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Bosonic String Theory and T-Duality

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1. Introduction

String theory has emerged as a promising candidate for quantum gravity. It assumes that all elementary particles are different vibrational modes of a one-dimensional string, where strings can be open or closed.

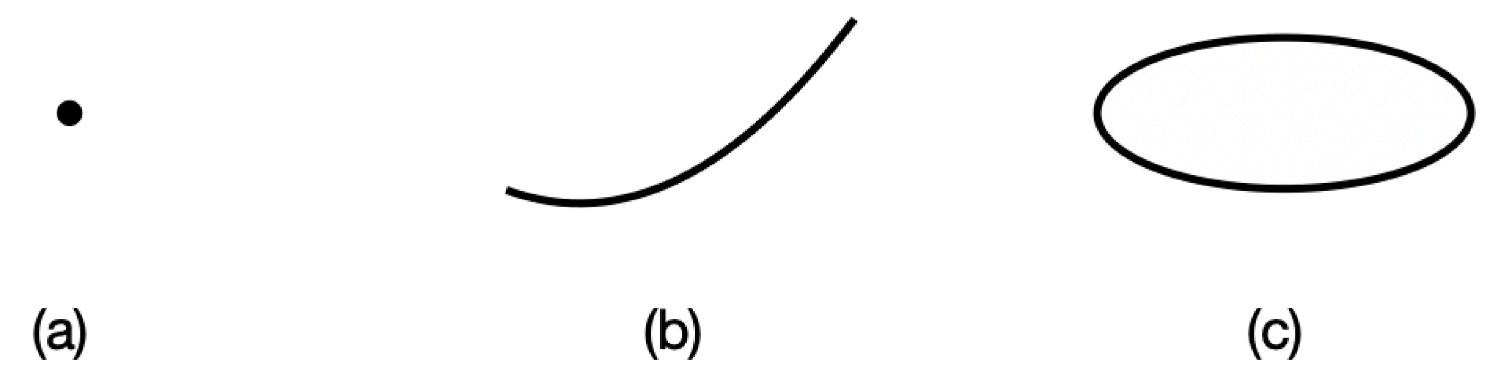

Figure 1: (a) Point particle (b) an open string (b) a closed string

While particles trace out a worldline in spacetime, a string traces out a two-dimensional worldsheet, parametrised by coordinates $X^\mu(\sigma,\tau)$, where σ represents the position along the string and τ is the proper time.

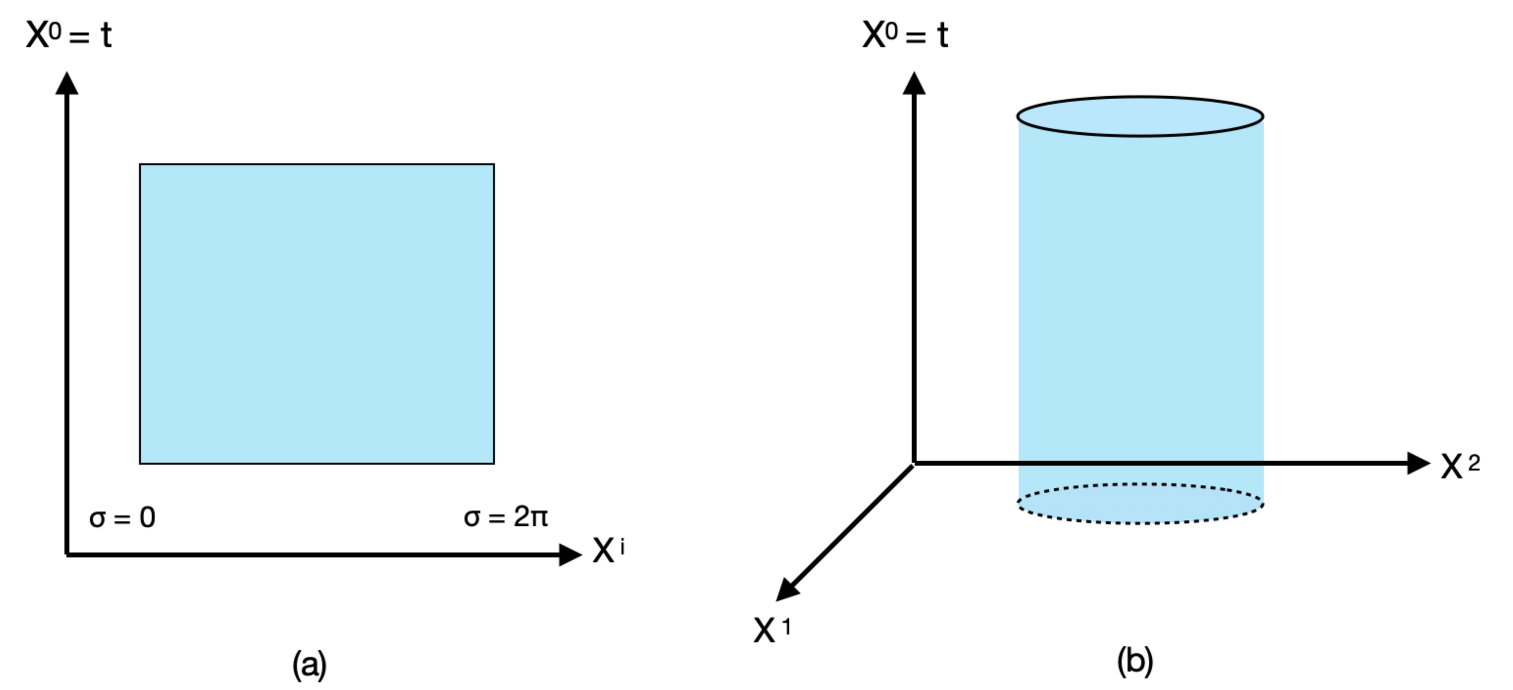

Figure 2: World sheet of (a) an open string (b) a closed string

The theory requires either 26 (bosonic) or 10 (superstring) space-time dimensions.

2. String Action

Relativistic mechanics of a point can be obtained by assuming the action is equal to the Lorentz invariant length of the worldline. The action of a string is given by the worldsheet area, known as the Nambu-Goto action:

$$S = -T\int \sqrt{-\dot{X}^2 X'^2 + (\dot{X}\cdot X')^2}\, \mathrm{d}\tau \mathrm{d}\sigma,$$

$$\dot{X} = \frac{\mathrm{d}X}{\mathrm{d}\tau}, \quad X' = \frac{\mathrm{d}X}{\mathrm{d}\sigma}$$

where T can be understood to be the string tension. This can be shown to be equivalent to the Polyakov action which, when a 'flat gauge' is used, has form

$$S = -\frac{1}{2}T\int \mathrm{d}\tau \mathrm{d}\sigma \partial_m X \cdot \partial^m X$$

and equations of motion given by

$$\partial_m \partial^m X^\mu = 0$$

Upon quantising the string, we find different vibrational modes that correspond to different elementary particles. String theory predicts one of these particles to have the exact properties of the hypothesised graviton.

3a. T-Duality

Since string theory suggests more than 4 spacetime dimensions, extra dimensions must be 'compactified'. T-duality considers compact spacetimes, with extra dimensions Y ~ Y+2πR, where R is the radius, Y = Ry.

$$S_Y = -\frac{1}{2}T\int \mathrm{d}\tau \mathrm{d}\sigma \partial_m Y \partial^m Y = -\frac{1}{2}TR^2 \int \mathrm{d}\tau \mathrm{d}\sigma \partial_m y \partial^m y$$

3b. T-Duality

Gauging the theory and then eliminating the gauge field results in an equivalent action

$$S_{\text{new}} = -\frac{T}{2}\frac{1}{R^2}\int \mathrm{d}\tau \mathrm{d}\sigma \partial_m y' \partial^m y'$$

Therefore, a compactified theory of radius R is the same as one with radius 1/R.

This approach can be generalized to a curved spacetime g with field B, which leads to the Buscher rules:

$$\tilde{g}_{yy} = \frac{1}{g_{yy}}, \quad \tilde{g}_{yi} = \frac{B_{yi}}{g_{yy}}, \quad \tilde{g}_{ij} = g_{ij} - \frac{g_{yi}g_{yj} - B_{yi}B_{yj}}{g_{yy}}$$

$$\tilde{B}_{yi} = \frac{g_{yi}}{g_{yy}}, \quad \tilde{B}_{ij} = B_{ij} - \frac{g_{yi}B_{yj} - g_{yj}B_{yi}}{g_{yy}}$$

These rules can be used to apply T-duality on a given spacetime background, e.g. a 3-torus.

4. Implications

The main motivation behind string theory is the fact that it's the only theory that consistently describes quantum gravity and is renormalisable.

Understanding all physics at the Planck scale is the key to making sense of quantum gravity and currently the only consistent framework is string theory.