Self-organisation out of disorder – L-H transition

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Thanks: James Heseltine, Schuyler Nicholson, Hanli Liu, Rainer Hollerbach

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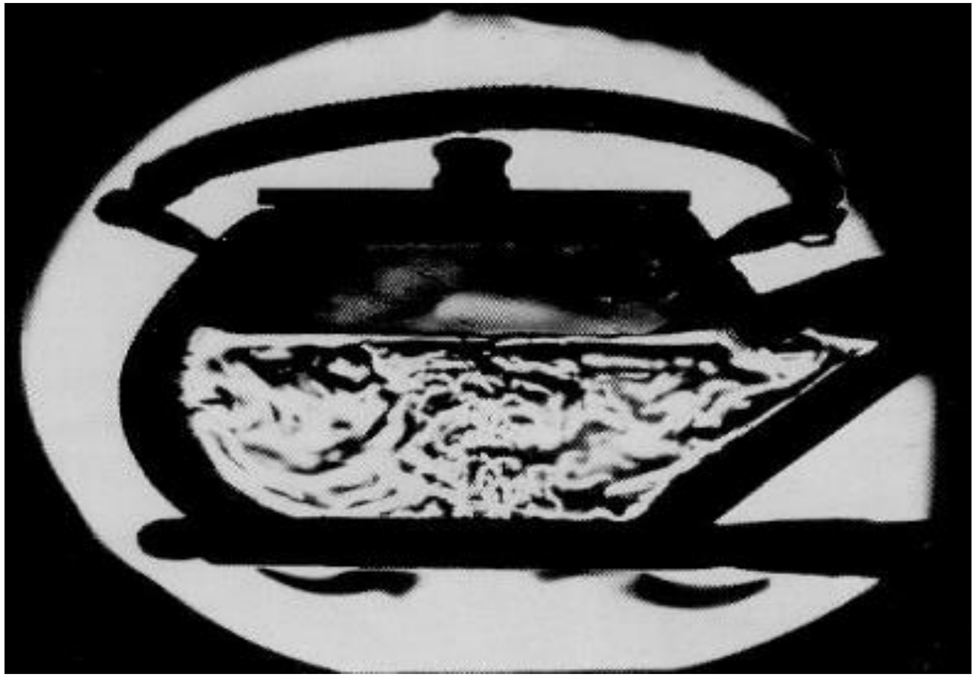
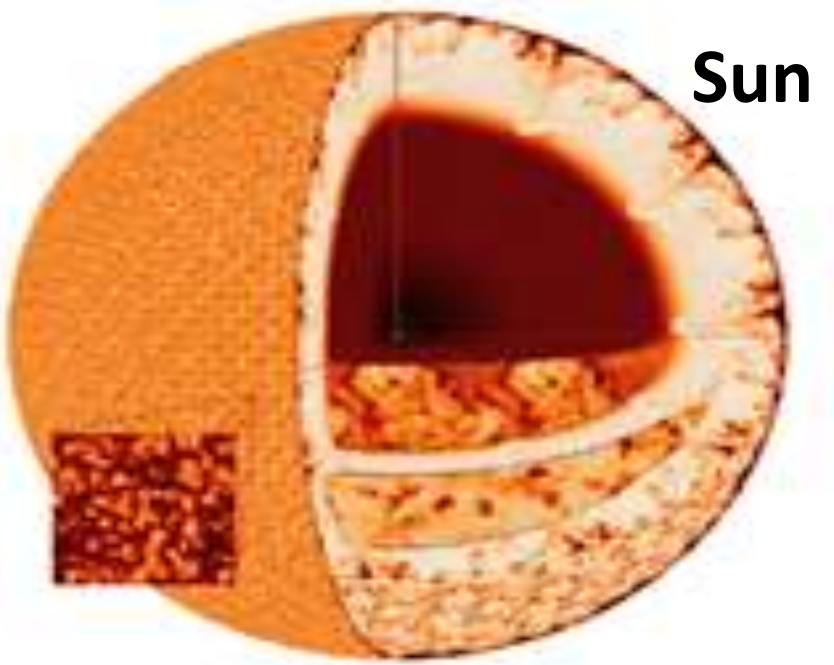
Outline

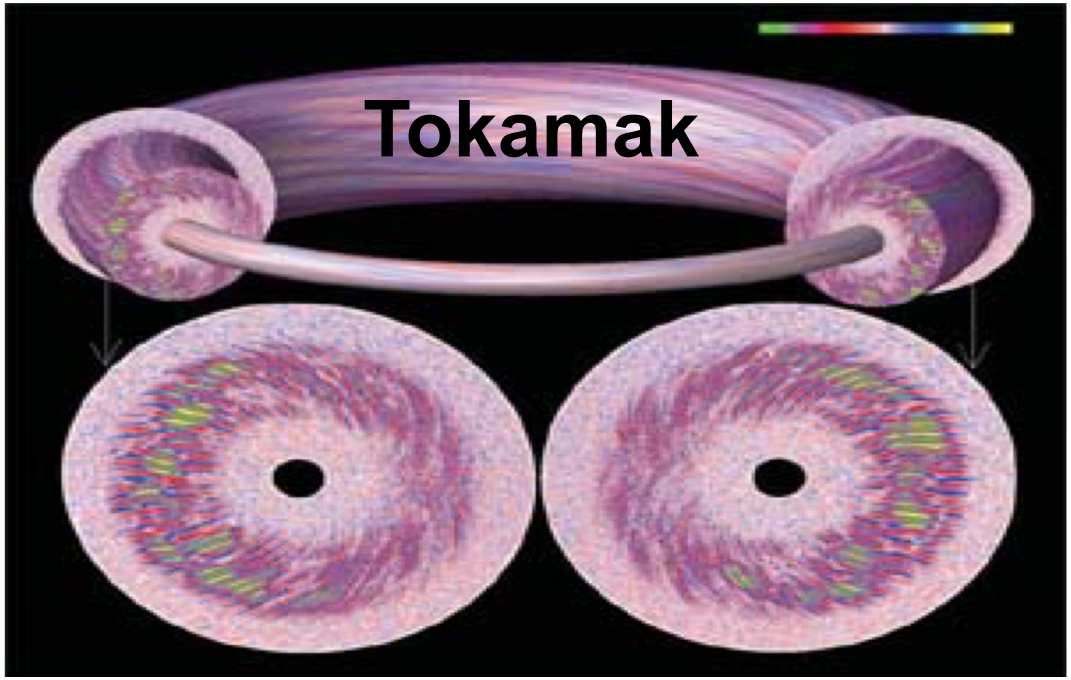
1. Overview
   1.1 Order/Structure out of Disorder/Complexity
   1.2 Self-organisation
2. The Low-to-High confinement (L-H) transition
   2.1 Deterministic model
   2.2 Stochastic model (information length)
3. Information length applied to
   3.1 Music
   3.2 Global circulations
4. Conclusion

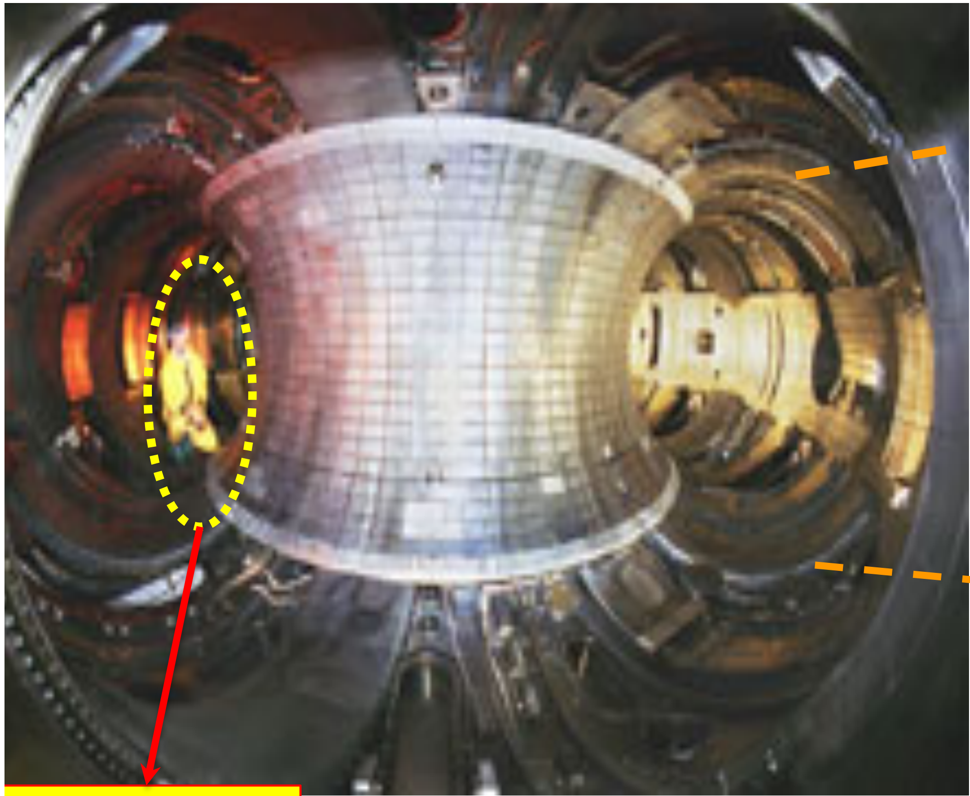
1. Overview
    1.1 Complexity/disorder and structure/order
    1.2 Self-organisation

1.1 Complexity/disorder (turbulence)

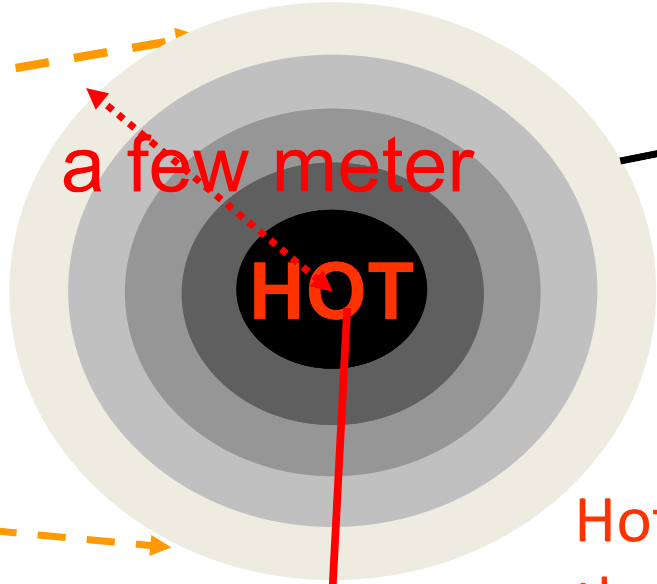








Yasmin
Andrew

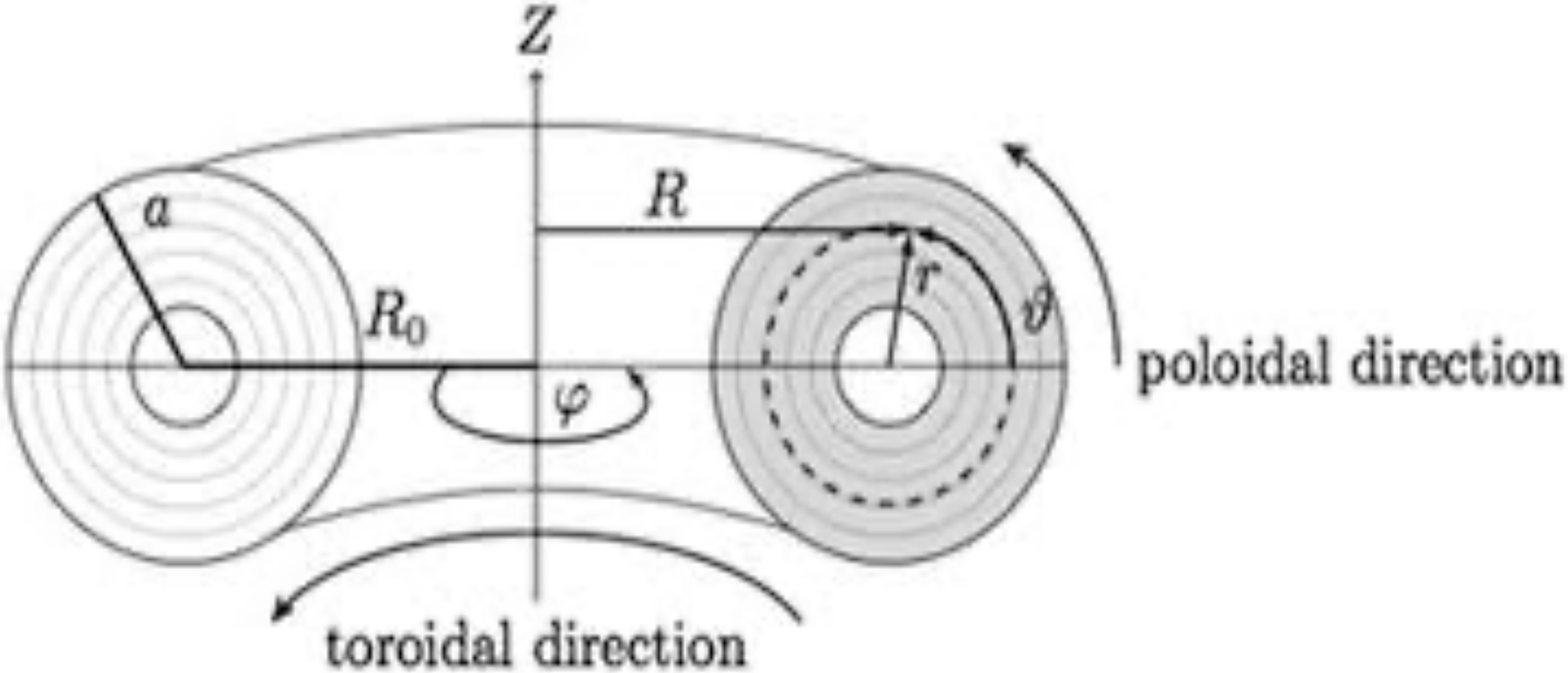


COLD

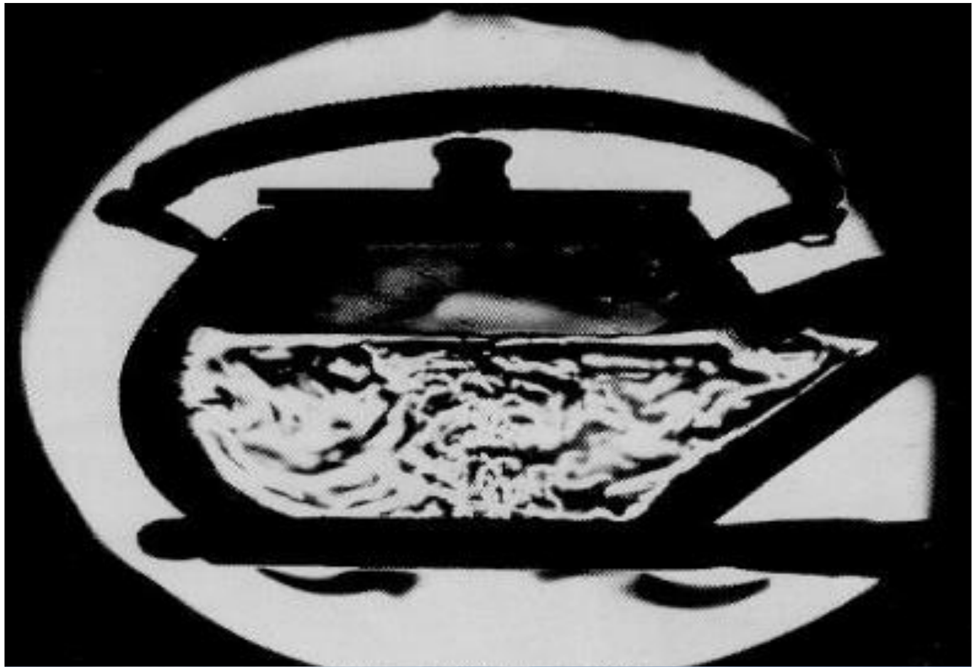
~300K

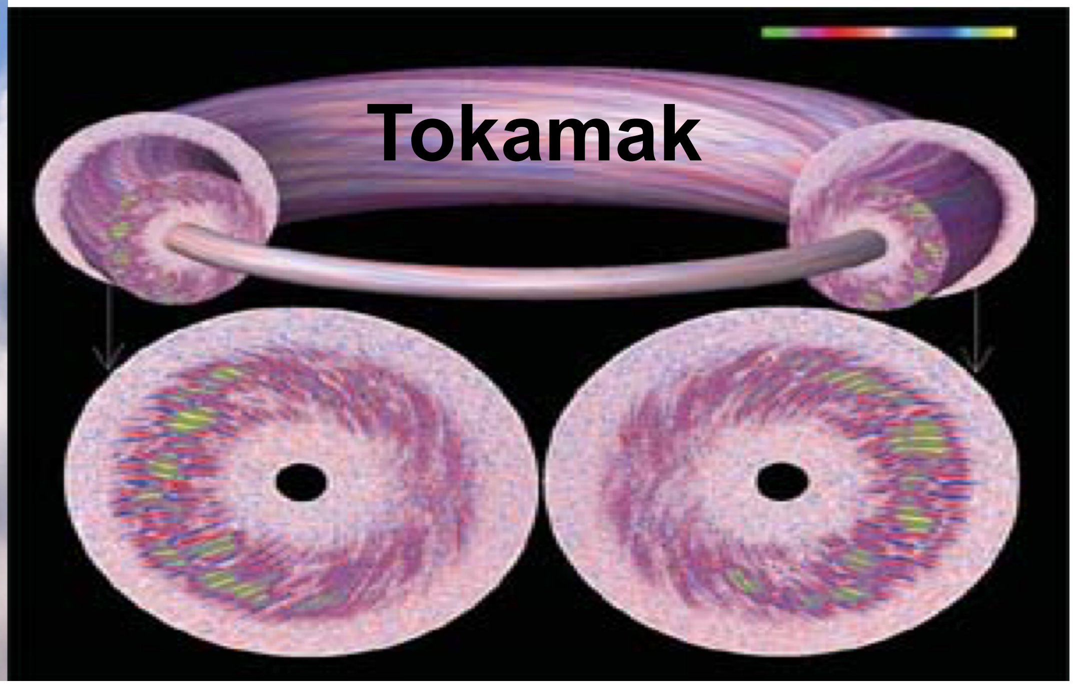
~10^8 K

Hotter than the centre of the Sun!



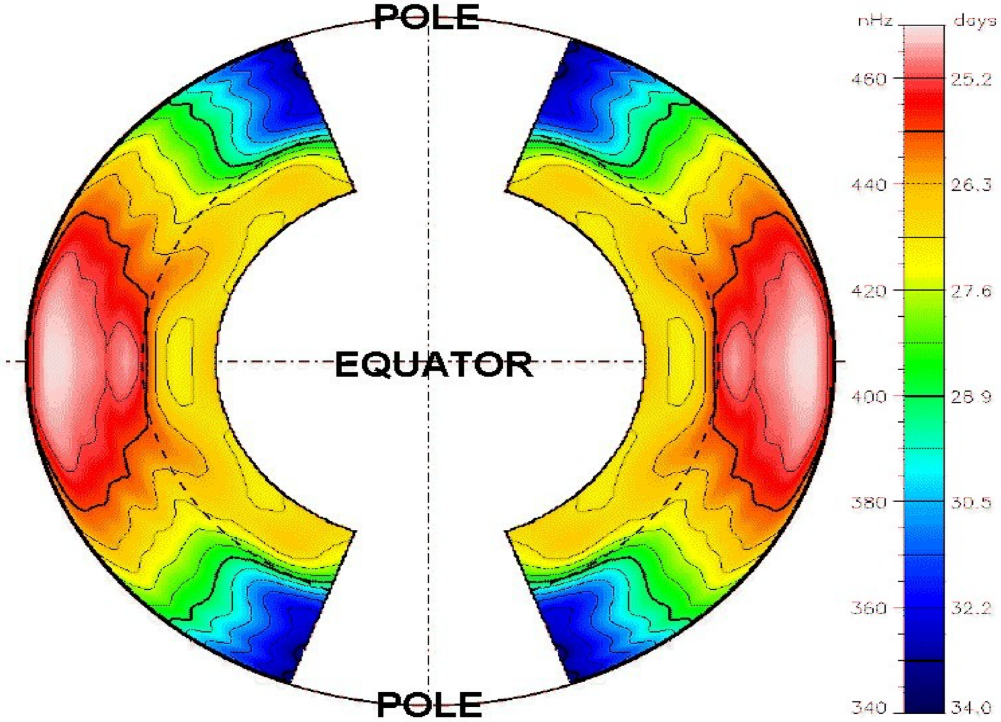
1.1 Complexity/disorder (turbulence)





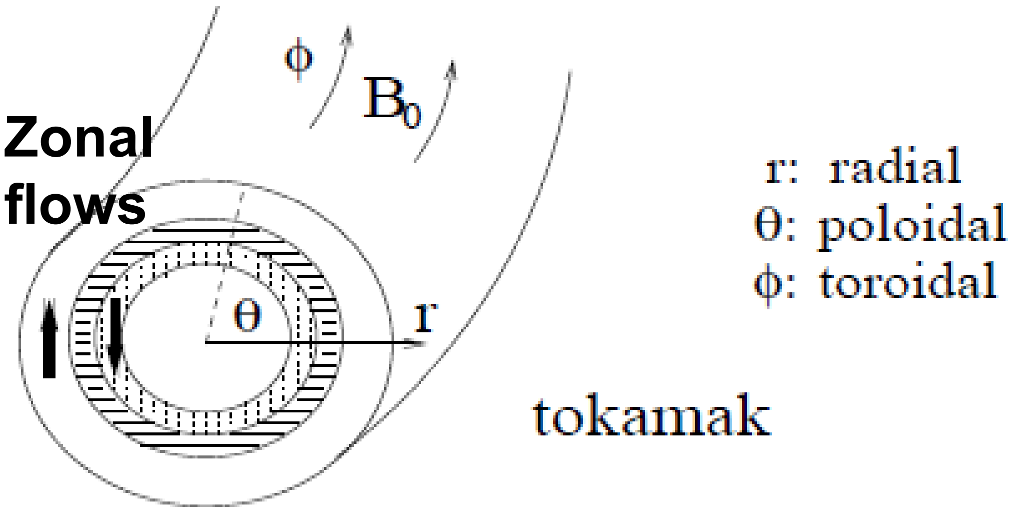
Order/Structure (shear, zonal flows)

Jovian zonal winds



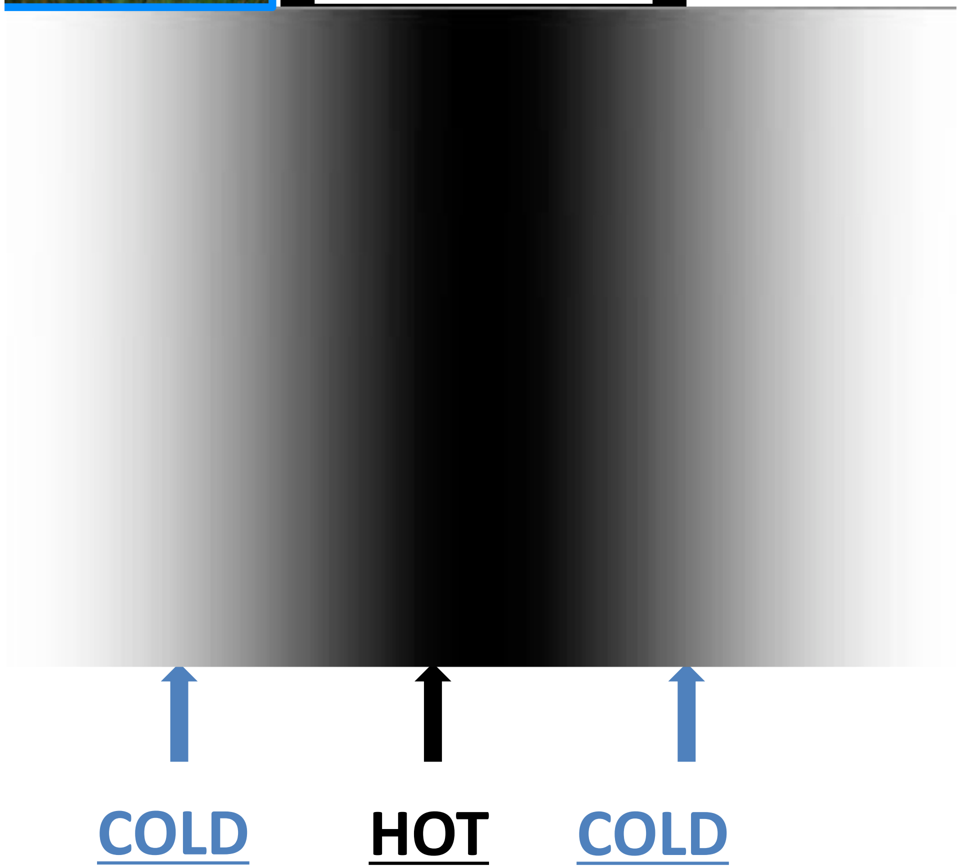
Solar differential rotation

Zonal flows

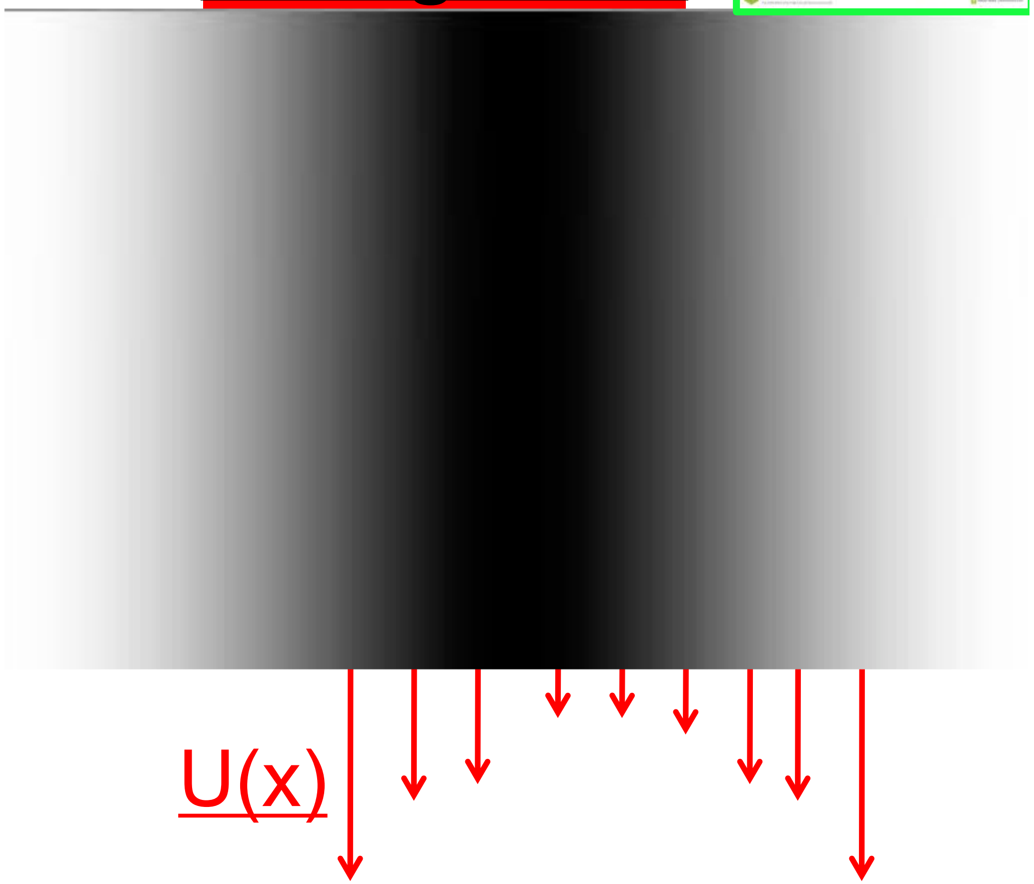


Reduction in turbulent mixing by shear flow

No shear



Strong shear



Turbulence enhances mixing but shear flow eats up turbulence, reducing turbulent mixing rate.

[Kim & Dubrulle 01,02; Kim 04,05,06,07; Kim et al 03,04,05,06; Leprovost & Kim 07,08,09,11; Numerical: Newton & Kim 08,09,11; Courvousier & Kim 09; Sood, Hollerbach & Kim 16, etc.]

Turbulence grows from instability

Shear flow grows from turbulence

Shear flow eats up turbulence

Less turbulence

Less shear flow

More turbulence
More shear flow
Less turbulence
Less shear flow

Rabbits grow by eating grasses

Lions grow by eating rabbits

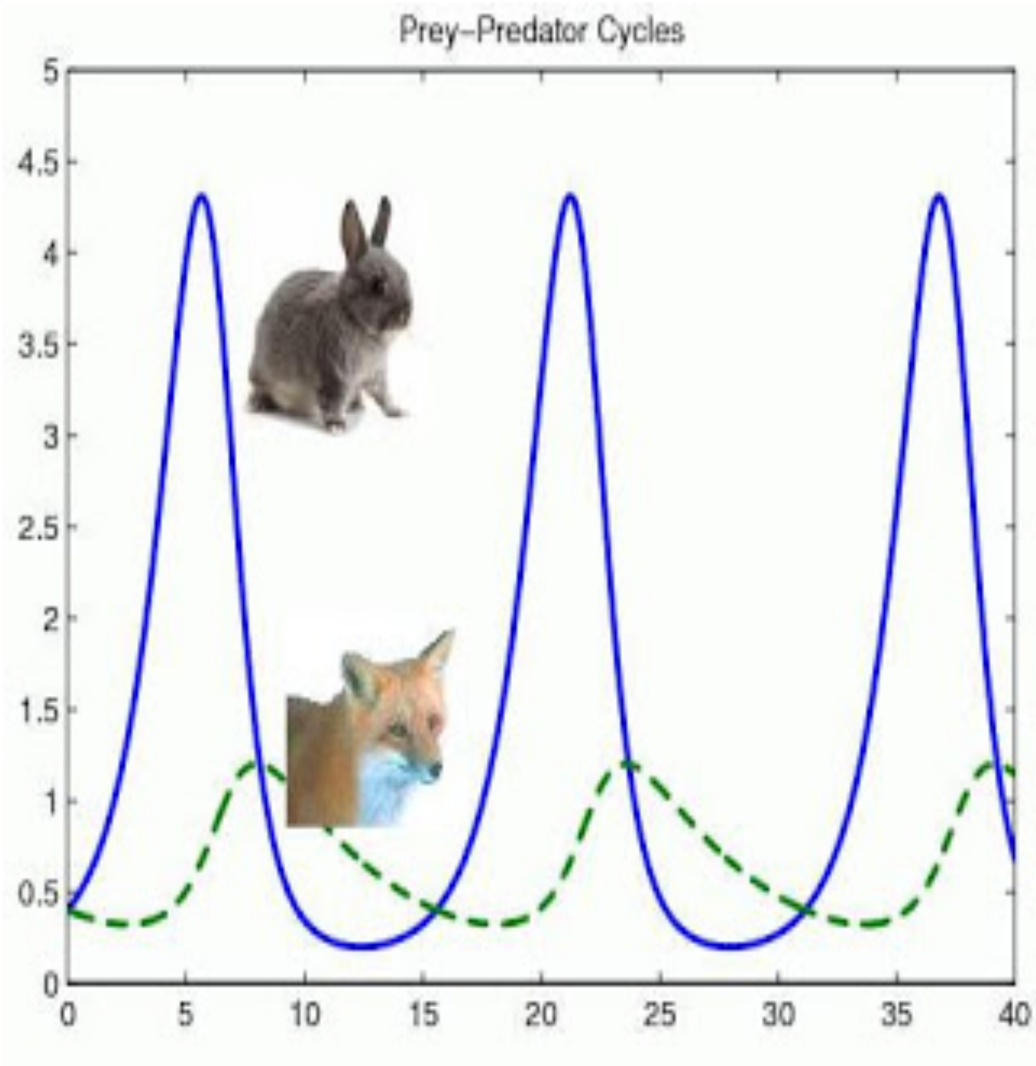
Lions eat up rabbits

Less rabbits

Less Lions

More rabbits
More lions
Less rabbits
Less lions

Prey-Predator



Predator:
Lion (zonal flow)

Prey:
Rabbit (turbulence)

2. The Low-to-High confinement (L-H) transition

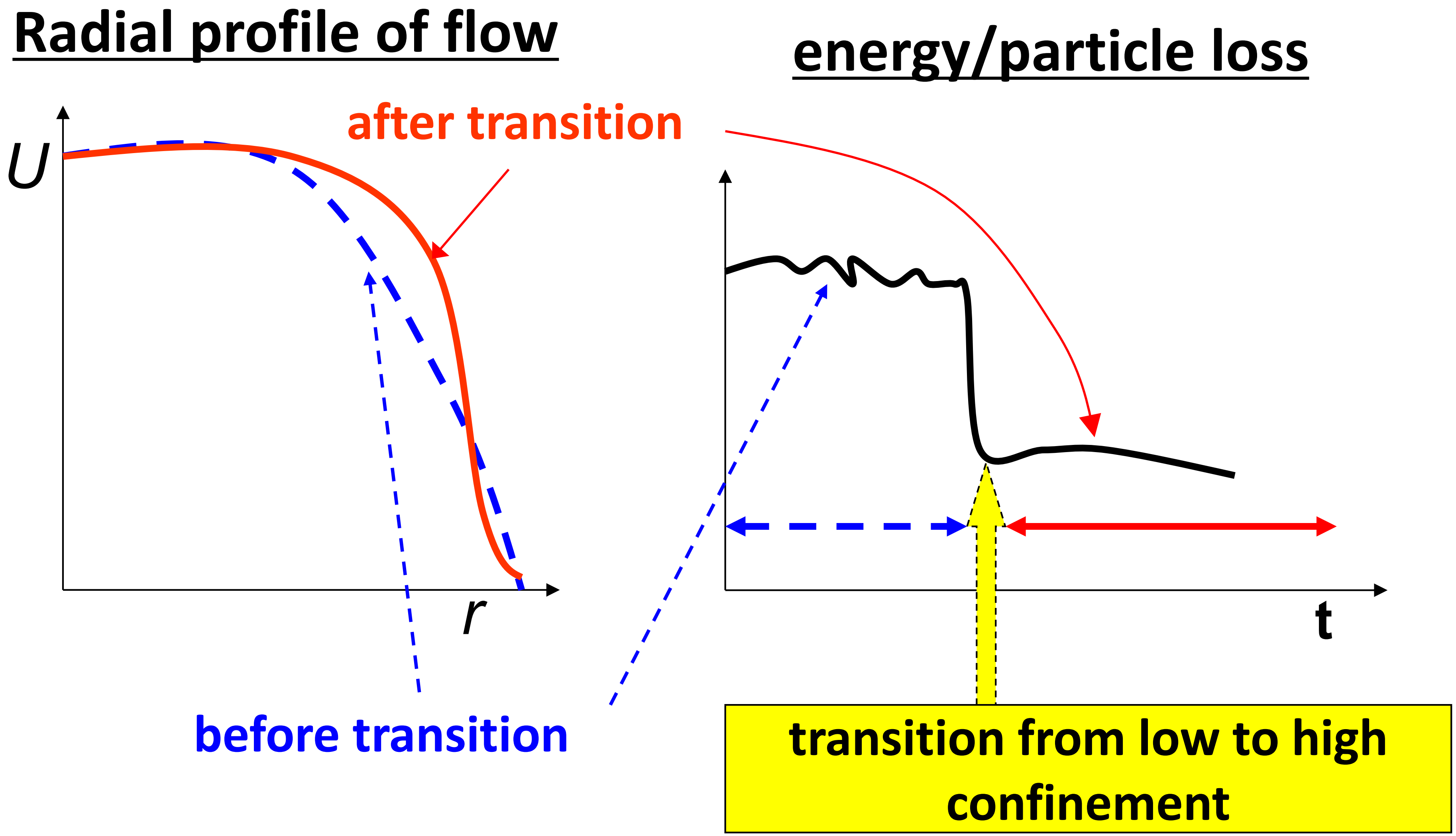
2.1 Deterministic model

2.2 Stochastic model

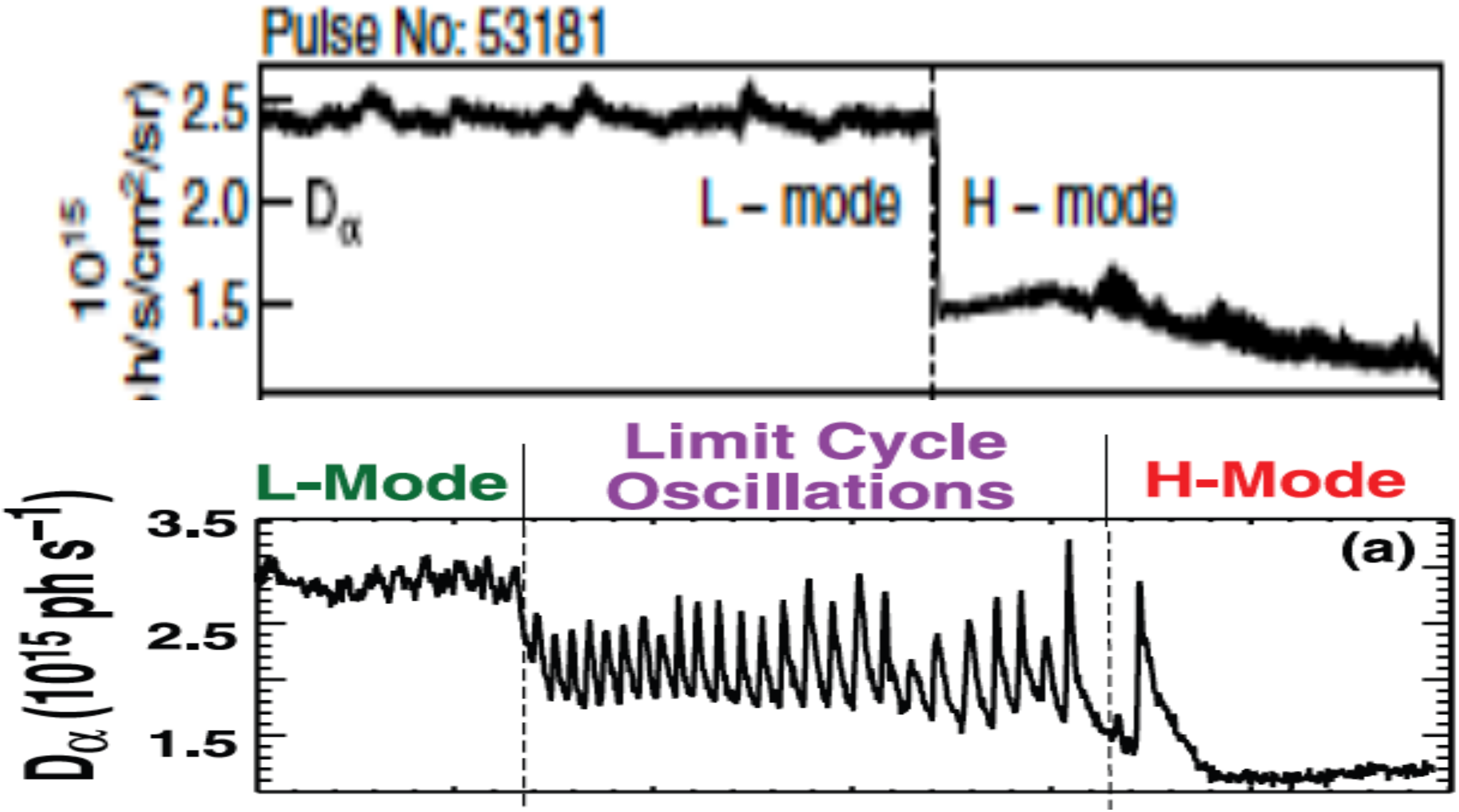
2. Low-to-high confinement (L-H) transition

⇨ **Spontaneous formation of shear flow**

⇨ **Improvement of confinement!**



JET: Andrew et al, 2006, PPCF 48, 479



DII-D: Schmitz et al, 2017, Nuclear Fusion 57, 025005

2.1 Deterministic model (Kim & Diamond, PRL 2003)

$$\partial_t \mathcal{E} = \mathcal{E}\mathcal{N} - a_1 \mathcal{E}^2 - a_2 V^2 \mathcal{E} - a_3 V_{\mathrm{ZF}}^2 \mathcal{E},$$

$$\partial_t V_{\mathrm{ZF}} = b_1 \frac{\mathcal{E} V_{\mathrm{ZF}}}{1 + b_2 V^2} - b_3 V_{\mathrm{ZF}},$$

$$\partial_t \mathcal{N} = -c_1 \mathcal{E}\mathcal{N} - c_2 \mathcal{N} + Q.$$

N: Temperature/density gradient (rabbit food: grasses)

Q: External heating (water/sunlight)

$\mathcal{E}$: Turbulence (prey: rabbits)

V_{ZF}: zonal flows (predator: lions)

$V = dN^2$: mean flows (super predator)

a_i, b_i, c_i, d: constant model parameters

From Kim & Diamond PRL 03

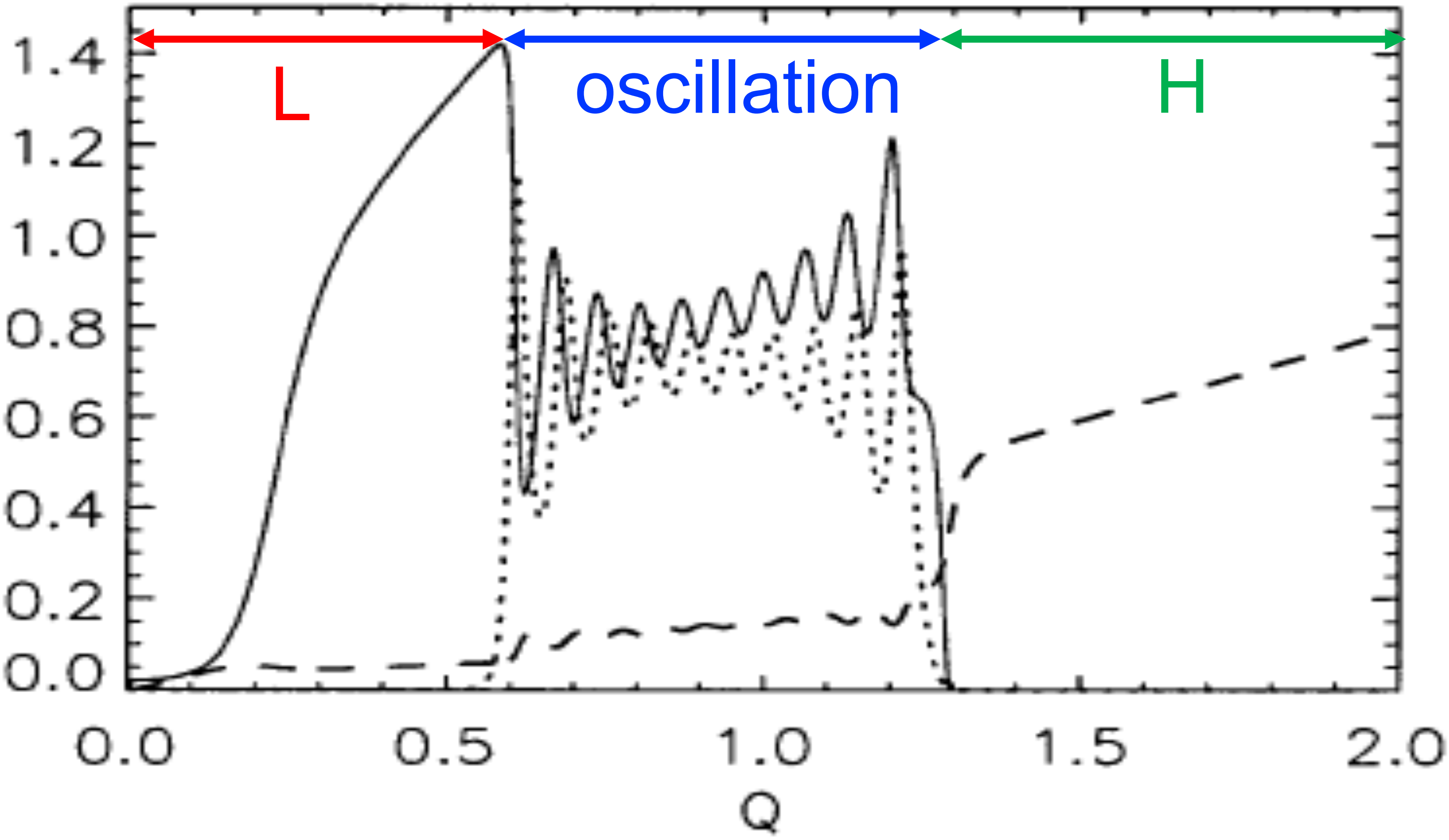


FIG. 1. Evolution of $\mathcal{E}$ (solid line), V_{ZF} (dotted line), and $\mathcal{N}/5$ (dashed line) as a function of input power $Q = 0.01t$. Parameter values are $a_1 = 0.2$, $a_2 = a_3 = 0.7$, $b_1 = 1.5$, $b_2 = b_3 = 1$, $c_1 = 1$, $c_2 = 0.5$, and $d = 1$.

2.2 Stochastic model (Kim & Hollerbach, PRL submitted)

$$\frac{dx}{dt} = f + \xi, \quad f = \frac{1}{2}\left[N - a_1 x^2 - a_2 V^2 - a_3 v^2\right] x,$$

$$\frac{dv}{dt} = g + \eta, \quad g = \frac{b_1 x^2 v}{1 + b_2 V^2} - b_3 v,$$

$$x = \pm\sqrt{\mathcal{E}}, \quad v = V_{ZF}, \quad N \sim \frac{Q}{c_1 x^2 + c_2}$$

$$\frac{\partial N}{\partial t} = 0$$

N: Temperature (rabbit food: grasses)

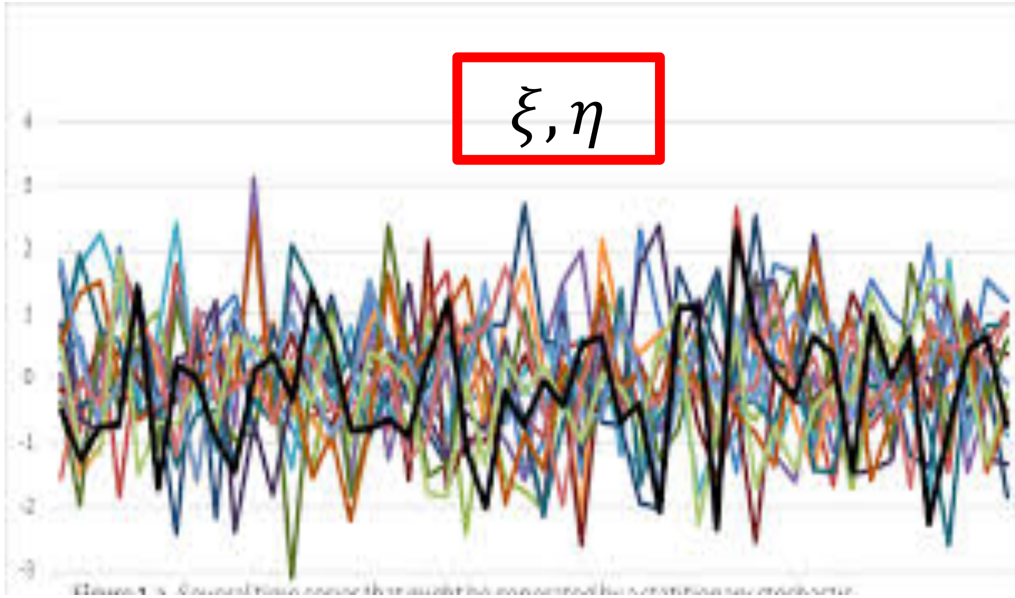
Q: External heating (water/sunlight)

$\mathcal{E}=x^2$: Turbulence (prey: rabbits)

$v = V_{ZF}$: zonal flows (predator: lions)

$V = dN^2$: mean flows (super predator)

a_i, b_i, c_i, d: constant model parameters



ξ and η are two independent Gaussian noise with a short correlation time

$$\langle \xi(t)\xi(t') \rangle = 2D_x \delta(t-t'),\ \langle \eta(t)\eta(t') \rangle = 2D_v \delta(t-t')$$

$$\langle \xi(t)\eta(t') \rangle = 0, \quad \langle \xi \rangle = \langle \eta \rangle = 0$$

Fokker-Planck equation for p(x,v,t)

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial v}(g\,p) - \frac{\partial}{\partial x}(f\,p) + D_x \frac{\partial^2 p}{\partial x^2} + D_v \frac{\partial^2 p}{\partial v^2}$$

Recall: Gaussian PDF

$$p(x,t) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{|x-\mu|^2}{2\sigma^2}\right) = \sqrt{\frac{\beta}{\pi}} \exp\left(-\beta |x-\mu|^2\right)$$

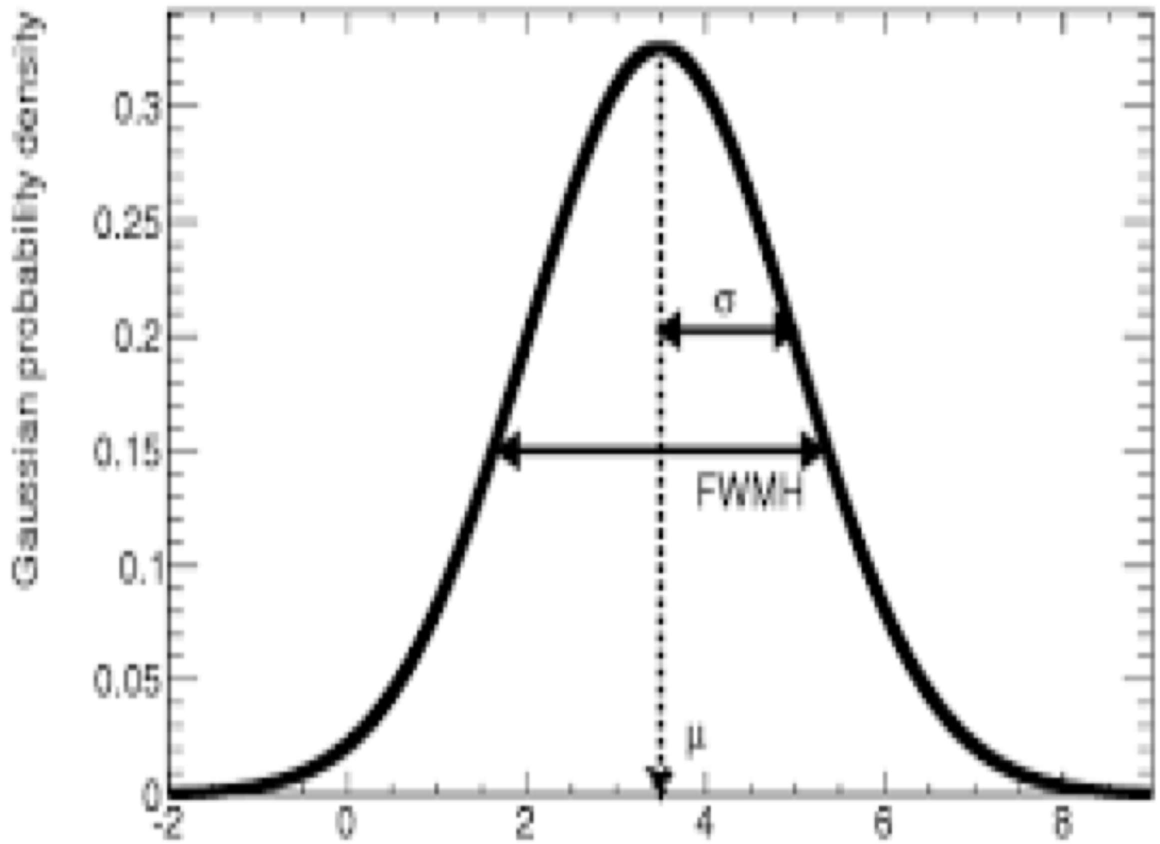
$\lambda^i = (\sigma, \mu)$ parameter

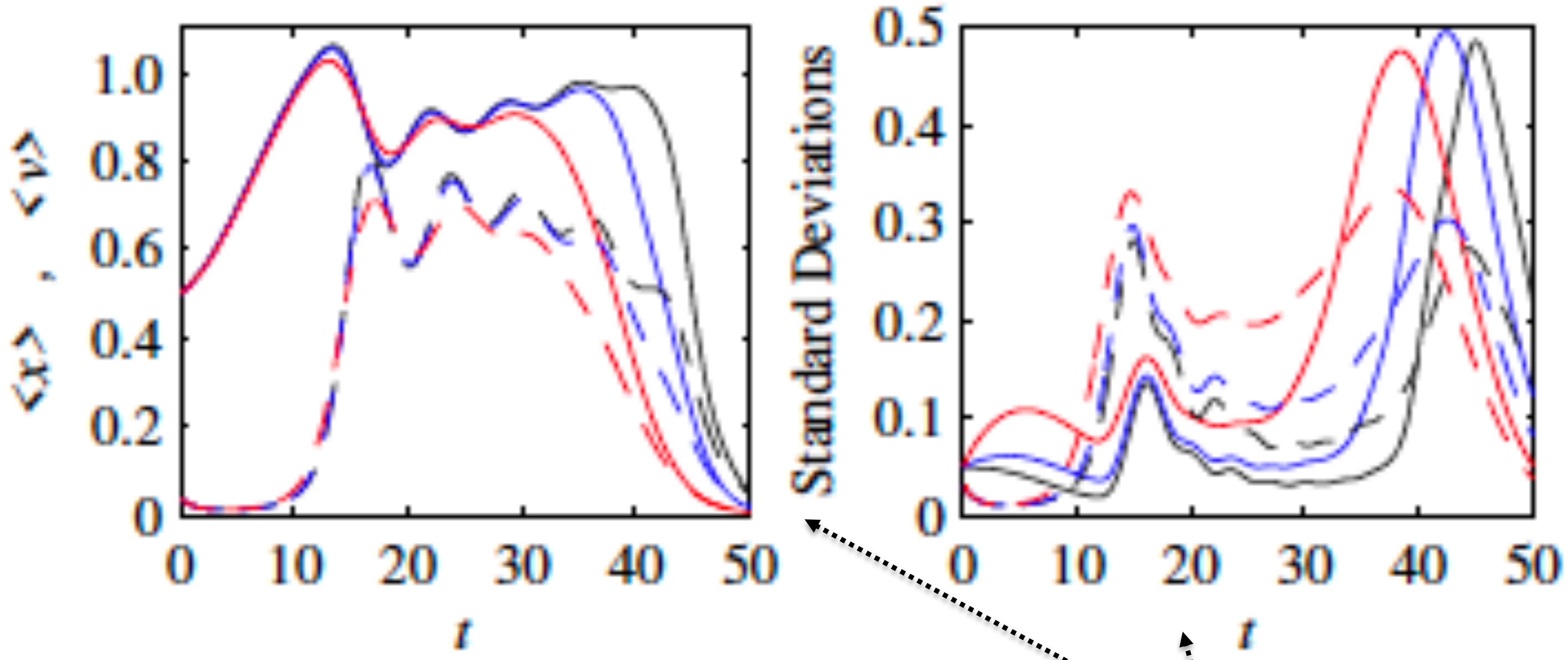
μ = mean value

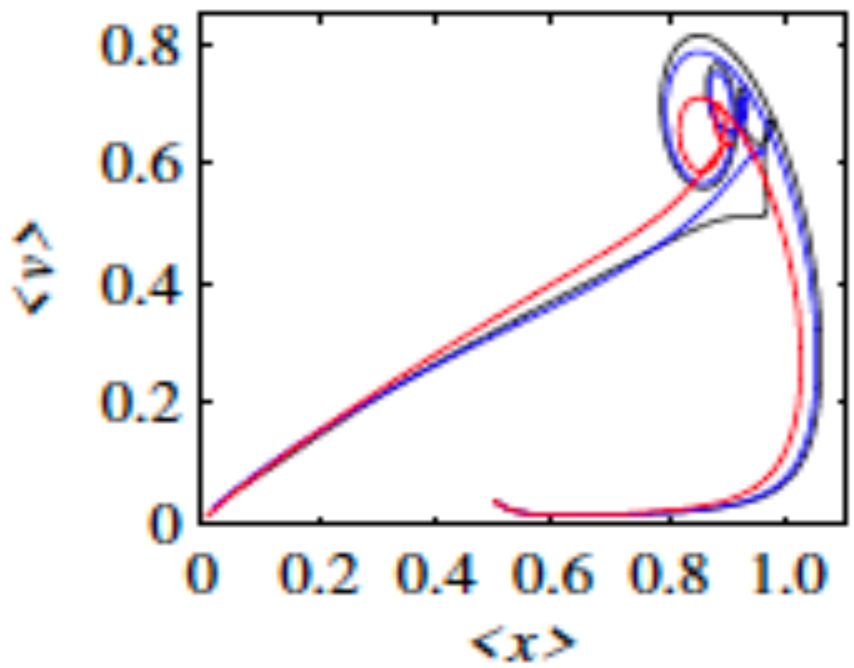
σ = standard deviation

$\beta = \dfrac{1}{2\sigma^2}$

= inverse temperature





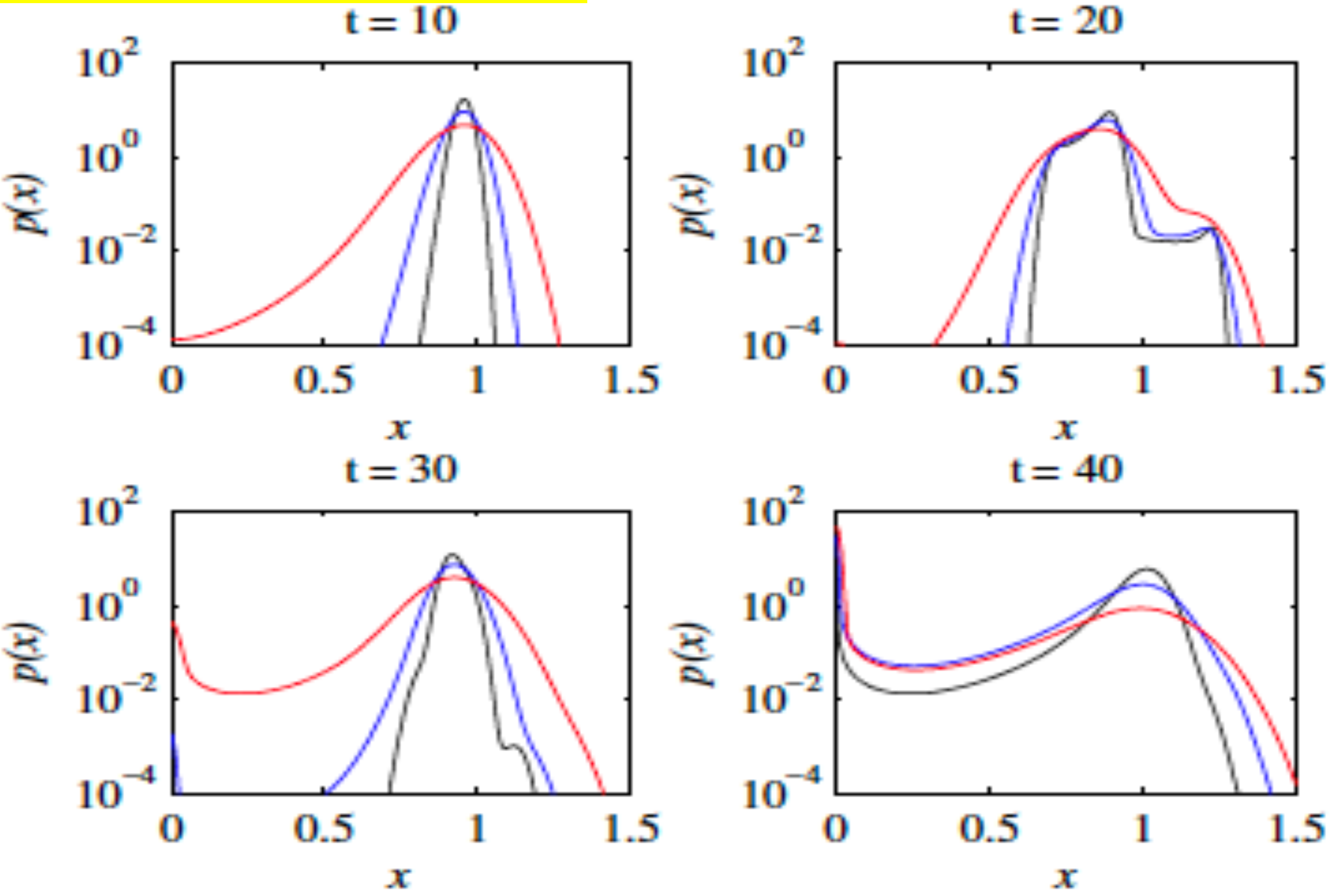


x: solid line (turbulence)
v: dashed line (zonal flow)

<*x*>: mean value of x
<*v*>: mean value of v

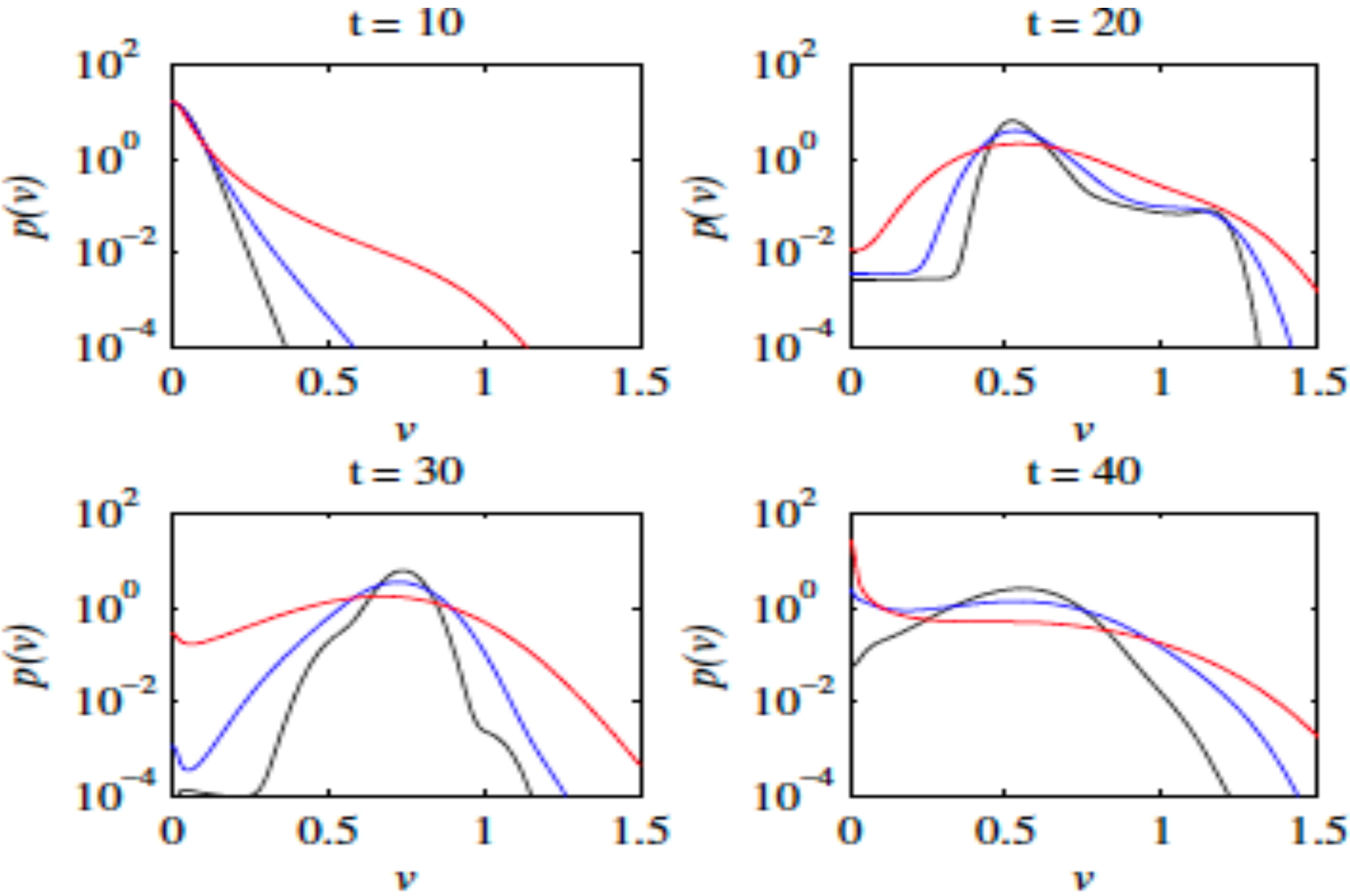
$$p(x,t) = \int dv\, p(x,v,t)$$

x: turbulence, *v*: zonal flow

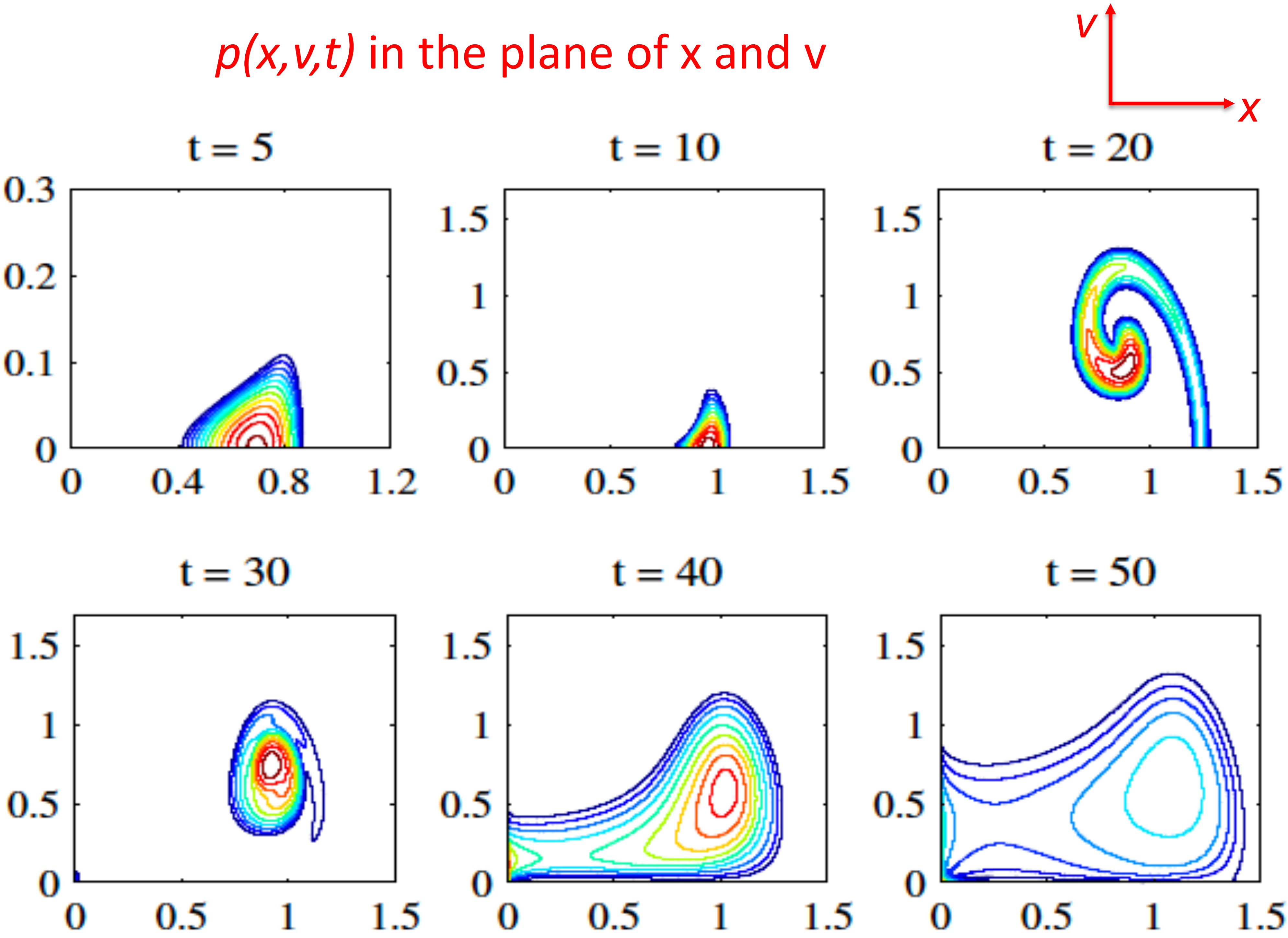


$$p(v,t) = \int dx\, p(x,v,t).$$

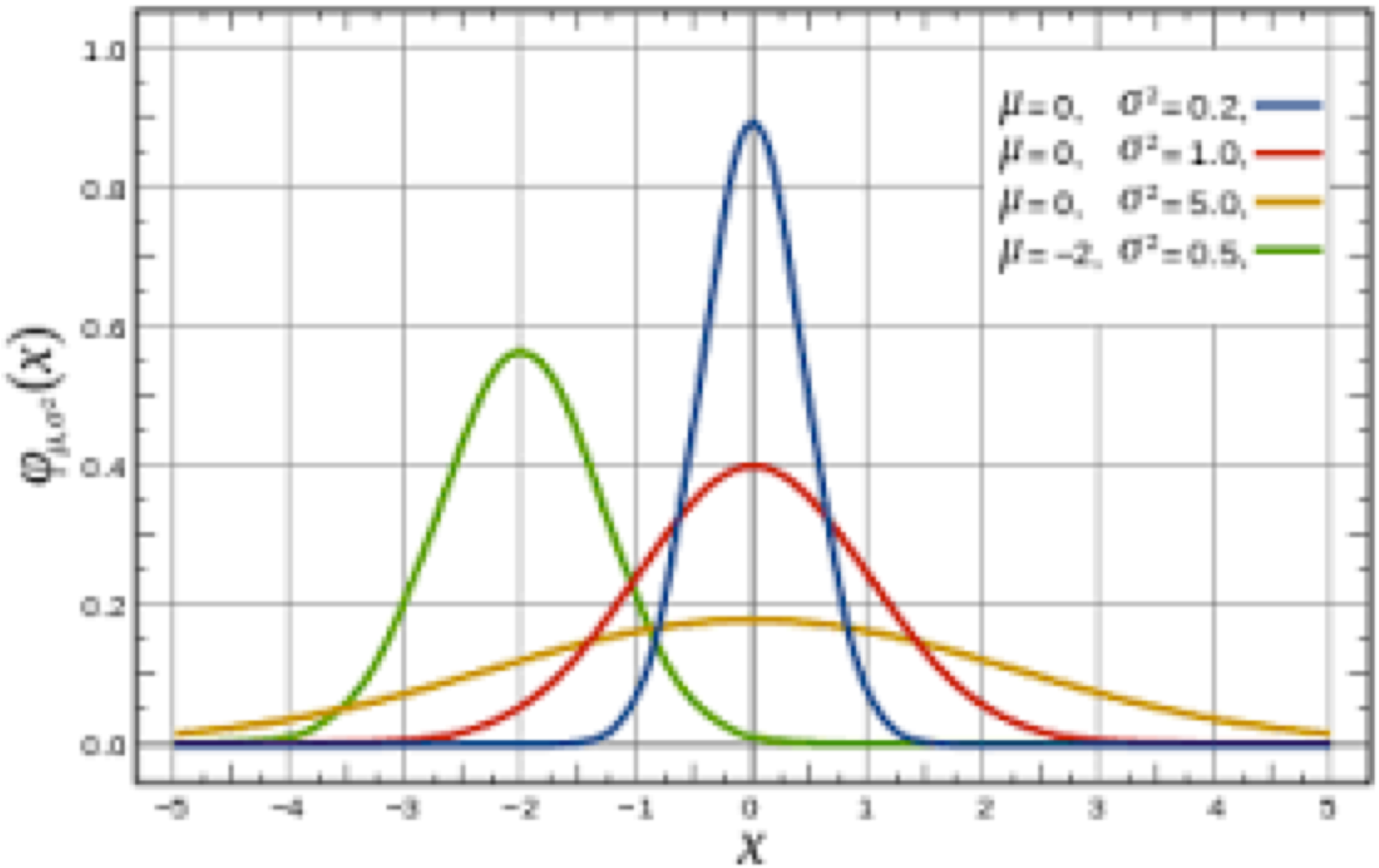
x: turbulence, *v*: zonal flow



p(x,v,t) in the plane of x and v



Information length

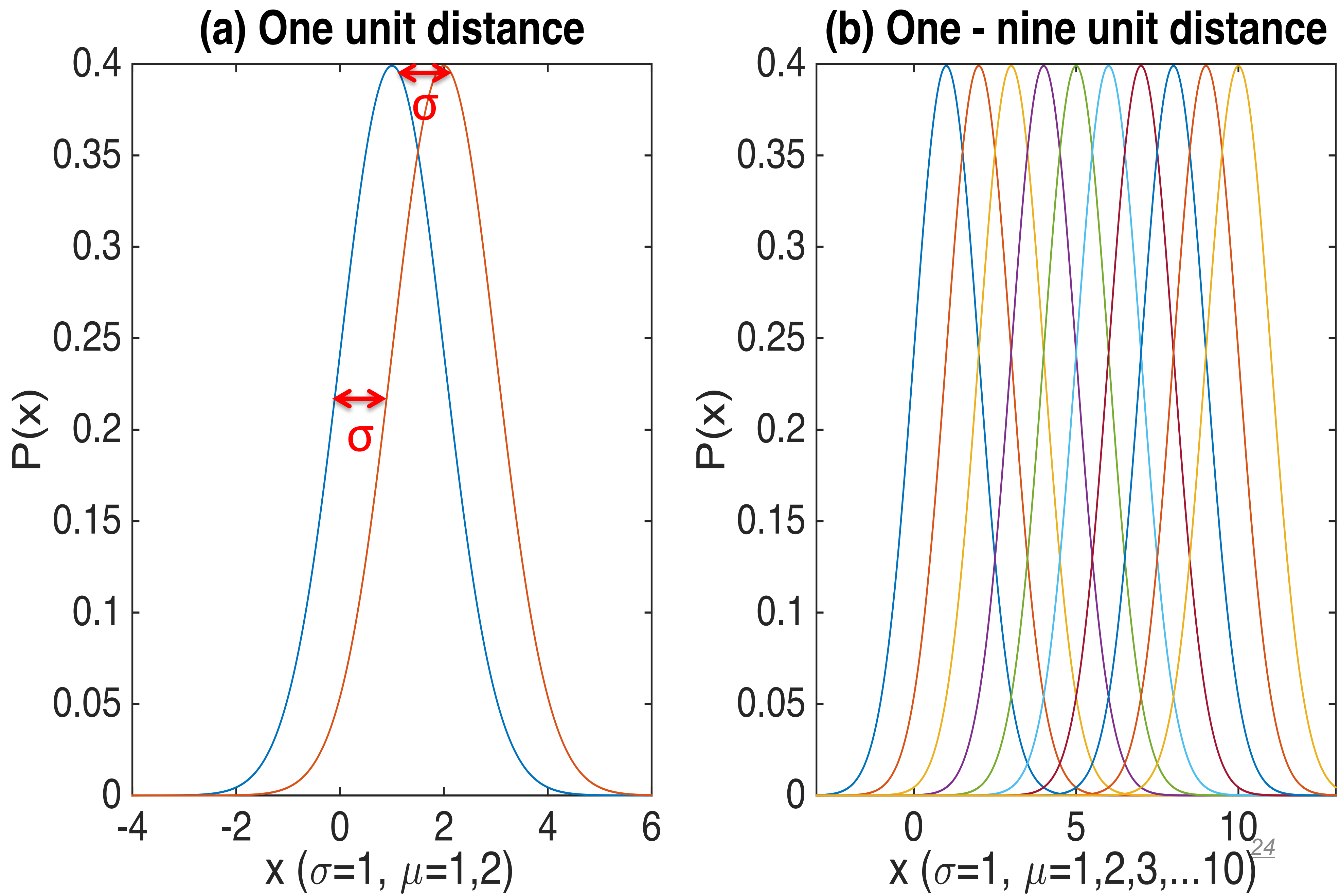


Q: Quantify similarity among PDFs by number (distance):

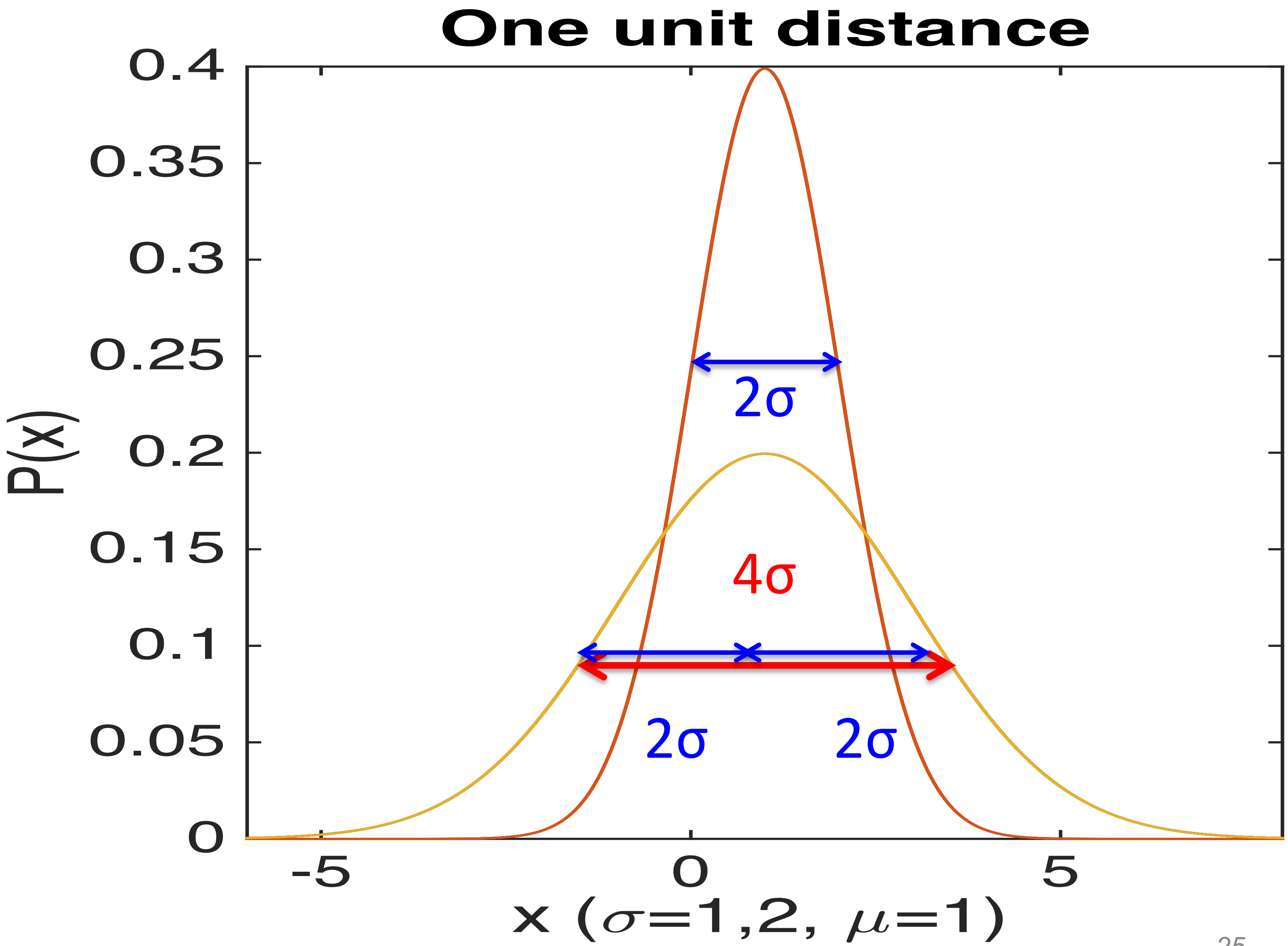
Smaller distance for similar PDFs

Larger distance for disparate PDFs

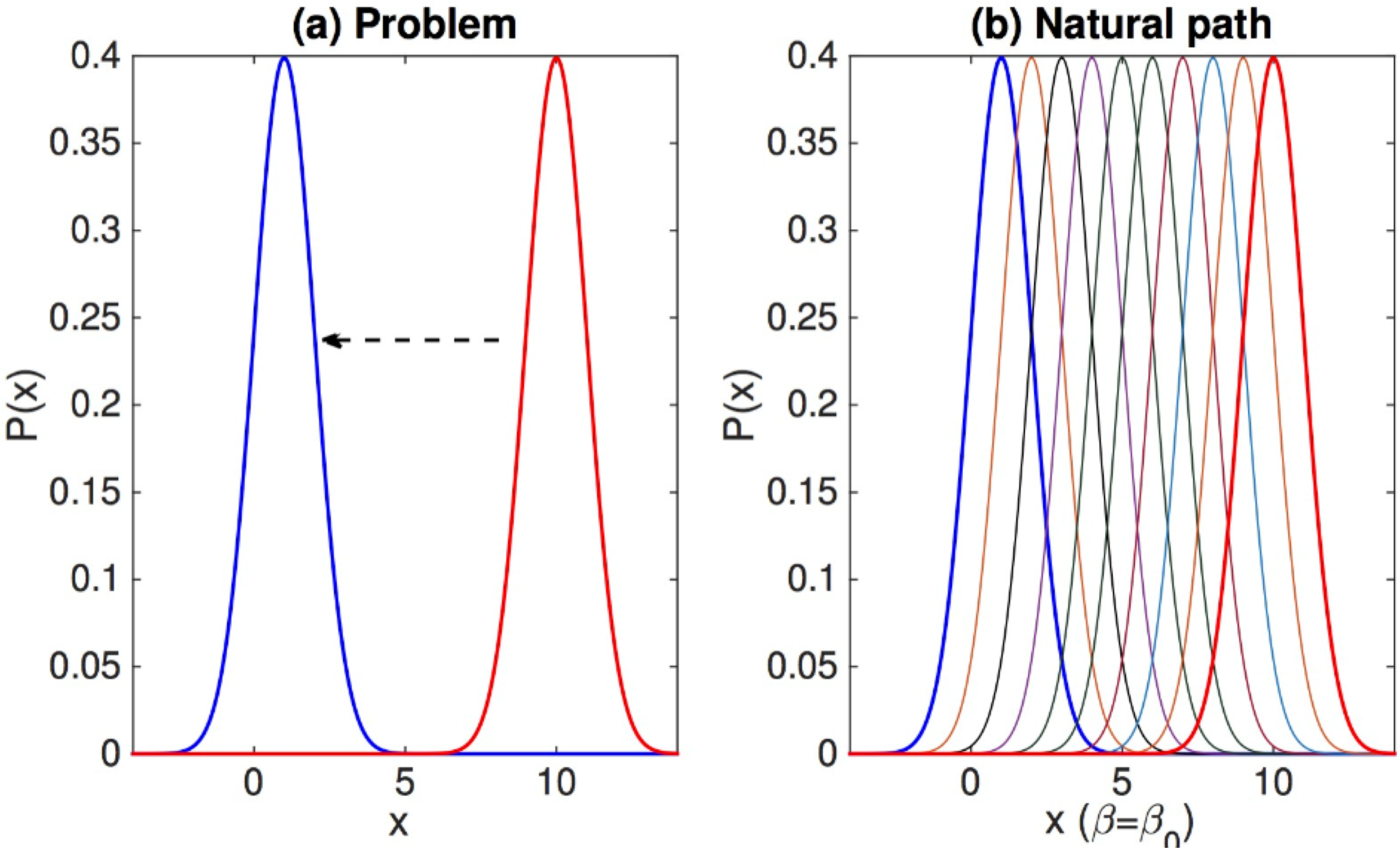
Unit of distance = width of PDF (variance σ)



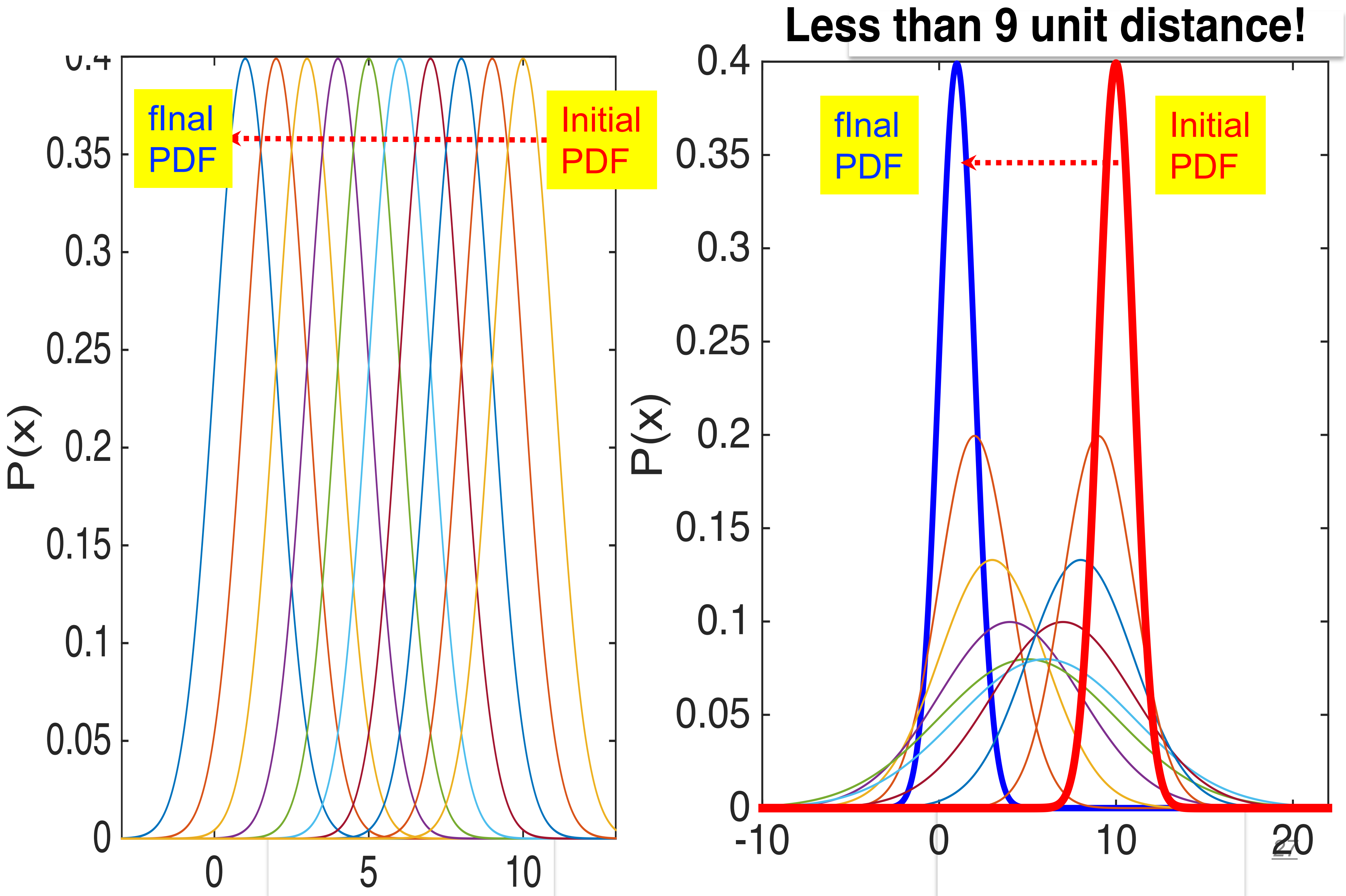
For different σ



Control experiment: Reducing x by constant σ



Time dependent problem: μ(t) and σ(t)



Time-dependent PDF

[Kim 18; Nicholson & Kim 15,16; Heseltine & Kim 16; Kim & Hollerbach 20]

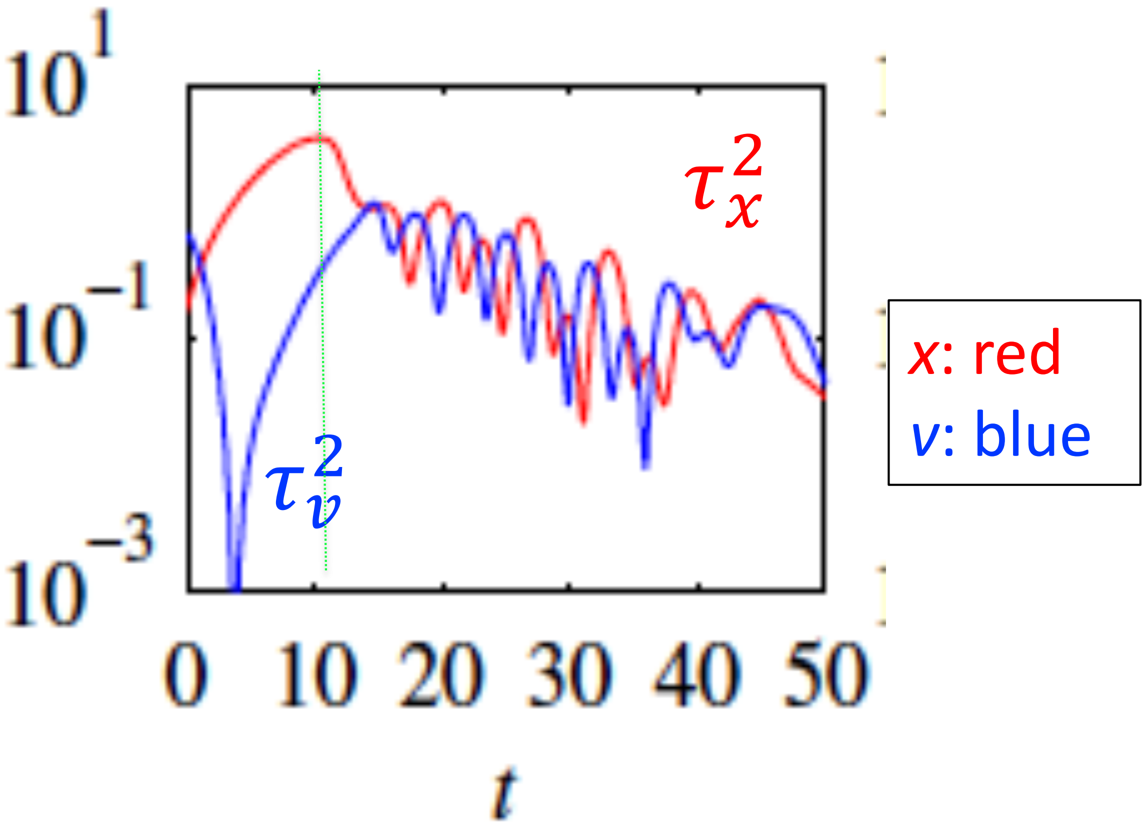
$$\left(\frac{dL}{dt}\right)^2 = \frac{1}{\tau^2(t)} = \int dx\ p(x,t)\left(\frac{\partial \ln p(x,t)}{\partial t}\right)^2$$

$$\text{rate of information change: } \tau^{-1} = \left|\frac{dL}{dt}\right|$$

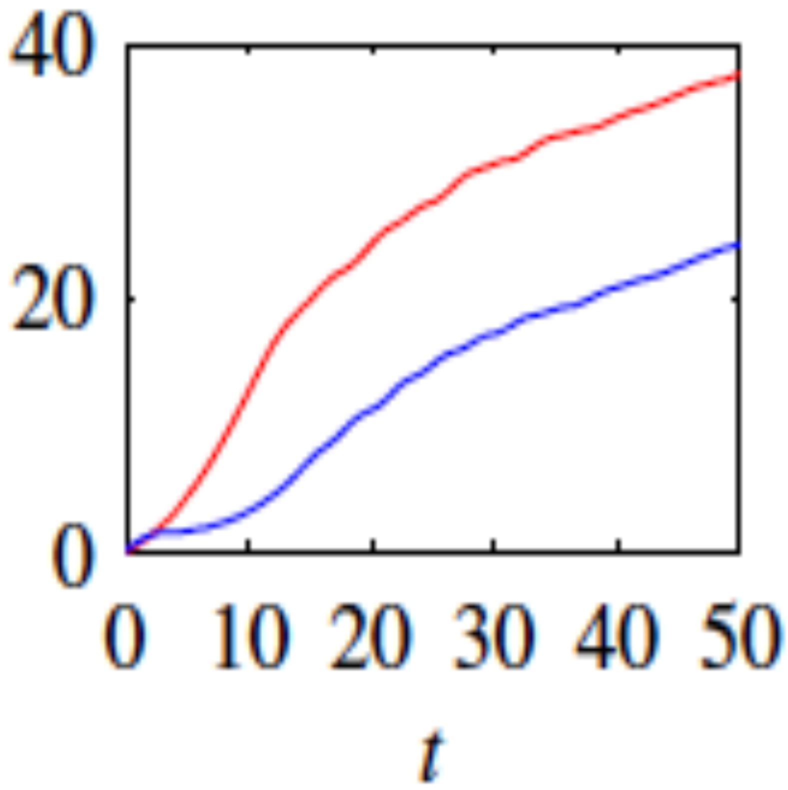
Information length L(t): dimensionless total number of different statistical states that a system evolves through in time (0,t)

$$L(t) = \int_0^t dL = \int_0^t \frac{dt_1}{\tau(t_1)} = \int_0^t dt_1 \sqrt{\int dx\ p(x,t_1)\left(\frac{\partial \ln p(x,t_1)}{\partial t_1}\right)^2}$$

$1/\tau^2$ vs time



Information length vs time



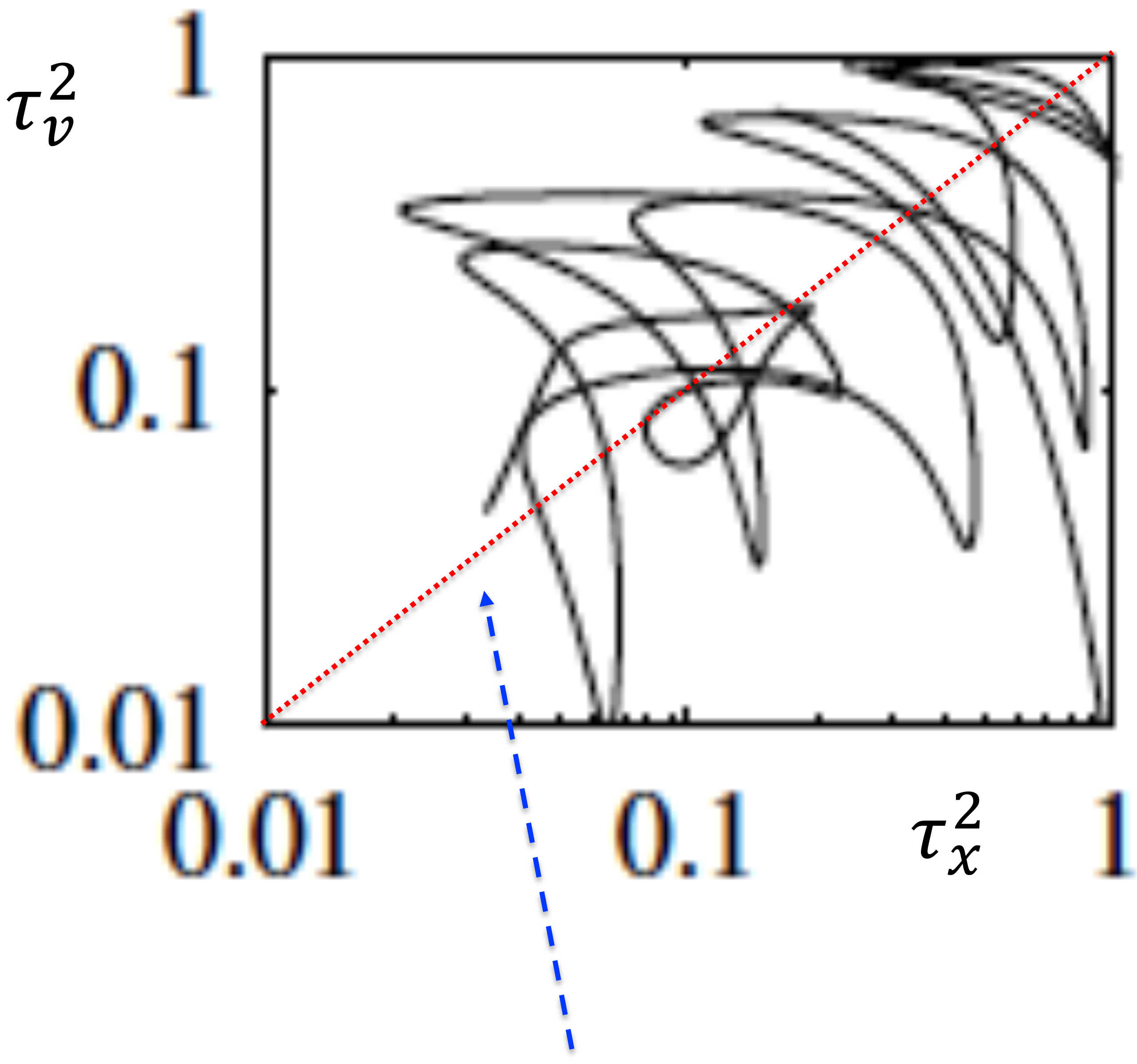
$$\frac{1}{\tau_x(t)^2} = \int_0^t dt' \frac{1}{p(x,t')} \left[\frac{\partial p(x,t')}{\partial t}\right]^2$$

$$L_x = \int_0^t dt' \frac{1}{\tau_x(t')}$$

$$\frac{1}{\tau_v(t)^2} = \int_0^t dt' \frac{1}{p(v,t')} \left[\frac{\partial p(v,t')}{\partial t}\right]^2$$

$$L_v = \int_0^t dt' \frac{1}{\tau_v(t')}$$

Information plane: τ_x^2 vs τ_v^2



Self-organisation: Oscillation around $\tau_x = \tau_y$

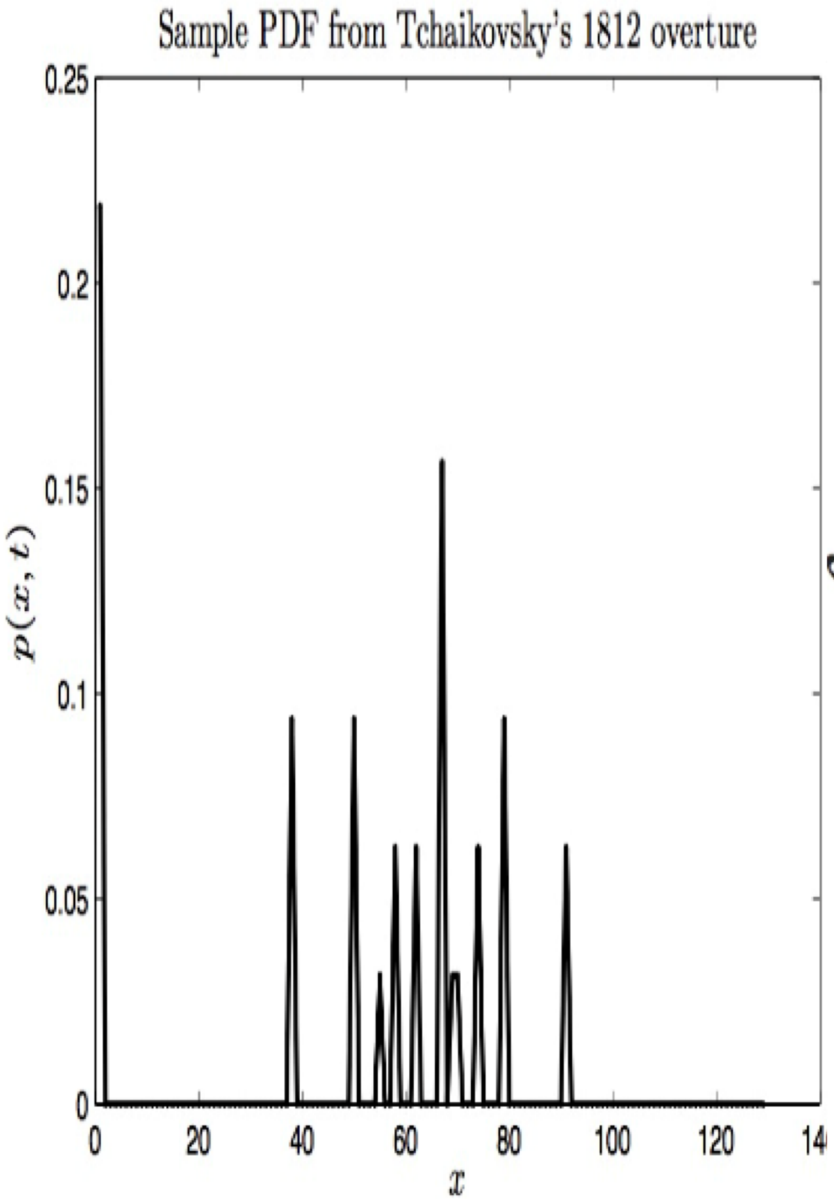
3. Other examples:

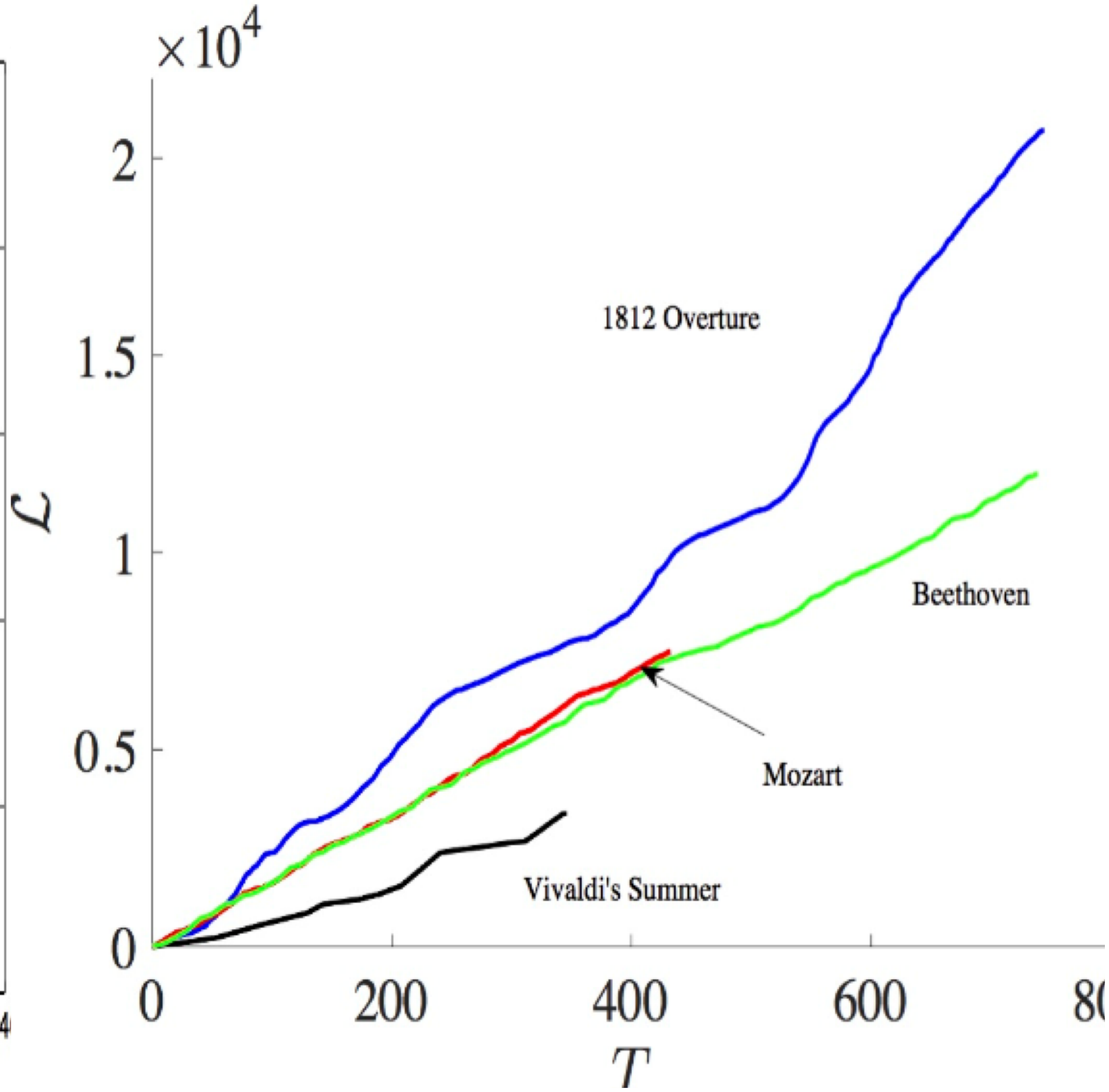
  3.1 Music

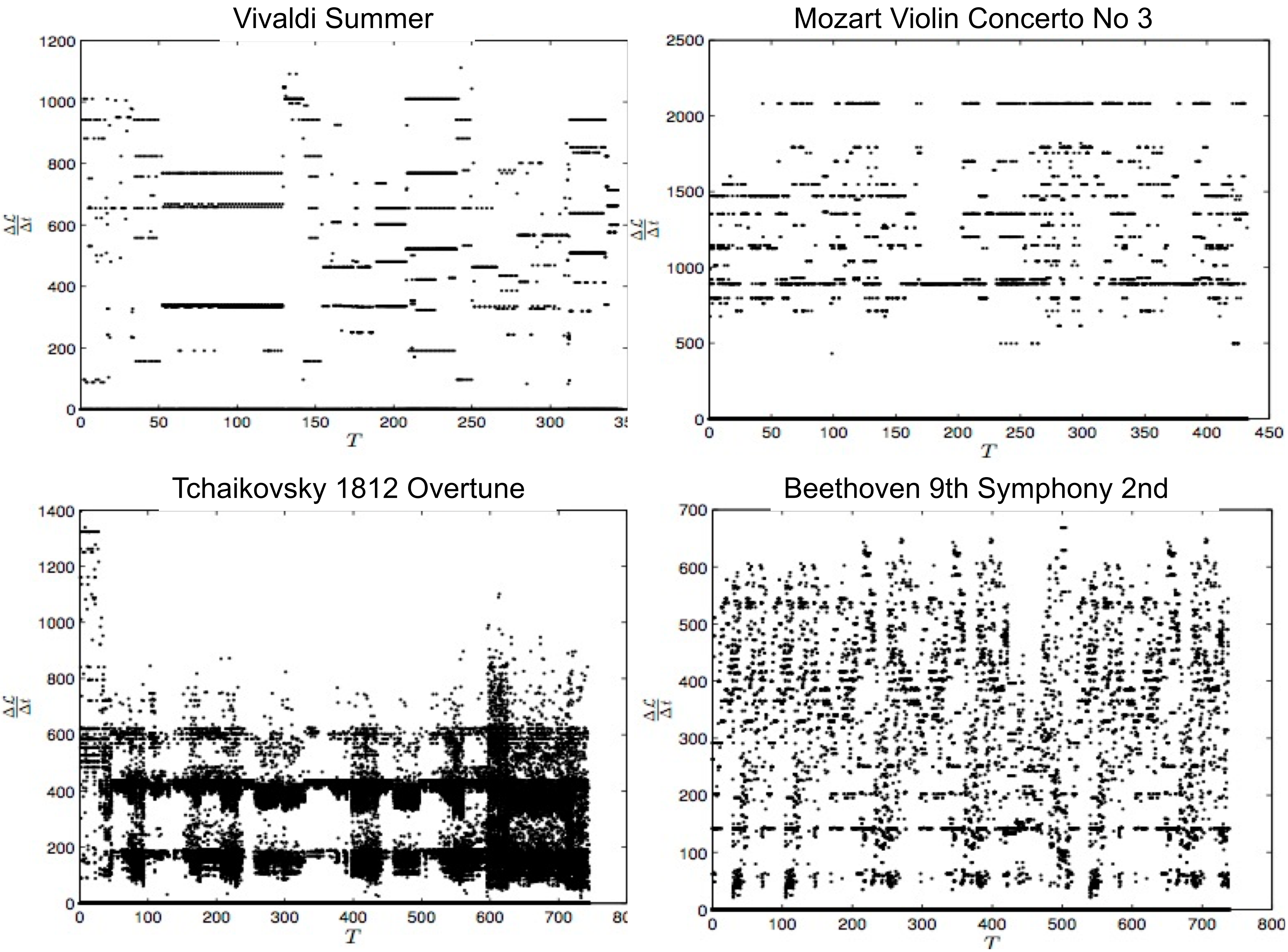
  3.2 Global circulations

3.1 Music: can we see music?

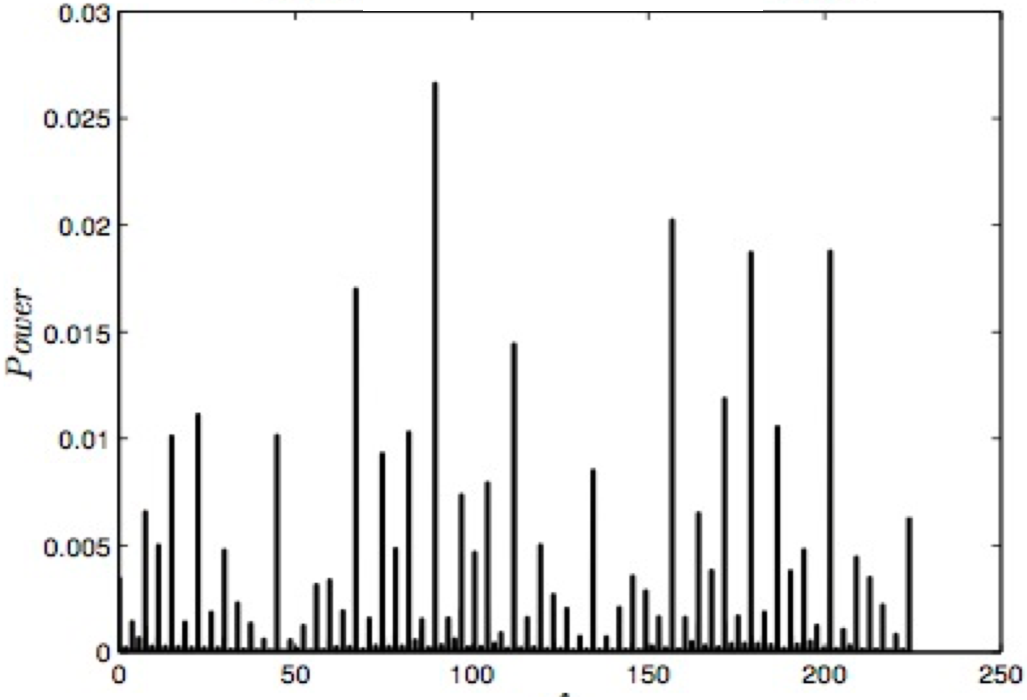
[Nicholson & Kim Entropy 2016]



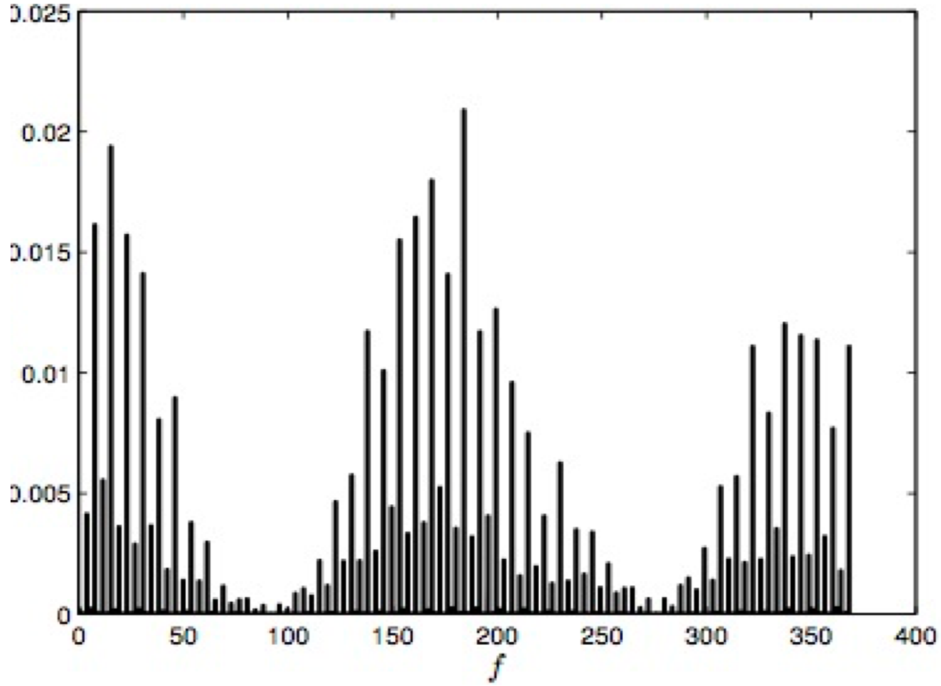



Vivaldi Summer
Mozart Violin Concerto No 3
Tchaikovsky 1812 Overtune
Beethoven 9th Symphony 2nd
$\frac{\Delta C}{\Delta t}$
T

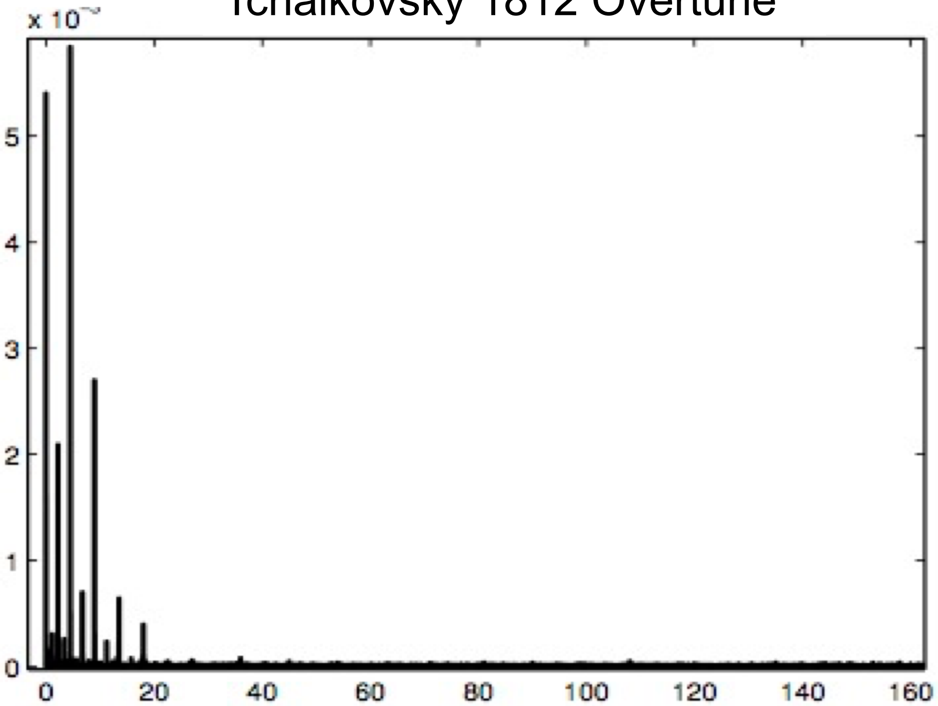
Vivaldi Summer



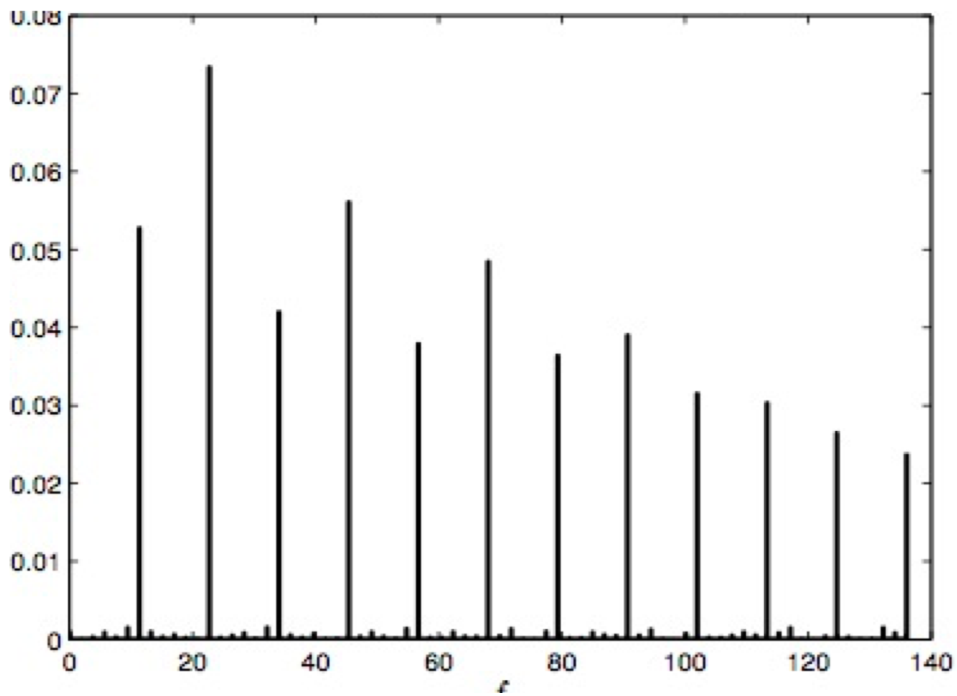
Mozart Violin Concerto No 3



Tchaikovsky 1812 Overtune

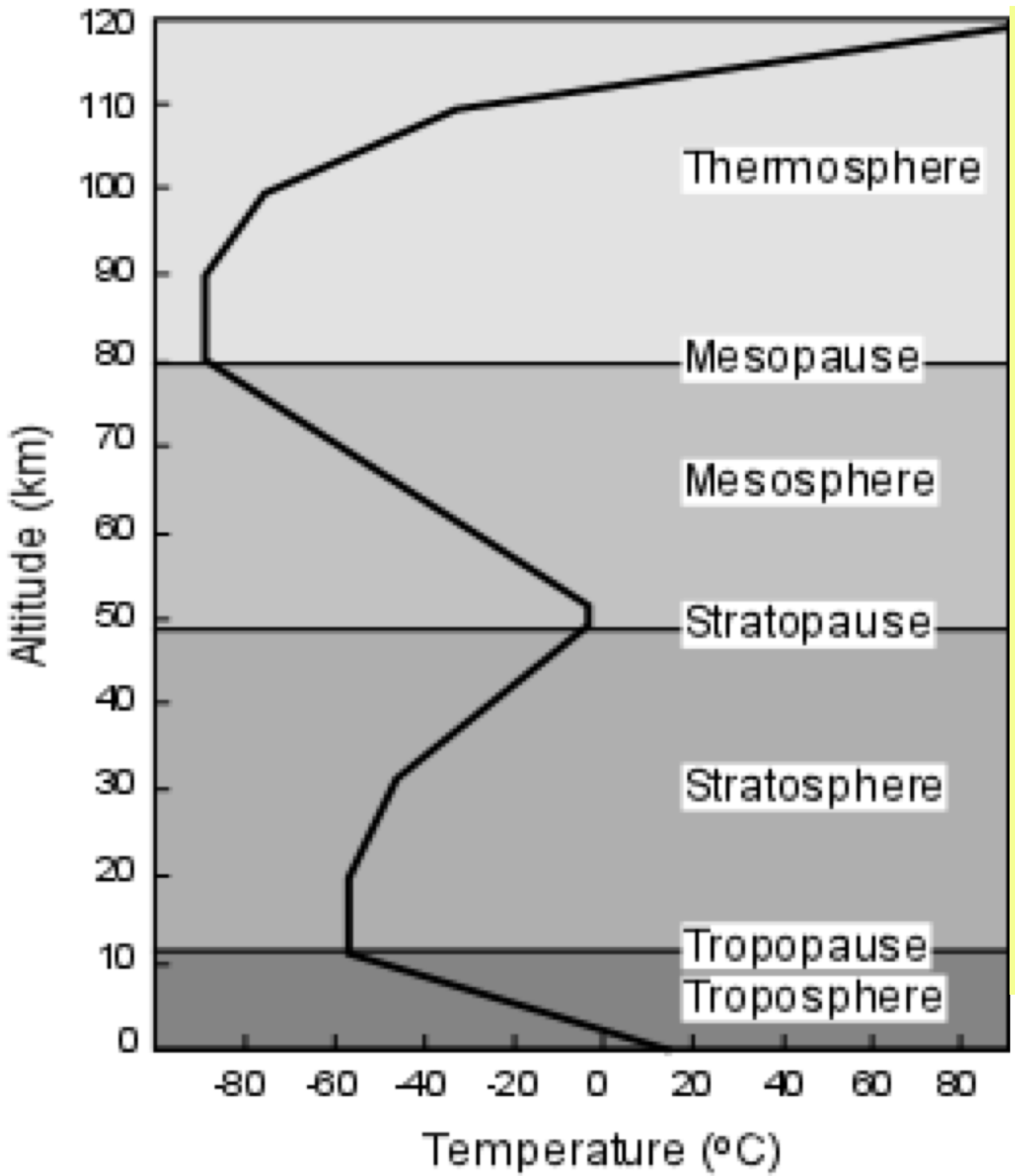


Beethoven 9th Symphony 2nd



3.2 Global circulation model

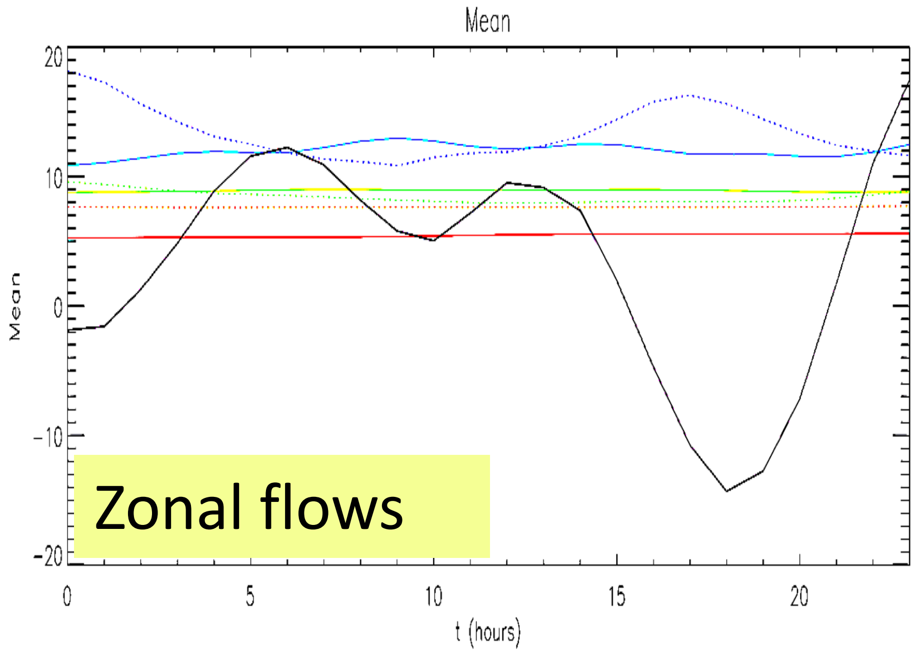
[Kim, Liu & Heseltine 2020]

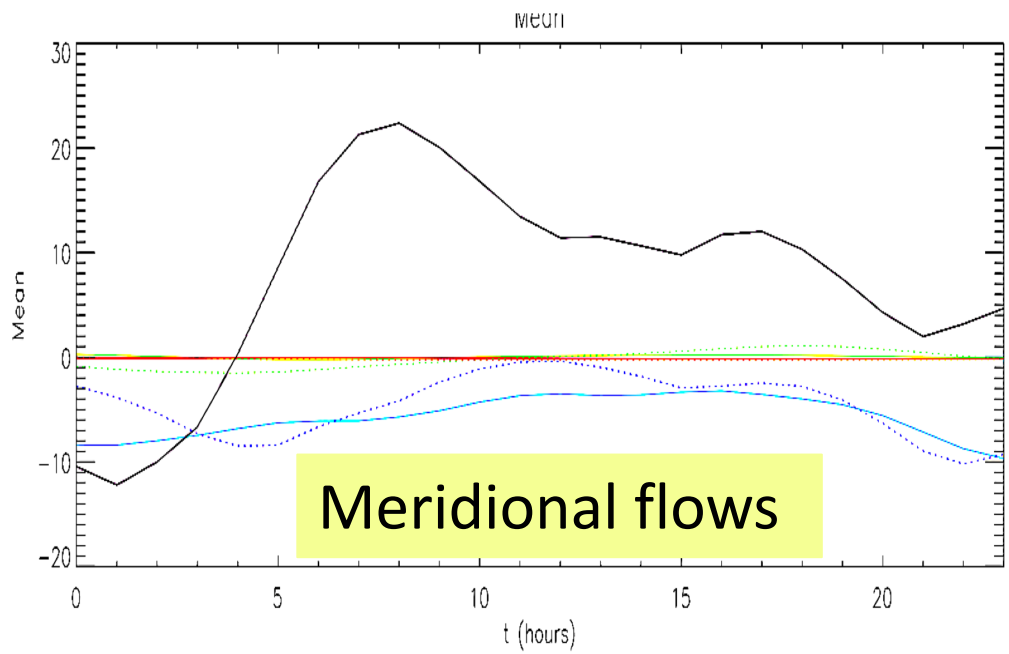


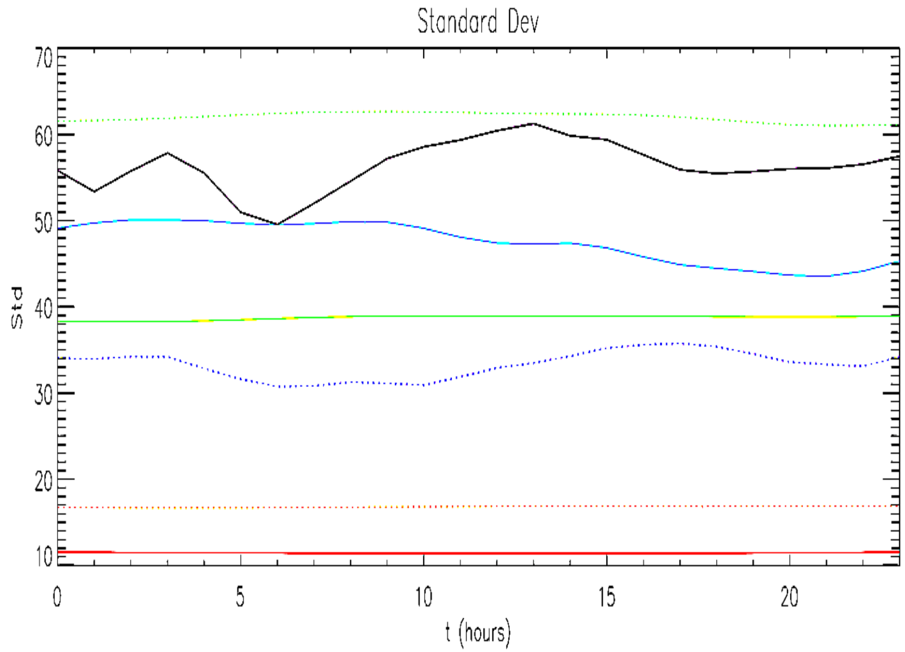
After Murck et al. (1997)

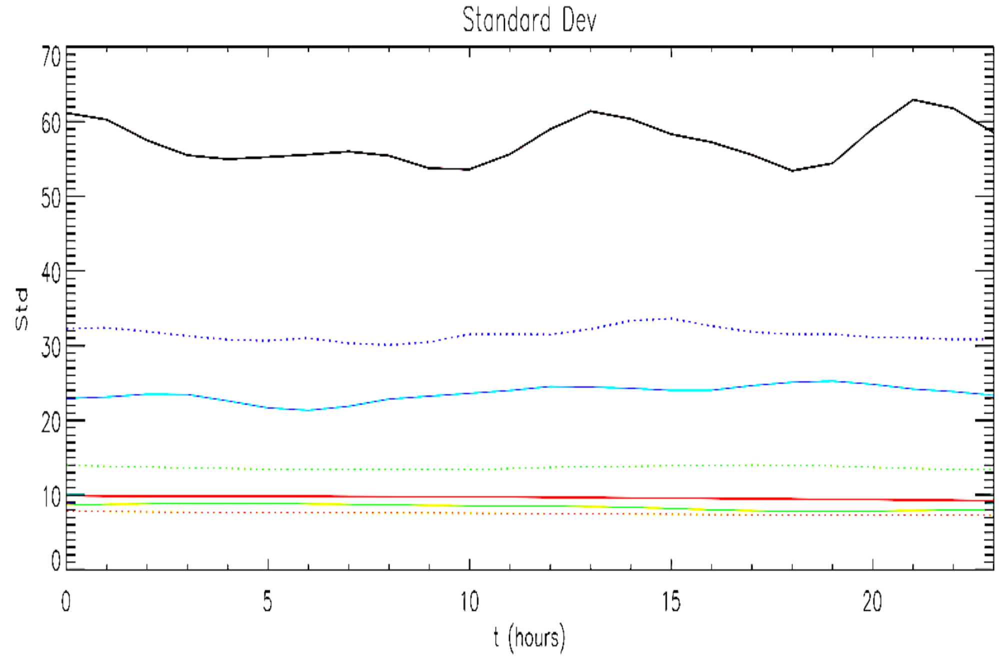
- Data from WACCM
- A few points around the middle of each sphere and pause
- Time dependent PDF from using data at these points, all longitude and latitude
- Information length L(t)

Mean and Standard deviation: zonal & meridional flows

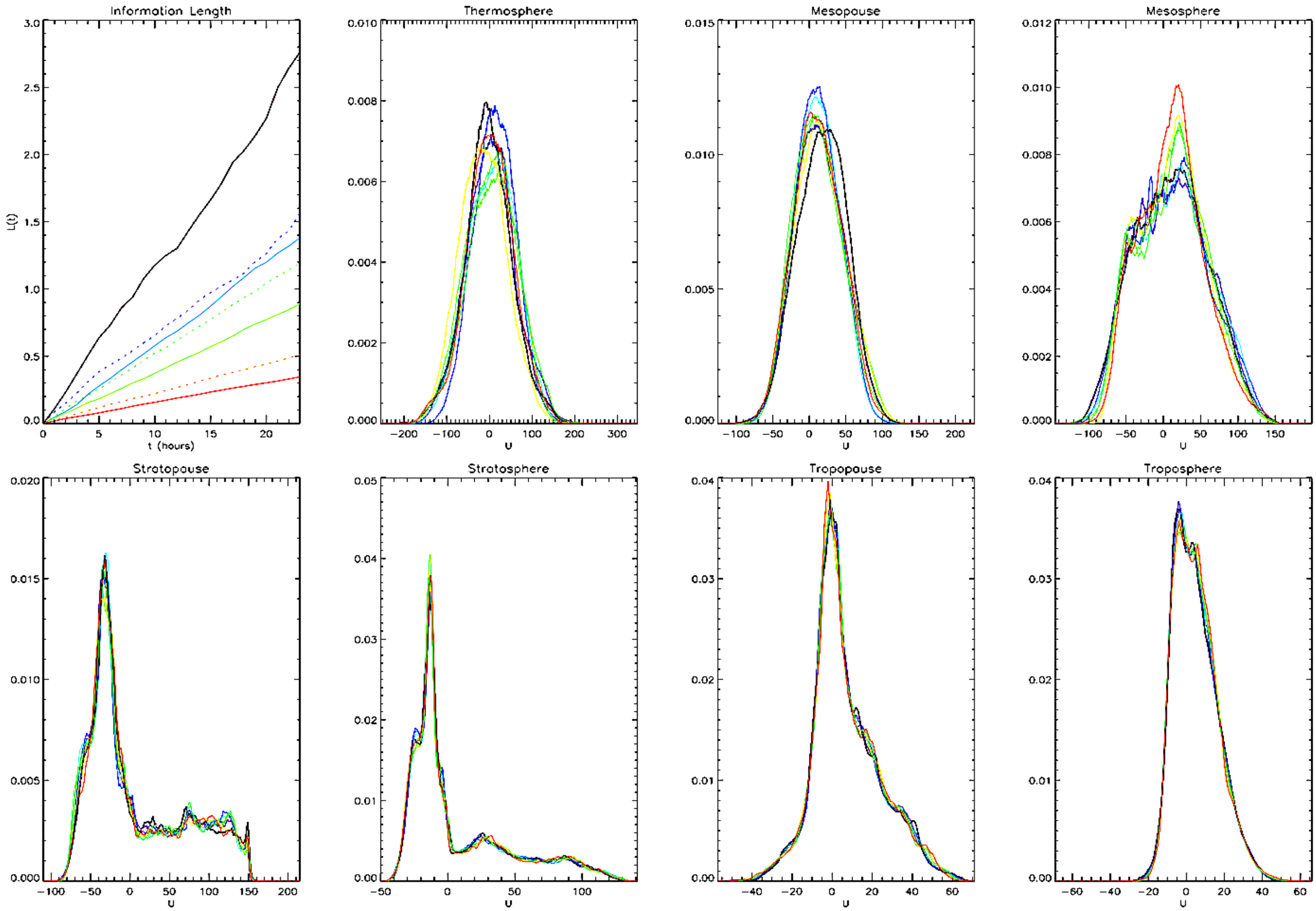




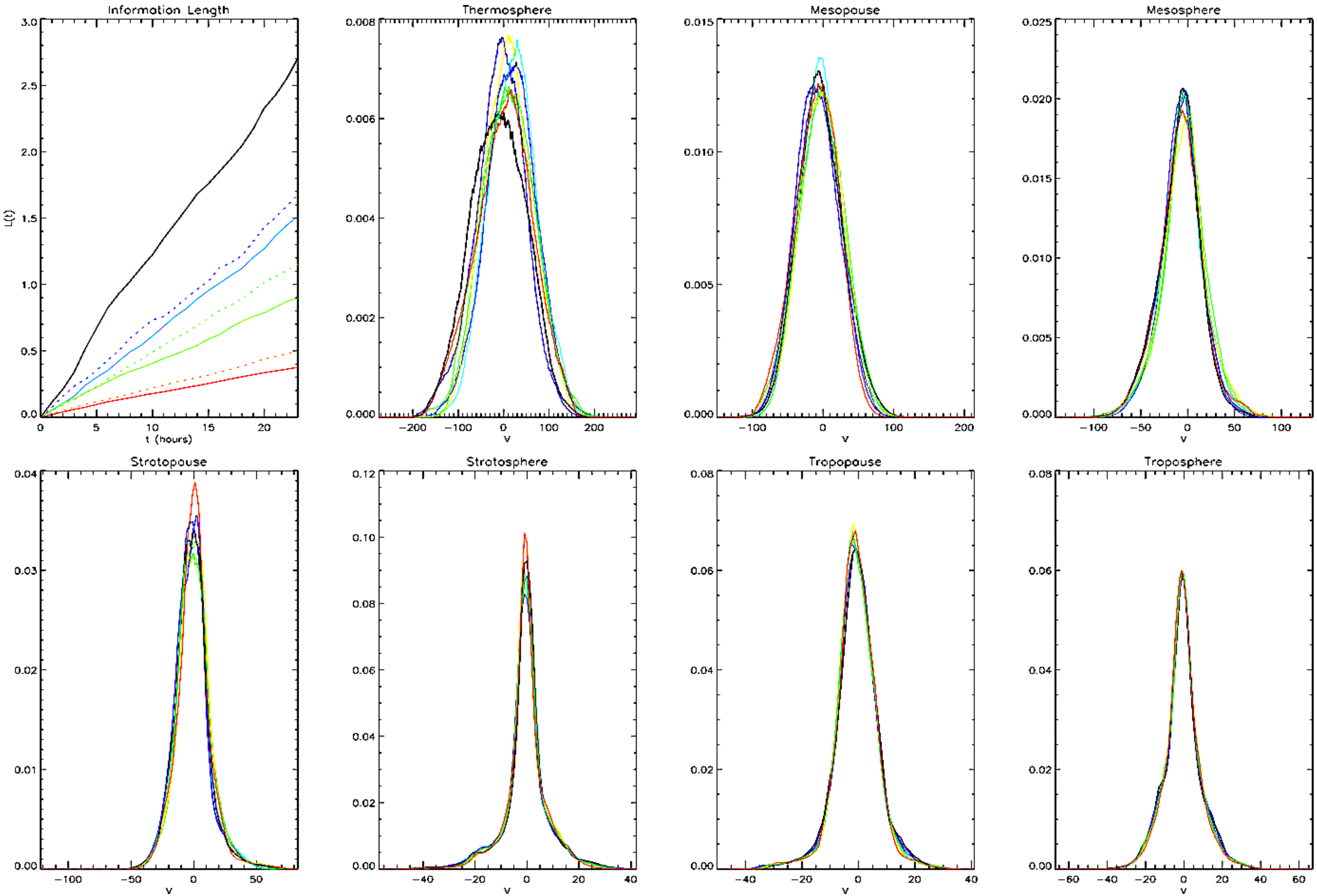




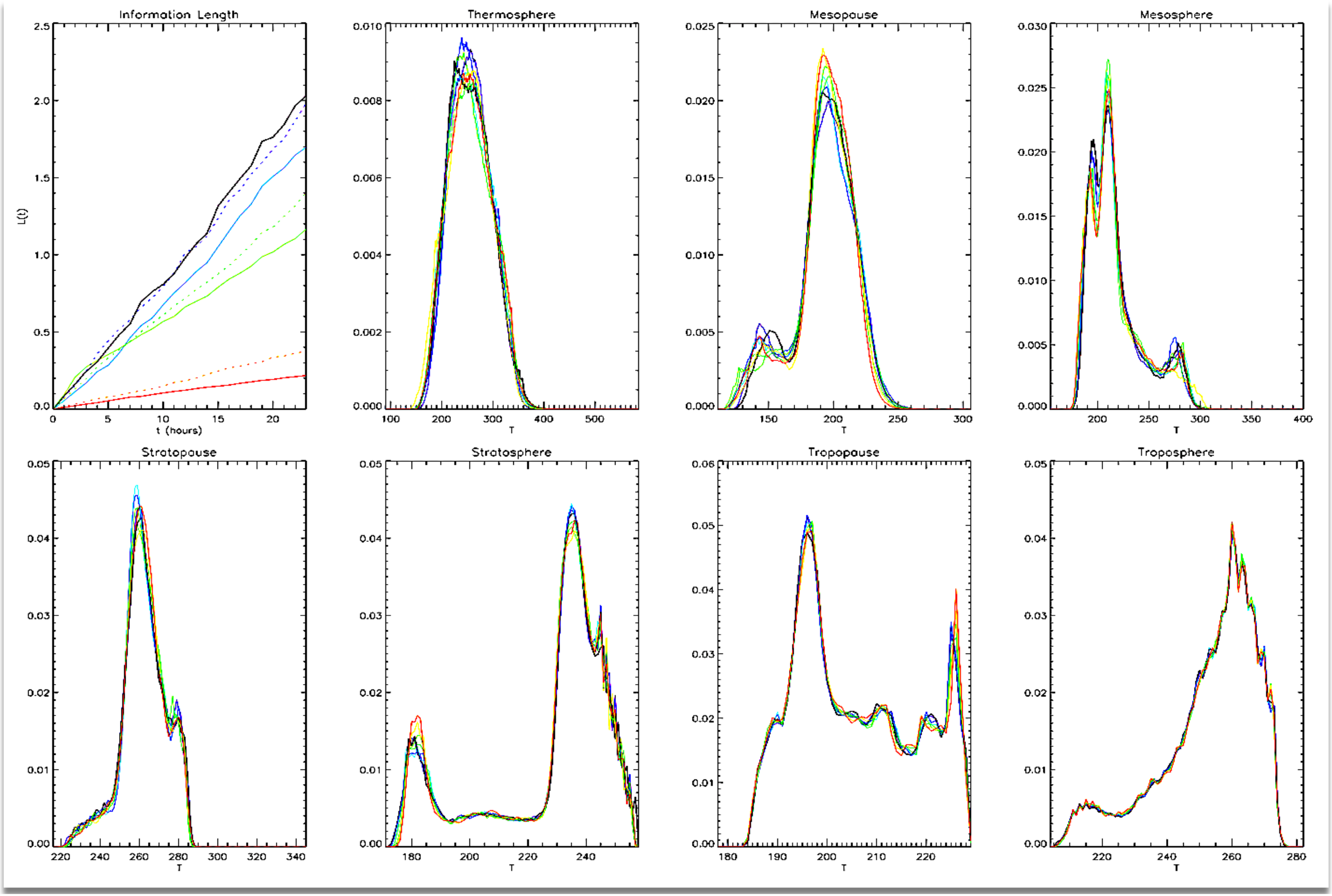
Information length and PDFs: Zonal flows

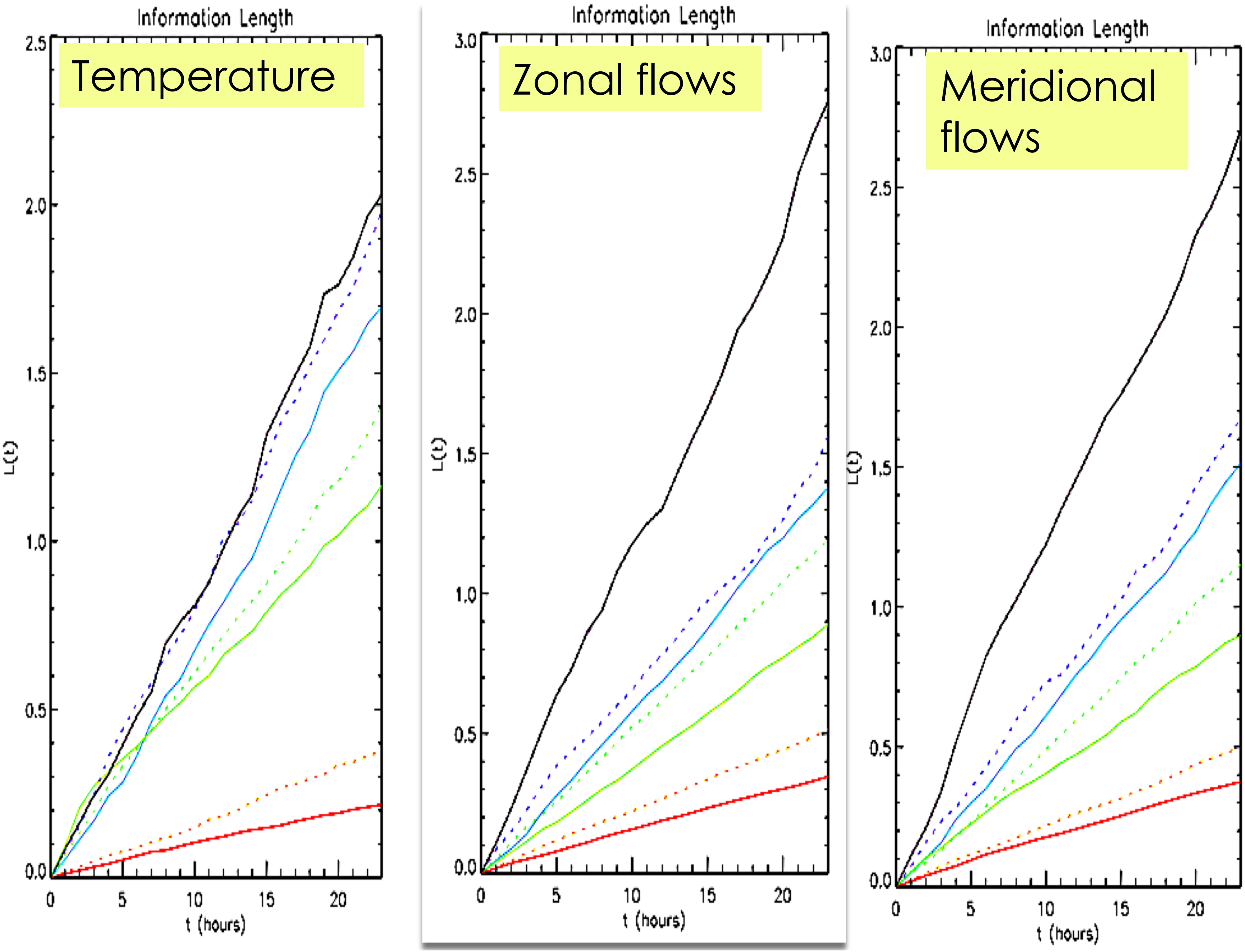


Information length and PDFs: meridional flows



Information length and PDFs: Temperature





Information Length
Temperature
L(t)
t (hours)
Information Length
Zonal flows
L(t)
t (hours)
Information Length
Meridional flows
L(t)
t (hours)

4. Conclusions

- Much scope for research on the L-H transition
- Limitation of mean value, standard deviation, Gaussian PDF
- Information length: the number of statistically different states that a system evolves through in time.
- It is dimensionless and invariant under (time-independent) change of variable.
- Useful to understand correlation in self-organising process.
- Applicability to different processes.
- Useful index to classify a growing number of data.

Imperial College London

The University Of Sheffield.

Coventry University

Thank you

- **Supported Summer Internships at Coventry**

- **Funded PhD studentship on turbulent plasmas**

https://www.findaphd.com/phds/project/turbulent-plasma-in-laboratory-and-space/?p118985

or

https://warwick.ac.uk/fac/sci/physics/prospective/postgraduate/pgintro/resourcesforapplicants

Contact: Prof Eun-jin Kim at ejk92122@gmail.com

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