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Black Holes and Branes in Supergravity

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Supergravity

Supergravity is a generalisation of GR which is (super)symmetric under a unitary 'rotation' of fermions into bosons and vice versa. The highest-dimensional consistent supergravity is the 11D 'maximal' supergravity, whose bosonic action is shown[1]. Upon dimensional reduction, it yields the classical type IIA supergravity.

$$S = \int_{\mathcal{M}_{11}} d^{11}x \sqrt{-g}\,\mathcal{R} - \frac{1}{2}\int_{\mathcal{M}_{11}} F_{[4]} \wedge * F_{[4]} + \frac{1}{3} F_{[4]} \wedge F_{[4]} \wedge A_{[3]}.$$

Branes

p-branes are solutions to the supergravity equations which generalise RN black holes to higher dimensions. They have Poincaré symmetry on their $p{+}1$-dimensional worldvolume and rotational symmetry around it. In D dimensions, their symmetry group is:

$$G = ISO(p-1) \times SO(D-p-1).$$

There are two solutions of this form to the equations of motion: the 'electric/elementary' 2-brane and the 'magnetic/solitonic' 5-brane. The branes become ' extremal' if we further impose that the solutions are partially supersymmetric.

Electric 2-brane

The 2-brane has 3-form gauge field $A_{[3]}$ coupled to the brane's 3-dimensional worldvolume, like Maxwell theory's 1-form $A_{[1]}$ couples to an electric monopole's worldline. Its metric is given by:

$$ds^2 = H(r)^{-2/3} dx^\mu dx^\nu \eta_{\mu\nu} + H(r)^{1/3} dy^m dy^m,$$

where Greek indices are worldvolume, Roman are transverse, r is the radial coordinate, and $H(r)$ is a specific harmonic function[2].

Magnetic 5-brane

We define a dual 7-form field strength $*F_{[4]}$ from a 6-form gauge field $A_{[6]}$. This is coupled to the brane's 6-dimensional worldvolume, analogous to a magnetic monopole. Its metric is given by:

$$ds^2 = H(r)^{-1/3} dx^\mu dx^\nu \eta_{\mu\nu} + H(r)^{2/3} dy^m dy^m,$$

where $H(r)$ is a different harmonic function from the electric case[3].

Field Content		Theory		
Name	Symbol	Maximal	Type IIA	N = 8
Graviton	$g_{\mu\nu}$	1	1	1
Gravitino	$\psi_{\mu\alpha}$	1	1	8
Vector	A_μ	0	1	28
Spinor	χ_α	0	1	56
Scalar	φ	0	1	70
p-form	$A_{[p]}$	1x3	1x3, 1x2	0

Table 1: Field content of supergravity theories dimensionally reduced from N=1, 11D SuGra.

Event Horizons

2-brane

- Timelike singularity
- Zero surface gravity
- Interpolates between $\mathbb{R}^{1,10}$ and $AdS_4 \times S^7$

5-brane

- Non-singular
- Zero surface gravity
- Interpolates between $\mathbb{R}^{1,10}$ and $AdS_7 \times S^4$

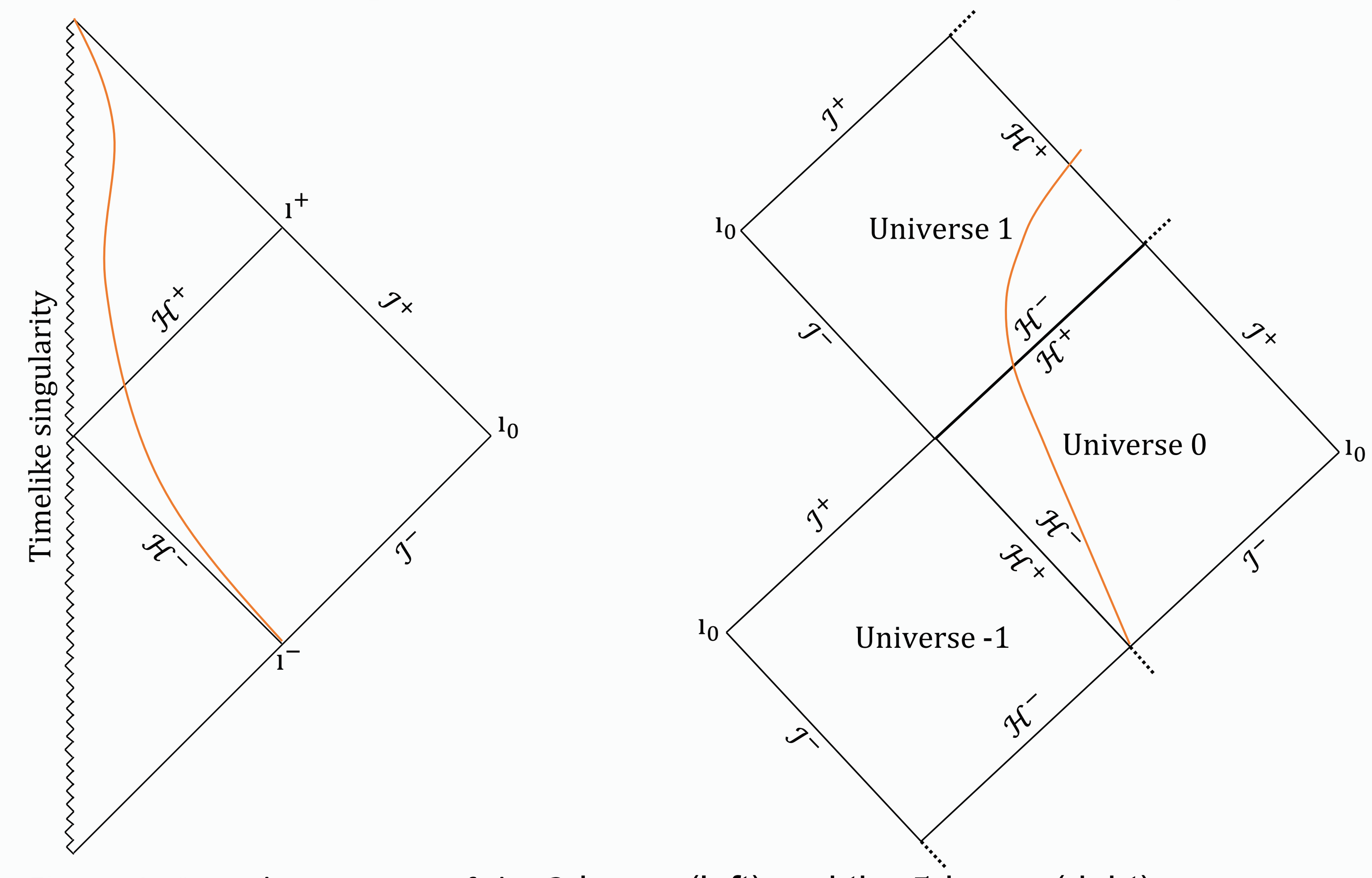

Figure 1: Causal structures of the 2-brane (left) and the 5-brane (right).

Supersymmetry

The super-Poincaré algebra gives raise to the (anti)commutators of the symmetry generators of supergravity, depending on mass $M = \sqrt{-P_A P^A}$ and charge $U = |U_{AB}|$ as shown below[4]. When the BPS bound $U \leq M$ is saturated, the state is partially supersymmetric and labelled 'extremal':

$$\{Q, Q\} = C(\Gamma^A P_A + \Gamma^{AB} U_{AB} + \Gamma^{ABCDE} V_{ABCDE}).$$

Extremal p-branes have zero surface gravity, thus may be 'stacked' and remain a solution. This is a result of the linearity of Laplace's equation – harmonic functions form a vector space.

References

[1] E. Cremmer, B. Julia, and J. Scherk. Supergravity Theory in Eleven-Dimensions. Phys. Lett. B, 76:409, 1978.
[2] M. J. Duff and K. S. Stelle. Multimembrane Solutions of D = 11 Supergravity. Phys. Lett. B, 253:113, 1991.
[3] K. S. Stelle. BPS Branes in Supergravity. In ICTP Summer School in High-energy Physics and Cosmology, 3 1998
[4] J.W. van Holten and A. van Proeyen, "N = 1 Supersymmetry Algebras in D = 2, D = 3, D = 4 mod-8," J. Phys. A15, 3763 (1982).