newton's laws are a hoax, allowing the plates to jump into spontaneous motion; or an invisible phantom hand has materialised and squeezed the plates together.

This article seeks to make physical sense of this phantom hand, delving into its historical discovery while explaining the quantum physics behind this illusive phenomenon. You'll be surprised to find out this effect has been used (somewhat cavalierly, depending on which physicist you speak to) to explain how ships behave at sea. We'll also uncover how this effect became a pest for nanotechnologists before being revolutionised into a tool in their armory for creating new technologies.

History behind the mystery

The phantom hand emerges

This eponymous effect was first theorised by the Dutch theoretical physicist, Hendrik Casimir, in 1948 [1]. In a brief, three page note entitled *"On the attraction between two perfectly conducting plates"* which accompanied another of his recent papers, Casimir used quantum mechanical reasoning to argue that two reflective plates in a vacuum should be subject to an attractive force pulling the plates together [1]. Perhaps with the optimism of a true theoretical physicist, Casimir concluded by postulating that empirical confirmation of this miniscule force would be attainable [1].



Mathematics. — *On the attraction between two perfectly conducting plates.* By H. B. G. CASIMIR.

(Communicated at the meeting of May 29, 1948.)

In a recent paper by POLDER and CASIMIR [1]) it is shown that the interaction between a perfectly conducting plate and an atom or molecule with a static polarizibility α is in the limit of large distances R given by

$$\delta E = -\frac{3}{8\pi}\hbar c \frac{\alpha}{R^4}$$

and that the interaction between two particles with static polarizibilities α_1 and α_2 is given in that limit by

$$\delta E = -\frac{23}{4\pi}\hbar c \frac{\alpha_1 \alpha_2}{R^7}.$$

These formulae are obtained by taking the usual VAN DER WAALS–LONDON forces as a starting point and correcting for retardation effects.

Figure 2: An exerpt from Casimir's original 1948 paper, where he sets the theoretical framework for this illusive phantom force [1].

A theory before its time

Despite Casimir's valiant suggestion that his phantom force could be observed through experiment, experimental contemporaries of Casimir found that their equipment was too rudimentary to provide any quantitative supporting evidence. There were a number of stumbling blocks inhibiting scientists at the time from obtaining a glimmer of experimental insight into this effect. The chief complication involved how perfectly parallel the plates must be to observe the effect, given they needed to be separated by around 1 micron in order for the magnitude of the force to be large enough to be measured [2]. In addition to this, if the two plates had even slightly different surface potentials, which can occur on the order of a few mV for metal plates of the same material, the phantom hand would not appear [3]. Moreover, the plates are required to be clean, to avoid debris on the plates from decreasing the attraction between them [3]. If this is done by rubbing the plates, static electricity can be built up, increasing the electrostatic attraction between the plates and adding another layer of uncertainty. All these factors were consequential when in 1958, physicist Sparnaay failed to measure the Casimir force with any inkling of precision. Unsatisfying as it is, the high uncertainty on the measured value of the force meant Sparnaay was merely able to conclude that his results "do not contradict Casimir's theoretical prediction" with effectively 100% uncertainty in the measurement [3].

In reaction to the experimental difficulty demonstrated by Sparnaay's paper, the search for empirical proof of this effect lay dormant for the best part of four decades.

Measuring the immeasurable

Then, in 1997, the paper *"Demonstration of the Casimir Force in the 0.6 to 6 μm Range"* emerged from the laboratories of the University of Washington, by the hands of Steve Lamoreaux [4]. The paper outlines how Lamoreaux succeeded in measuring the Casimir force to within 5% of the theoretically predicted value. How was the issue of perfectly parallel plates overcome? Lamoreaux opted to substitute one of the conducting plates for a spherical conductor, eliminating the issue posed by parallelism; the system is simply defined at the point of closest approach between the sphere and plate [4]. Casimir's original equation describing the force had to be modified to fit this new topology [4]. However, the underlying physics was able to be carried over in this adjustment [4]. To detect the force, the plate was moved towards the sphere by the experimenters while its motion was measured with excruciating precision - to the nearest 0.5 μm. They observed that the plate did not only move the amount dictated by the experimenters; it overstepped. The measurements showed the plate moved the exact amount extra that would be expected by the force predicted by Casimir all those years ago.

This discovery marked the beginning of a new era of Casimir force measurements. Armed with Lamoreaux's plate-sphere technique, researchers across the globe began pumping out measurements of the Casimir force with extraordinary precision. Just three years after Lamoreaux's intial observation, the likes of Thomas Ederth of the University of Stockholm found empirical evidence in agreement with theory to within 1% [5].

The torrent of successful observations in recent years can be thought of as a tribute to the importance of perseverance in science. The contributing factor to the multitude of studies was merely the fact that the researchers now *knew* it was possible, thanks to Lamoreaux's perseverance to observe what was before thought to be unobservable.

You may be wondering whether Casimir's initial set-up of two parallel plates has ever been empirically realised. The issue of parallelism remains to this day a difficult hurdle to overcome. One of the few observations of this parallel plate configuration was carried out by researchers at the University of Padova in Italy, in 2002. The uncertainty on this measurement is high, approximately 15%, a far cry from the more commonplace spherical geometry configurations [6]. Having said that, the degree of progress achieved since the early days of Casimir force measurements is truly a scientific triumph.

Timeline: From proposal to observation

- 1948: Hendrik Casimir is the first to theorise the effect, in his publication *"On the attraction between two perfectly conducting plates"*
- 1958: Sparnaay makes a first unsuccessful attempt to measure the Casimir effect
- 1997: Steve Lamoreaux measures the Casimir force to within 5% of the theoretical value
- 2001: The Casimir Effect is observed to within 1% uncertainty
- 2002: A parallel plate set-up is used successfully to measure the Casimir effect

The science behind the phantom force

All of this history begs the question: How does this curious effect come about? If Newton's laws have not been broken, what *is* the physics behind it?

Are vacuums really empty?

Luckily for Newton, the plates do in fact move in reaction to the imbalance of forces acting on the plates. But what kind of force could possibly be acting in such a way in a vacuum?

To answer this, we have to come to grips with the fact that our classical idea of a vacuum is, simply put, wrong. Classically, we imagine a vacuum to be completely and perfectly empty. This notion was shattered by the likes of Dirac and de Broglie who brought quantum field theory (QFT) to the forefront of physics in the late 1920s [7] [8]. QFT sought to unify two fundamental fields of physics: quantum mechanics and electrodynamics [9]. Simply put, it extends quantum mechanics, which deals with singular particles, to fields containing an infinite number of degrees of freedom [9]. A rudimentary way to visualise this is to imagine a field as an expanse of infinitely many points. Each point oscillates as a simple harmonic oscillator, giving us the infinite number of degrees of freedom a field is required to contain [10]. The simple harmonic oscillators we are used to dealing with in classical physics are defined in terms of a single frequency. However, quantum mechanics stems from the notion that our world is a probabilistic one. Hence, the simple harmonic oscillators are not defined in terms of an exact single frequency, but rather by a spectrum of frequencies, which take a Gaussian distribution [11].

Definition: Quantum field theory

Quantum field theory is a modern approach to areas of physics such as elementary particle physics, condensed matter physics and statistical mechanics. The theory allows physicists to deal with both particles and fields with a uniform theoretical understanding, which wouldn't be possible using quantum mechanics alone [9].

In a similar way, the energy of the oscillator is defined by a spectrum, almost like the universe has a Gaussian-shaped bag of values from which it can choose an energy and assign it to the point in space. Crucially, this means that the energy in the vacuum *cannot* always be zero: it *must* fluctuate.

Now that we've given the vacuum energy some conceptual consideration, why don't we try calculate it? It's not quite as scary as you might think: we simply need to sum up the energies of each constituent oscillator. So to begin, let's figure out the energy of a single oscillator.

Corresponding to our understanding of classical physics, the total energy E of the quantum harmonic oscillator must be, at least, the sum of its kinetic and potential energies

$$E = \frac{\Delta p^2}{2m} + \frac{1}{2} m\omega^2 \Delta x^2, \tag{1}$$

where Δp is the momentum uncertainty, m is the mass of the system, ω is the angular frequency of the oscillator, and Δx is the position uncertainty [12].

The Heisenberg uncertainty principle can be written as

$$\Delta x \Delta p = \frac{\hbar}{2}, \tag{2}$$

in its lower limit, where $\hbar$ is the reduced Planck constant. This lower limit corresponds to the lowest energy state of the oscillator [12]. We can confidently take this lower limit when dealing with the vacuum energy because we expect a vacuum to have the lowest amount of energy, since there's not matter in it at all! Using Equation 2, we can rewrite Equation 1 in terms of position uncertainty, obtaining

$$E = \frac{\hbar^2}{8m(\Delta x)^2} + \frac{1}{2} m\omega^2 (\Delta x)^2. \tag{3}$$

By taking the derivative of this expression for the energy with respect to Δx and setting the whole thing equal to zero, we should be able to find the spread in position required for minimum energy. If you try it yourself, you'll find

$$\Delta x = \sqrt{\frac{\hbar}{2m\omega}}. \tag{4}$$

Then, we simply need to substitute this into Equation 3 to obtain the minimum value of energy E_{min} in terms of Δx. This simplifies neatly to this eloquent expression for the zero-point energy:

$$E_{min} = \frac{\hbar\omega}{4} + \frac{\hbar\omega}{4} = \frac{\hbar\omega}{2} \tag{5}$$

And *voila*, we have the minimum energy allowed for each of our oscillators. Simply by summing these up, over all the possible points in the field we have

$$\sum_{n=1}^{\infty} E_n = \sum_{n=1}^{\infty} \frac{\hbar\omega_n}{2}, \tag{6}$$

which is clearly non-zero. This more mathematical proof once again goes to show that a vacuum is not quite as empty as we thought.

Definition: Zero-point energy

This is the background energy present in all quantum mechanical systems, calculated by summing up the energy of the infinite simple harmonic oscillators in a field.

What does this mean for our plates?

You're probably thinking, this all sounds like interesting physics, but how on Earth does that translate to two plates moving towards each other out of thin air!

The key here is to bear in mind our thinking about fields, and apply it to the fields surrounding the plates. The region outside the two plates lacks an important quality: boundary conditions. Without these boundary conditions, all frequencies of the oscillators can be present, making it a so-called 'free-vacuum' [13].

Definition: Boundary conditions

Boundary conditions occur when the value of a dependent variable is specified at two different points within a system [14].

This is not the case for the region between the plates. The two reflecting plates are conductors, which means the electric field must be zero inside the conductors themselves. So, just as with a standing wave on a string fixed at both ends, the field between the plates must have nodes at the conductors. This means that only integer multiples of half wavelengths can fit between the two reflecting plates. Consequently, all the other possible wavelengths, and thus frequencies, are supressed, as shown in Fig. 3 [13].

Consider counting the contributions from each mode outside the plates pairwise with the equivalent mode between the plates. You can see that for every suppressed mode inside the plate, which has no contribution to the zero-point energy of the field, the field

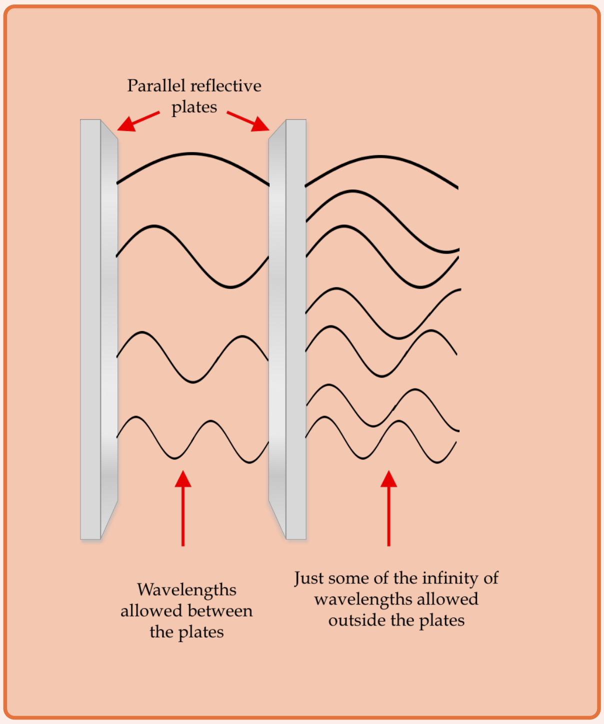


Figure 3: A visualisation of the wavelengths allowed between the plates, compared to those outside the plates. You can see that the inner region is missing modes of certain wavelengths because they have been suppressed by the boundary conditions.

outside the plates *will* get an energy contribution from the mode. When we sum up over all the possible frequencies inside and outside the plates, we find there is more energy in the field outside the plates than inside the plates [1].

So, now, how does this energy transfer into the momentum of the plates? An interesting facet of electromagnetic waves is that they do in fact carry momentum, unlike other types of waves. This can be shown by considering the relativistic formula for energy

$$E^2 = p^2c^2 + m^2c^4. \tag{7}$$

Since the electromagnetic wave has no mass, this equation reduces to

$$E = pc. \tag{8}$$

In other words, in order for the energy to be non-zero, the momentum must also be non-zero.

In this way, the waves of the electromagnetic field transfer momentum to any matter they encounter, which in our case is the plates. The pressure the waves exert on the plates is known as the *field radiation pressure*. Momentum being proportional to energy implies that the field radiation pressure on the outside of the plates will be greater than on the inside, due to the energy imbalance we derived before. Now we've got to the bottom of it, Newton survives: there is a net force acting on the plates, pushing them together!

Using this ideas and his previous work on the van der Waals forces, Casimir carried out the non-trivial calculation of the force per cm^3, finding it to be

$$F = \hbar c \frac{\pi^2}{240} \frac{1}{a^4}, \tag{9}$$

where a is the distance between the plates [1]. This corresponds to a very, very, *very* small force. For a plate separation of $6\,\mu\text{m}$, the force has a magnitude of merely $2.17 \times 10^{-8}\,\text{N}$. That's less than the force holding an electron to a hydrogen atom. Hopefully now you have some sympathy for the experimentalists who found it so difficult to measure!

Physicists not all in the same boat

It is still up for debate as to whether Casimir's phantom hand could crop up in macroscopic physical phenomenon. The most historically renowned of these alleged classical analogs suggests that ships at sea could in fact be pushed together due to a force not dissimilar from Casimir force.

Figure 4: Ships at sea in P. C. Caussé's sailors' manual. [15]

In 1936, a French Royal Navy captain P. C. Caussé, wrote a sailors' manual detailing how to deal with a variety of potentially dangerous situations at sea. Caussé warns that "a certain attractive force" pulls ships towards each other in a long swell [15].

While this reeks of pure sailors' superstition, some physicists, such as Boeserma in 1996, have argued that the Casimir effect is to blame [16]. They claim the ships act as Casimir plates and the waves play the role of the field. The idea is that if the wavelength of the waves is large compared to the distance between the ships, then the ships will rock side to side in phase (i.e. starboard of one ship in phase with starboard of the other, and likewise for the port sides). This means, however, that the one ship's starboard side will be in opposite phase to the other ships port side which are closest to each other, as depicted in Fig. 5. The waves emitted by the sides of each ship will superpose and cancel, resulting in less energy between the ships than outside - just like there is for Casimir plates [16]. In a similar fashion to the conducting plates, the rolling ships will be drawn together by Casimir's phantom hand.

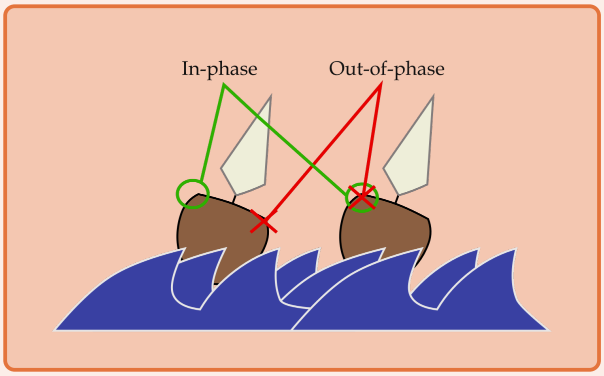


Figure 5: A cartoon schematic of ships at sea. The green circles indicate the port side of each ship, which are in phase in their rocking motion. The red crosses mark the starboard side of the left-hand ship and the port side of the other. These points are out of phase in their motion; one is moving towards its highest point, while the other is moving towards its lowest.

However, other physicists refute this claim. In 2006, former NASA Scientist, Fabrizio Pinto, got ahold of P. C. Caussé's book and found that Boeserma had the situations mixed up: Caussé was actually claiming the ships were pulled together in the *absence* of waves, rather than when waves were present [17]. This leads to the motivation behind Boeserma's work being unfounded. Naval architect Jason Smithwick says that he could imagine this being possible, although it would require a very specific set of circumstances and is definitely not something he has ever heard of or encountered [17].

It is unclear whether this explanation is simply a product of physicists' desire to keep the 'folklore' associated with the subject alive and kicking, or whether it is truly a result of the quantum Casimir effect emerging in the macroscopic world.

Quantum problems require quantum solutions

You may now be wondering how something as small as the Casimir effect could have any ounce of importance to our daily lives?

The answer is nanotechnology. This term, first coined by Norio Taniguchi in 1974, describes scientists' and engineers' manipulation of matter on a nanometer scale for humanity's own technological benefit [18]. The small scale of the Casimir effect means it is both a pest and a saviour in the field of nanotechnology.

One key complication is that the attractive Casimir force can introduce an indirect phantom frictional force in microelectromechanical systems [19]. The attractive force between micro-structures can hinder their relative motion and sometimes even stick them together entirely [19]. Evidently this can put you in a rather *sticky* situation if you're in the nano-manufacturing business.

Nanotechnologists have, however, figured out how to harness the contactless nature of the effect for their own benefit, for example in high sensitivity force sensors, structures requiring controlled self-assembly, and contact-free nanomachines [20]. In 2009, physicists succeeded in creating a repulsive Casimir force using a gold coated sphere and a substituting the traditional gold plate with one made of silica [21]. By combining this repulsive Casimir force with the attractive one, they created a Casimir equilibrium, capable of performing quantum levitation [21] [20].

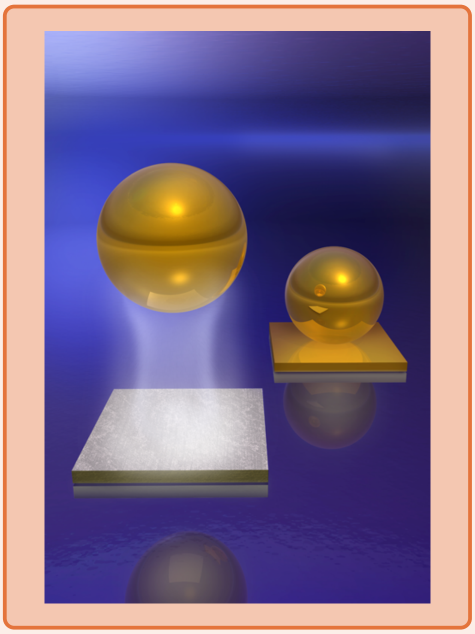


Figure 6: An artist's impression of quantum levitation due to the combination of repulsive and attractive Casimir forces. (Image credit: Jay Penni and Federico Capasso)

The yet-unharnessed applications of the Casimir effect are plentiful, leaving the field wide open for future generations of innovative physicists. Along with more ingenious uses of the effect in nanotechnology, researchers are investigating the possibility of translating the effect to different topologies and even harnessing it to explain some of the quantum questions shrouding the early universe [22].

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