Imperial College London

Black Holes and Branes in Supergravity

BPS and Extremal Reissner-Nordström Solutions

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Motivations

Supergravity is a unifying theory that combines general relativity and supersymmetry (SUSY). Besides being the low-energy limit of M-theory and having ties to string theory, it is strongly motivated by its potential to:

- resolve the hierarchy problem.
- propose many dark matter candidates.
- fix gauge coupling unification such that the Standard Model naturally descends from a higher symmetry group (Figure 1).

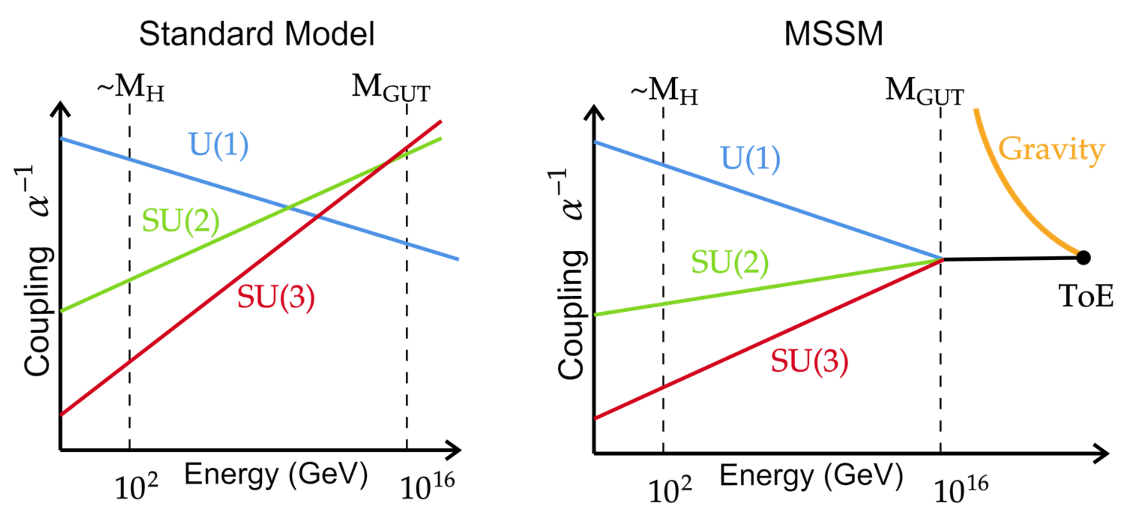

***Figure 1.** SUSY gauge coupling unification in the Minimally Supersymmetric Standard Model (MSSM) compared to the Standard Model. Quantum gravitational interactions may couple at even higher energies and unify to a Theory of Everything (ToE)*

Supergravity theories can be generalised to an arbitrary number of dimensions $D \leq 11$ (constrained by SUSY). However finding general solutions is difficult without imposing many assumptions and ansätze. This project deals with deriving such solutions and investigating their properties in order to understand the implications of supergravity.

Aims and Objectives

- To derive brane solutions of a general supergravity theory by imposing certain ansätze.
- To understand the properties of these branes, especially their degree of SUSY.
- To investigate their connections with black hole solutions in general relativity.
- To extend the solutions to more general cases and explore other areas of supergravity.

Supergravity Model

We work in the bosonic sector of a general supergravity theory in D spacetime dimensions, containing a scalar field ϕ and a q-form field strength $F_{[q]}$ (antisymmetric rank q tensor).

$$I = \int d^D x \sqrt{|g|}\left(R - \frac{1}{2}\nabla_M\phi\nabla^M\phi - \frac{1}{2q!}e^{a\phi}F_{[q]}^2\right)$$

Objects embedded in this spacetime may only span p spatial dimensions. These are called p-branes. Stokes' theorem and the equations of motion can be used to determine which brane solutions are expected. In $D = 11$ supergravity for instance, which contains a 4-form field strength, we find a 2-brane and a 5-brane solutions.

p-Brane Ansatz

Assume Poincaré invariance along the brane worldvolume ($d = p + 1$ spacetime dimensions) and rotational symmetry in transverse directions i.e., $(\text{Poincaré})_d \times SO(D-d)$ symmetry and a metric ansatz

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(r)}\delta_{mn}dy^m dy^n,$$

where $r = \sqrt{y^m y_m}$. Index convention is summarised on Figure 2. It is also useful to assume the linearity condition

$$dA + \tilde{d}B = 0$$

where $\tilde{d} = D - d - 2$. This condition causes the brane to satisfy the Bogomol'nyi-Prasad-Sommerfield (BPS) bound [1]

Mass density = Charge density

and to preserve half the SUSY. BPS solutions have many applications in SUSY and supergravity. Determining the metric requires solving the Ricci tensor and field equations. Both Weyl transformations and vielbein are sufficient methods.

Weyl Transformations

Apply the following Weyl (conformal) transformation:

$$g_{MN} = e^{2A}\hat{g}_{MN}.$$

The spacetime can then be written as a product manifold where the metric is decomposed as $\hat{g} = \hat{g}_{\mu\nu}(x) \oplus \hat{g}_{mn}(y)$. The Ricci tensor is solved using the new metric

$$R_{MN} = \hat{R}_{MN} - (D-2)\hat{\nabla}_M\hat{\nabla}_N A + (D-2)\hat{\nabla}_M A\hat{\nabla}_N A$$
$$-\hat{g}_{MN}\hat{\nabla}^2 A - (D-2)\hat{g}_{MN}\left(\hat{\nabla}A\right)^2.$$

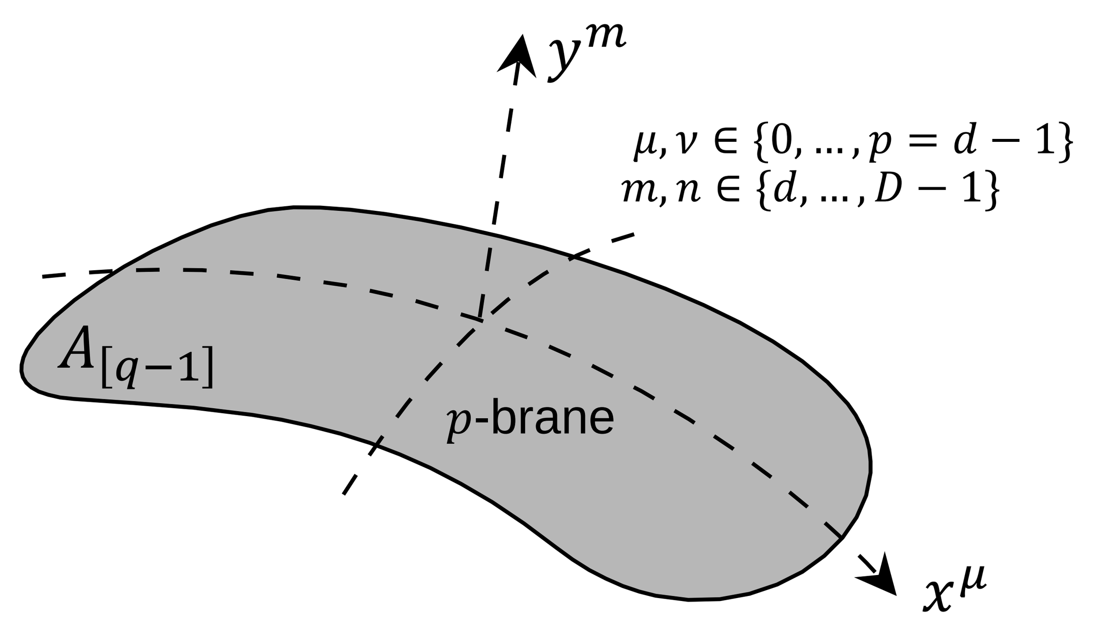

***Figure 2.** The p-brane embedded in a D-dimensional spacetime charged under a $(q-1)$-form gauge potential $A_{[q-1]}$ satisfying $F_{[q]} = dA_{[q-1]}$ and $d = q - 1$. Indices are divided into $M = (\mu, m)$ where x^μ are the brane worldvolume coordinates and y^m denotes the transverse space coordinates.*

Vielbein Formalism

To avoid general coordinate transformations (GCTs) of spinors, consider working in the tangent spaces at each point in Minkowski spacetime. Vielbein $e_\mu{}^a$ then define an orthonormal non-coordinate basis

$$g_{\mu\nu}(x) = e_\mu{}^a(x)e_\nu{}^b(x)\eta_{ab}.$$

Instead of the usual procedure for calculating the Ricci tensor ($g \to \Gamma \to R$), construct the spin connection ω and use the Cartan structure equations for zero torsion.

$$\omega_\mu{}^a{}_b = e_\nu{}^a e^\lambda{}_a\Gamma^\nu_{\mu\lambda} - e^\lambda{}_b\partial_\mu e_\lambda{}^a$$
$$0 = de^a + \omega^a{}_b \wedge e^b$$
$$R^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b$$

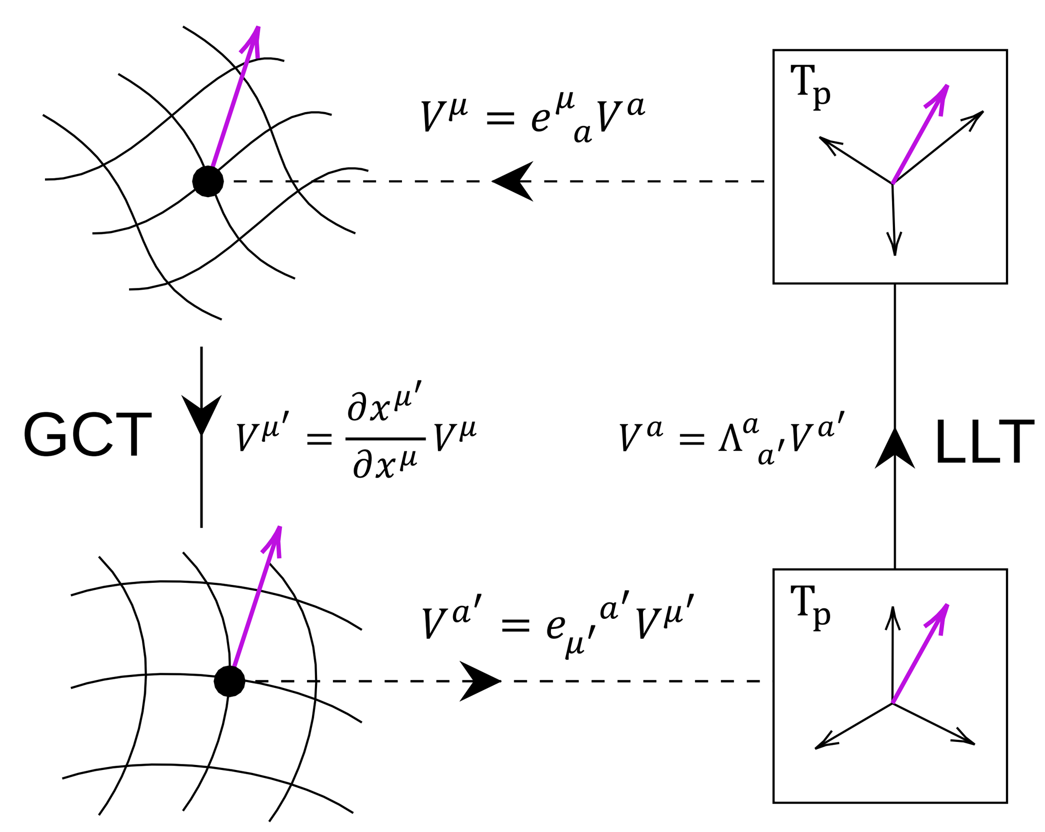

***Figure 3.** Vielbein transformations of a vector V^μ. In the tangent space T_p, one can do local Lorentz transformations (LLTs) and convert back to general coordinates using the inverse vielbein.*

p-Brane Solutions

By deriving the Ricci tensor and solving the field equations, we find the metric solution

$$ds^2 = H^{\frac{-4\tilde{d}}{\Delta(D-2)}}\eta_{\mu\nu}dx^\mu dx^\nu + H^{\frac{4d}{\Delta(D-2)}}\delta_{mn}dy^m dy^n$$

where $H(r) = 1 + kr^{-\tilde{d}}$ is a harmonic function, $d = q - 1$, and $\tilde{d} = D - d - 2$. The quantity

$$\Delta = a^2 + \frac{2d\tilde{d}}{D-2}$$

is related to the fraction of preserved SUSY on the brane.

Extremal Reissner-Nordström Black Holes

Notably, our solution can be mapped to the extremal Reissner-Nordström black hole (charge Q and mass M). It satisfies $M = Q$ and has two concentric horizons at $r_\pm = M$. Its metric is given by

$$ds^2 = -\left(1 - \frac{M}{r}\right)^2 dt^2 + \left(1 - \frac{M}{r}\right)^{-2} dr^2 + r^2 d\Omega^2,$$

where $d\Omega^2$ is the metric on a unit two-sphere.

In general, one can show that SUSY black holes satisfying BPS are extremal [2]. As such, our brane solution has applications in black hole physics, which has become an important sector for studying beyond standard model physics.

Extended Work

Following topics are extended questions we have dealt with during the remainder of the project:

Generalised metric – can the underlying metric be generalised from being flat to Ricci flat?

Non-extremal branes – what are the features of non-extremal solutions, where the linearity condition $dA + \tilde{d}B = 0$ is relaxed?

Cosmology – what happens if a cosmological constant is added to the theory? Can one find de Sitter spacetimes on the brane, which can describe an accelerating universe embedded in higher dimensions?

References

[1] K. S. Stelle, "Brane Solutions in Supergravity," in *Particles and Fields*, pp. 507-607, World Scientific, Nov. 2002.
[2] T. Mohaupt, "Black Holes in Supergravity and String Theory," *Class. Quantum Grav.*, vol. 17, pp. 3429-3482, Sept. 2000. arXiv: hep-th/004098