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The Physics of Crying

The *birth, life* and *death* of a falling teardrop

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(Bitmoji, 2019)

Introduction

Even the most macho of men may find themselves shedding a tear; may that be while listening to James Blunt during a recent heartbreak, watching an emotional movie or even merely cutting an onion. This article will discuss the underlying physical effects of a falling teardrop. Although on first glance this may appear to be a purely biological phenomenon, by dwelling deeper we shall discover a fascinating physical perspective into the archaic practice known as crying!

Figure 1: Barack Obama, the 44th president of the United States of America, crying *emotional* tears after the death of his grandmother, Madelyn Dunham (Cafe Arjun, 2008).

Why do we Cry?

(Biological Physics behind Crying)

cry [**krahy**]

verb (used without object), **cried, cry·ing.**

1. to utter inarticulate sounds, especially of lamentation, grief, or suffering, usually with tears.
2. to weep; shed tears, with or without sound.

crying [**krahy**-ing]

adjective

1. demanding attention or remedy; critical; severe:
   a crying evil.

(Dictionairy, 2019)

The dictionary provides a useful initial insight to the reasons for which we cry. Per the definition, to cry is 'to utter inarticulate sounds, especially of lamentation, grief, or suffering, usually with tears.' Crying is the fundamental method in which we display emotion – from the moment we are born we burst out into tears as a method of communication to our mothers of wonder, fear and life.

There are three types of tears: basal, reflex and emotional. The tears are produced by the lacrimal glands, and subsequently drained by the lacrimal sac. Basal tears are constantly in our eyes; they provide lubrication, nourishment and protection for the cornea. Reflex tears are the result of an environmental influence, such as our previous example of cutting an onion. Any potentially damaging irritant will be washed away by the surge of basal tears produced by the lacrimal gland. Emotional tears, as the name suggests, are a result of emotional stress and physical pain (Figure 1). As the limbic system and hypothalamus experience the emotional/physical stress the autonomic nervous system is activated, which causes the lacrimal gland to secrete more tears. The interesting part about emotional tears is that they contain a stress hormone

(ACTH) and a natural painkiller (Leu-enkephalin) – scientists believe this to be the reason why it sometimes feels good to shed a tear, and the reasons we feel more calm and relaxed after crying (Yuhas, 2016).

Humans are in fact the only animal to shed emotional tears when subject to intense emotions (Diamond, 2017). Researchers believe this to be an evolutionary advantage to gather social support of friends and family through displays of struggle. Furthermore, it shows signs of submissiveness to potential attackers which may result in a more empathetic outcome (Yuhas, 2016).

The Journey of the Teardrop

The Birth

Like with humans, a trickling tear has a birth. The breaking of the surface tension of the tear film located on the surface of the eye, due to excess fluids amalgamating from the lacrimal gland, causes a teardrop to be expelled.

Surface Tension

Surface Tension in a water-based medium is the result of the attraction of molecules due to hydrogen bonding. The intermolecular forces of attraction between the surrounding molecules behave differently at the surface of the liquid than deeper within it, see Figure 2. Molecules located deeper within the liquid are fully surrounded by water molecules, this will cause them to experience an equal force of attraction in all directions and therefore a net force equal to zero (cohesion). Conversely, molecules on the surface will only feel attractive forces on one side; as water molecules are attracted to each other with a far greater strength than with the surrounding air molecules – this will result in an inwards net force (adhesion). In addition, due to the lack of available water molecules, the outermost molecules on the surface will have a stronger bond of attraction with each other. These effects cause the surface to behave as if it is a stretched membrane and shrink to a minimum area due to the internal pressure – hence why certain objects, such as a pin, can 'float' on water given its higher density (Labman, 2012).

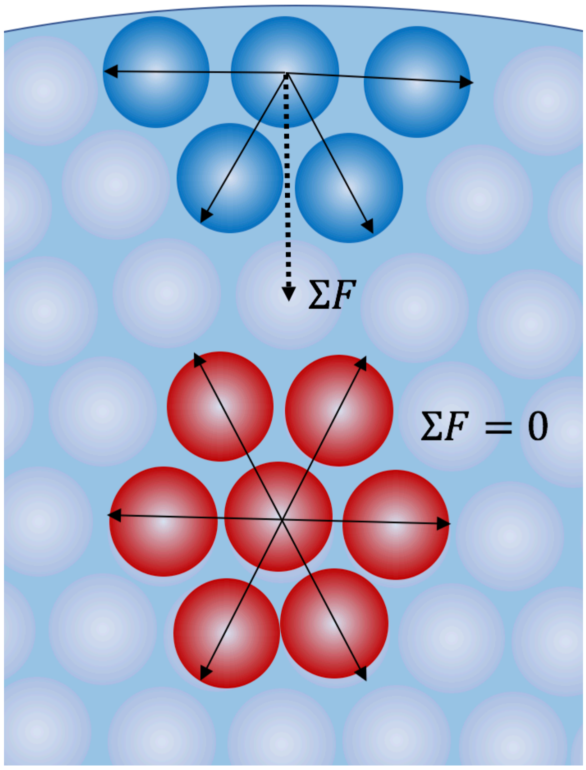


Figure 2: A visual representation of the physics behind surface tension. The blue molecules feel an attractive force towards the liquid, while the red molecules feel no net force of attraction as they are attracted equally in all directions.

Expulsion Requirements

The tear film consists of three non-distinct layers: the inner mucus layer, the middle aqueous layer, and the outer lipid layer (AAO, 2017). The tears produced by the lacrimal gland aid in increasing the volume of the aqueous layer – the incremental increase in the volume of the aqueous layer can cause a strong enough force to break the surface tension of the tear film. The average surface tension of the tear film is in fact 42-46*mN/m*, this correlates to a volume of roughly 6.5μl to break the surface tension (J. M. Tiffany, 1989). In fact, the tear expulsion volume may lie in the range of 3.4 to 10.7μl (Wolfgang Scherz, 1974).

This large range is due to numerous reasons which are mainly dependent on the host. For example, the tear film usually consists of 98.2% water and 1.8% proteins, however a variation in the amount of proteins can cause a noticeable change in the surface tension. In addition, dry eyes, conjunctivitis and other serious eye diseases further alter the value of the surface tension. The surface tension value isn't the sole determinant for the expulsion volume; many factors such as humidity, temperature, eye-size and genetics can affect it. Tears can even be emitted preemptively by blinking with enough force. This increases the chance of a lower volume tear to be excreted, as one is effectively minimizing the volume the tear film has to occupy, causing the excess liquids to be drained via the lacrimal sac, or by escaping through gaps within the eye-lid due to a pressure difference.

The Life

Figure 3: A sketch of a teary eye (Itodorovski, 2003)

No human is the same, similarly no tear is the same. A trickling tear can be defined by numerous variables, the most significant being the shape and volume of the tear, the expulsion location from the eye, and the physical properties of the host.

Shape and Volume

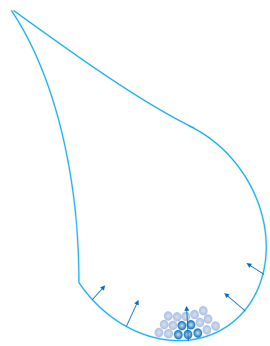


Figure 4: An illustration of the shape of a teardrop. Surface tension and gravity are responsible for the lopsided orientation.

Tears are shaped as… you guessed it – teardrops! The physics behind the iconic shape sparks a reasonable amount of interest. Due to surface tension and gravity a lopsided shape is immediately created after expulsion from the eye. An equilibrium is met in which the force of gravity experienced by the weight of the teardrop is equal and opposite to that of the force exerted by the surface tension. The elastic properties resulting from the high surface tension creates an almost sack-like capsule for the water. Imagine filling a balloon with water and hanging it from the neck – this is essentially what is happening with a teardrop. The weight of the water causes the skin of the balloon to stretch, and this results in most of the water being situated closer to the ground (lopsided shape).

The volume of the tear diminishes with distance travelled as it lubricates the skin on the face with its composition. This allows for quick speed in its descent down the face, due to a reduced magnitude in the opposing frictional force – hence a minimized deceleration. One can model the volume loss with a differential equation.

If a secondary tear is emitted, it can prolong its distance travelled by using the lubrication provided by the initial tear; however, if its volume (mass) is greater than the initial tear - due to its excess momentum - it may branch of the path at a curve or facial contour and create its own route to the jaw. Another way of visualising this is by considering the width of the lubrication trail – it diminishes with distance. This means a greater-volume secondary tear will struggle to stick on the path as it will have less of an influence on the teardrops trajectory. In fact, this is generally the case even if the volume is lower than the initial tear. The secondary tear is losing less volume with respect to distance than the initial tear (as the path the tear is travelling on has already been lubricated, it is travelling on a surface with a lower coefficient of friction - so less energy is being lost to frictional work), this means that at a certain point in its journey the secondary tear could have a greater volume than the larger initial tear had at that same point. Reversing the argument, it is easy to see how a secondary tear with a much lower volume will in fact follow the pre-lubricated path. The mechanical analogy of a potential well can be implemented here, with respect to the latter case, the tear will 'lack energy' to escape the route.

Expulsion Locations from the eye

For a long time, experts believed that tears were only expelled from the corners of the eye – partly due to the greater thickness of the tear film caused by the meniscus. However, research conducted by Dr Richard Braun at the University of Delaware proved otherwise. By creating a mathematical model of how fluid travel in the eye, and accounting for gravity, surface tension and viscosity, researchers found that tears can in fact escape from the centre of the eye. This phenomenon occurs if there is greater than normal amount of fluids excreted from the lacrimal ground at a slower rate of production. For this to occur there must be a break up in the surface tension in the centre of the eye (Braun, 2015).

Physical Properties of the Host

The properties of the host heavily impact the life of the tear. The speed at which the tear travels on the face is largely dependent on the distance between the eye and the point of departure from the face. Assuming the ideal case, all the gravitational potential energy will be converted into kinetic energy. This means that the speed of the tear will be proportional to the root of the distance travelled. However, we don't live in an ideal world, in fact the human face contains hair, stubbles, make-up, sweat, spots… a large friction-based obstacle course for the tear to travel! This will drain the energy of the tear, and reduce its speed appropriately as:

Gravitational Potential Energy =
Kinetic Energy + Work Done by Friction.

The Death

The tear can end its life in one of two ways – diminishing its volume to zero at some point on the face of the host, or falling of the face and plummeting to the ground! We will come to the see that the latter sparks some very interesting physics!

Raindrop Approximation

The analysis of the latter termination of the teardrop is analogous to the analysis of a falling raindrop under appropriate parametrisation.

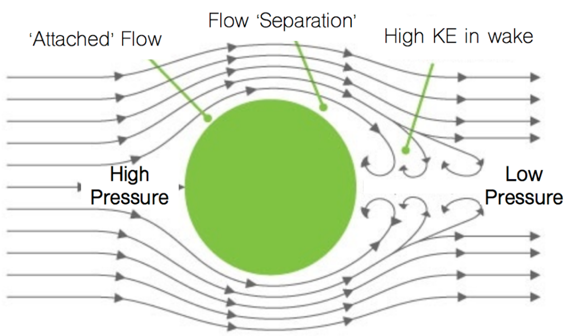


Figure 5: An illustration of the concept of pressure drag on a sphere. The random turning of the eddies at the wake are of lower pressure than the front side of the sphere – this causes the raindrop to become slightly flattened (University of Waikato, 2011).

A common misconception in the public's mind is that raindrops are shaped as teardrops. This is in fact false! The shape of a raindrop is dependent on the size of said raindrop. For raindrops $< 1mm$, surface tension, cohesion and adhesion cause the drop to exhibit a spherical shape. For drops of greater diameter, the shape still tends towards a spherical orientation, however due to the greater influence of air pushing against it as it falls it becomes slightly flattened (oblate spheroid) (NASA, 2015). This can be explained in terms of a pressure difference – air flowing against the raindrop creates a backwards force called pressure drag due to a greater pressure at the bottom of the raindrop than at the top, see Figure 5.

The physics of a *free*-falling teardrop is very like that of a raindrop with appropriate diametrical constrictions, albeit the slight deviation of a few variables (mainly the different compositions of the two). We can conclude that a tear of diameter $< 1mm$ will occupy a spherical shape upon descent to the ground, however a slightly larger tear, will fall with the shape of an oblate spheroid.

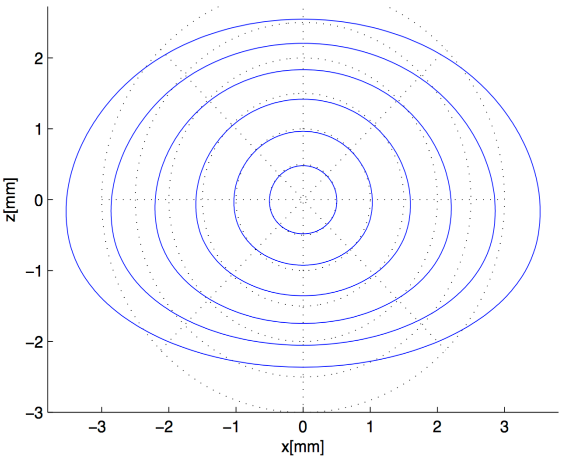
Mathematical Modelling

It is in fact possible to mathematically model and derive the shape of a falling raindrop, and hence a teardrop. There are five factors which are generally considered to affect the shape of a falling teardrop: surface tension, hydrostatic pressure, aerodynamic pressure, internal circulation and electric stress. The mathematical derivation, which is far beyond the scope of this article, involves the exploitation of the Laplace-Young equation for pressure balance

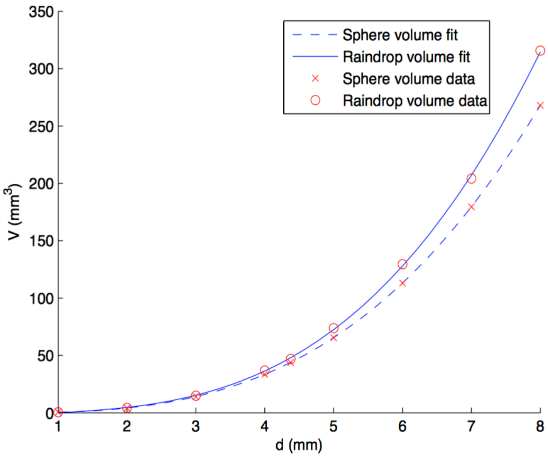
$$\Delta p = \gamma\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

where Δp is the Laplace Pressure, γ is the surface tension, and R_1 and R_2 are the radii of curvature.

Using a tangential coordinate system one can integrate this equation from 0 to π and the shape can be determined (Lim, 2006). Although this sounds rather simple, there are numerous variables and coefficients to be accounted for which makes this equation extremely tedious and difficult to solve. With the aid of a computational program calculating millions of points, the results depicted in Figure 6 are produced for values of the diameter of the teardrop in the range of $[1,6]mm$. Figure 6a visualises the greater influence of the pressure drag at greater values of the diameter; this is best illustrated in Figure 6b where we can see the deviation in the model with respect to the idealised spherical model.



(a)



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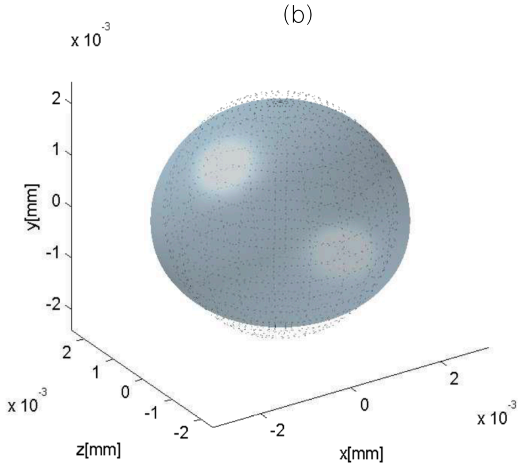

(c)

Figure 6: A mathematical model of the shape of a falling raindrop derived by Brian Lim (Lim, 2006). (a) The drop shape from diameters in the range of $[1,6]mm$ with the idealised spherical model illustrated with dashed lines. (b) A graphical depiction of the variation between the model fit and the spherical fit. (c) A 3D illustration of a drop with diameter 4.38mm.

Mathematical models of teardrops may not be so important and useful; however, the mathematical modelling of raindrops are! Knowing the shapes of raindrops on satellites can aid us in knowing the intensity of rain and even clouds from satellite images (Lim, 2006).

Volume Loss via Evaporation

As the tear makes its descent to the ground it also losses volume via diffusion into the surrounding air. Using a first order differential equation, we can calculate the rate of evaporation in terms of the diameter of the teardrop:

$$\frac{dD}{dt} = \left(\frac{M_V}{M_A}\right)\left(\frac{D_V}{D_a}\right)\left(\frac{\rho_a}{\rho_w}\right)\left(\frac{\Delta P}{P_a}\right)k$$

where D is the diameter of the teardrop in μm, M_V is molecular mass of the diffusing vapour, M_A is the molecular mass of air, ρ_a and ρ_w are the densities of air and water, ΔP is the difference in pressure between the air and the surface of the teardrop, P_a is the partial pressure of air and k is a constant (dependent on the Reynolds and Schmidt number) (Thorel, 2015).

Eulogy

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With our beloved tear having passed away we must look back on its life. From breaking the surface tension of the tear film, trickling down the friction obstacle course that is the hosts face, and plummeting down to the ground, the tear experienced many mathematical and physical phenomena's during its short life. Although lots more could have been explored, we can appreciate the interesting underlying physics in a relatively obtuse location. So, next time you have a cry, don't focus on the negatives, focus on your tears, trickling down your face, and the fascinating physics involved!

Acknowledgements

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