1-Loop Quantum Corrections in String Theory

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Introduction

AdS/CFT Correspondence

First proposed by Maldacena in 1997, this conjecture states that there exists a map between dynamics of certain string theories in anti-de Sitter (AdS) space and certain conformal field theories (CFTs) living on the conformal boundary of that space.

Wilson Loops

The Wilson loop (WL) is an important operator encapsulating all of the CFT dynamics for given gauge field *A,* defined over curve *C* as

$$W[C] = \mathrm{Tr}\ P\exp\left[\int iA_\mu d\xi^\mu\right],$$

which is path-ordered and traced over.

The VEV of this WL corresponds to the string partition function defined via the following path integral

$$\langle W\rangle = \int [dx][d\theta][dg]\ e^{-S},$$

where S is the string action, and the integral measure is over scalar fields, spinor fields and the metric tensor.

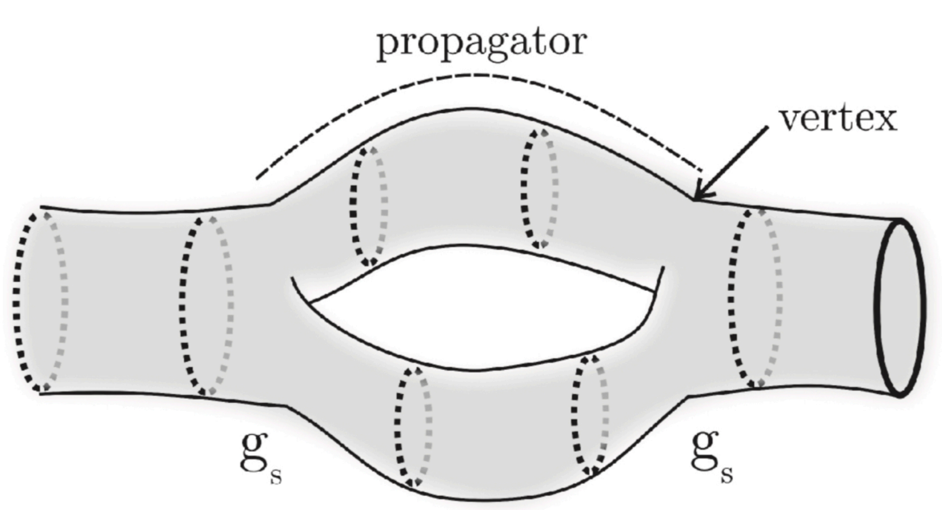

Fig.1: Example of a 1-loop string interaction [1]

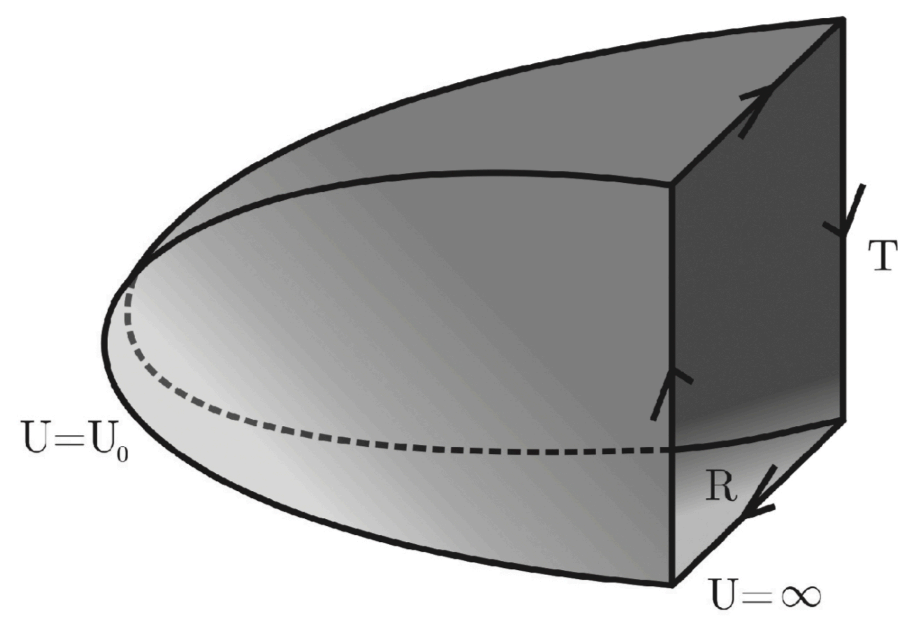

Fig.2: String minimal surface ending on a rectangular Wilson loop [1]

Aims

- To find strong coupling limit VEV of a circular WL corresponding to a string minimal surface using classical bosonic dynamics in the presence of a two-form *B*-field on a curved $\mathrm{AdS}_3 \times S^3 \times M^4$ background.
- To extend this result to second order (1-loop) quantum corrections which can test important aspects of the AdS/CFT correspondence beyond the classical level.

Classical Solution

The classical bosonic action of the string can be found as

$$S_B = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\left(\sqrt{g}g^{ij}G_{\mu\nu} + i\epsilon^{ij}B_{\mu\nu}\right)\partial_i X^\mu \partial_j X^\nu,$$

where *g* is the worldsheet metric, *G* the target metric whose AdS_3 part is

$$\mathrm{d}s^2 = z^{-2}(\mathrm{d}r^2 + r^2\mathrm{d}\phi^2 + \mathrm{d}z^2),$$

and the two-form *B*-field defined by

$$B = iqz^{-2}r\mathrm{d}r\wedge\mathrm{d}\phi.$$

To find the equations of motion of such a string ending on a circle, we utilise periodic ansatze and reduce this multidimensional problem to a one-dimensional integrable system [2].

Interestingly, we find the resulting minimal surface to retain its AdS_2 geometry even after introducing the *B*-field term. As one can deduce from the pullback metric, this extra term only changes the radius of the space

$$\mathrm{d}s^2 = \frac{1}{1-q^2}\ \frac{1}{\sinh^2\sigma}(\mathrm{d}\tau^2 + \mathrm{d}\sigma^2).$$

Finally, the zeroth order part of the action was found to be

$$S_B = -\frac{1}{\alpha'}\left(1 - q\,\mathrm{arctanh}\,q\right).$$

Quantum Corrections

The full supersymmetric Green-Schwarz action describing this particular type IIB string theory is highly non-linear, yet one can use perturbation theory.

If we covariantly expand the bosonic action and only keep the quadratic fluctuation terms, we should arrive at the following general expression consisting of the kinetic term with covariant derivatives and mass matrix term

$$S_{2\mathrm{B}} = \frac{1}{2}\int d^2\sigma\sqrt{h}\left(h^{ij}D_i\zeta^{\bar{a}}D_j\zeta^{\bar{a}} + \bar{X}_{\bar{a}\bar{b}}\zeta^{\bar{a}}\zeta^{\bar{b}}\right).$$

The fermionic part of the action to quadratic order consists of two sixteen-component Majorana-Weyl spinors, which can later be set to be equal using the kappa-symmetry

$$S_{2\mathrm{F}} = \frac{i}{2\pi\alpha'}\int d^2\sigma(\sqrt{g}g^{ij}\delta^{IJ} - \epsilon^{ij}s^{IJ})\bar{\theta}^I\rho_i D_j\theta^J.$$

Once the Lagrangian densities are brought to a simpler form, we can perform the Gaussian path integral to find the semiclassical partition function which can then be evaluated using the method of functional determinates [3].

Conclusion

- We have found the classical minimal surface ending on a circle on a $\mathrm{AdS}_3 \times S^3 \times M^4$ background with *B*-field.
- We have also outlined the procedure of obtaining the 1-loop quantum corrections to this surface which will be carried out in the near future.
- Once these corrections are calculated, it will be interesting to compare the final semiclassical partition function to the one in the absence of *B*-field.

References

1. Năstase, H. (2015) *Introduction to the AdS/CFT Correspondence*. Cambridge: Cambridge University Press.
2. Drukker, N. and Fiol, B. (2006) *On the Integrability of Wilson Loops in AdS_5×S^5: Some Periodic Ansatze.* JHEP01(2006)056.
3. Drukker, N., Gross, D.J. and Tseytlin, A.A. (2000) *Green-Schwarz String in AdS_5×S^5: Semiclassical Partition Function.* JHEP04(2000)021.