Many Curious Roads of Λ

Patching 120 Zeroes with Modified Gravity

Jiaqi Bao

Word count: 2905

Many Curious Roads of Λ — Patching 120 Zeroes with Modified Gravity

Modern fundamental physics has been an extremely successful field. General relativity (GR) and quantum field theory (QFT), which underlie our understanding of the physical world, are both astonishingly accurate theories [1], sometimes matching experimental results by more than 10 significant figures [2] — a truly amazing achievement. But you see, success in physics is boring. It indicates that there's nothing new, that progress has gone stale. And we don't like that.

So let's talk about failure instead. In this case, a very big one.

120 Zeroes

The cosmos is vast, and gravity dominates at large scales. So to model the cosmos, we use GR:

$$G_{\mu\nu} + \boldsymbol{\Lambda g_{\mu\nu}} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

$G_{\mu\nu}$, the Einstein tensor, describes the curvature of spacetime. $T_{\mu\nu}$, the stress-energy tensor, is essentially a generalized version of mass. Ignoring the $\Lambda g_{\mu\nu}$ in the middle, this gives us a clear picture of gravity: mass decides how spacetime curves and spacetime decides how mass moves.

$\Lambda g_{\mu\nu}$ is a special *extra* term. It was observed in 1998 that the universe is expanding at an ever-increasing rate, and GR doesn't have a solution that can explain this phenomenon. $\Lambda g_{\mu\nu}$ was thus *artificially* added to (1), as the resulting modified equation can indeed produce a universe with accelerating expansion, provided that Λ is a *positive and very small constant*.

Λ is known as **the cosmological constant**, and it remains the simplest and most effective way to model cosmic acceleration. And it is indeed very small[1] [3]:

$$\Lambda \sim 10^{-66}\,\mathrm{eV}^2 \text{ (in natural units)}. \tag{2}$$

But what is the physical meaning of Λ? We observe that Λ directly multiplies $g_{\mu\nu}$, which is the metric tensor that *describes spacetime itself*. Therefore, Λ should represent an intrinsic characteristic of spacetime that can somehow cause cosmic acceleration. There is a great candidate for this: vacuum energy.

In QFT, vacuum isn't empty; it is filled with quantized fields, such as electromagnetic field and the Higgs field[2], each of which has a zero-point energy, not unlike the non-zero ground state

[1]We only quote the order of magnitude here, since it suffices for the discussion, and calculating Λ precisely requires the Hubble constant which has a serious crisis itself.

[2]Every field in QFT corresponds to a known particle in the standard model. In fact particles are just excitations of

energy of a quantum harmonic oscillator. This energy has negative pressure, creating a repulsive gravitational field, causing the universe to expand. And since this energy has constant density, being a property of vacuum, the more the universe expands, the greater amount of this energy there is, and the faster the universe expands. Bingo.

So, a reasonable conclusion is that Λ is vacuum energy, and we should be able to predict Λ this way. From QFT, we have [4]

$$\Lambda \sim 10^{53}\ \mathrm{eV}^2 \tag{3}$$

Hmm.

Notice something wrong?

This, compared with (2), is (roughly speaking) **120 orders of magnitude** too big.

Oops.

This is the **cosmological constant problem**, and is often described [5] as "the biggest problem in fundamental physics."

Sounds terrible, doesn't it? Well, for physicists, it's Christmas. Big failure often signals that new physics is on the horizon. And nothing is more exciting than that.

Taylor expanding gravity

Undeniably, the cosmological constant problem reveals that our understanding of both gravity and quantum physics is flawed. Solving the problem probably requires advancements on both fronts, or even better, a theory of quantum gravity. Meanwhile, many do believe that we can still make significant progress on this problem by considering a group of theories called **modified gravity**, which simply modify GR but remain classical.

Why? The answer is one of both logic and pragmatism. On one hand, nobody knows what quantum gravity looks like; proposals exist but so far none works. On the other hand, GR already has a potential problem: it's not very stringently tested at very large scales [6], exactly where the cosmological constant problem arises. This leaves room for modification of GR, and many hope that such modifications can indeed eliminate those 120 troublesome zeroes.

How, then, do we modify GR? Let's first look at something we are all familiar with: **Taylor expansion**. We often use concatinated Taylor series to approximate functions. For example, $\sin x = x - x^3/3! + x^5/5! - x^7/7! + \cdots$. When $|x| \ll 1$, first-order approximation is enough:

fields, so you can basically think of them as the same thing, and we can talk about properties like the spin and mass of a field just like the spin and mass of a particle. For example, the Higgs field corresponds to the Higgs boson, and you can describe both as massive and spin-0.

$\sin x \approx x$. As $|x|$ becomes larger, we add higher-order terms so that the approximation remains precise.

The spirit of modified gravity is much the same. Here we're concerned with *action*, a variable which encodes all the information about a physical system and from which the whole theory can be derived through *the principle of stationary action*[3]. The action for GR looks like this (knowing its appearance is sufficient for this discussion; don't worry about the math):

$$\mathcal{S} = \frac{1}{2\kappa} \int d^4x \sqrt{-g}\, R\,, \tag{4}$$

where $\kappa = 8\pi G$ and g is the determinant of $g_{\mu\nu}$.

$\mathcal{S}$ only contains the first order of R, the Ricci scalar, a quantity relating to spacetime curvature. We can then think of $\mathcal{S}$ as a first-order approximation of an unknown, "complete" theory of gravity, akin to what x is to $\sin x$ in the above example, and go on to modify GR by throwing in extra terms:

$$\mathcal{S}_{\text{Modified}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g}\, (R + f_{\text{extra}})\,. \tag{5}$$

What is slightly different from the Taylor expansion analogy is that f_{extra} may or may not be a function of R, unlike the expansion of $\sin x$, but it is still constrained to follow the symmetries we need from the theory. The hope is then that $\mathcal{S}_{\text{Modified}}$ results in something that better approximates the "complete" theory, thereby giving us additional insights to gravity that might just solve the cosmological constant problem[4].

This is a fascinating area of research, filled with wild imaginations and challenges, with the prospect of addressing one of the most important physics problems of all. So, let's dive in and have a tour, shall we?

Going around the solar system's precision cut

Physics is an experimental science, and no physics theory can be accepted if they don't agree with what we observe. For modified gravity theories, this means that they all face a stringent test right off the bat: **the solar system**.

[3] This is very similar to how we find stationary points of a curve and basically means $\delta(\text{action}) = 0$, where δ means a small change. As an example, in GR we can recover equation (1), without the Λ term, by requiring $\delta\mathcal{S} = 0$. Read ahead to see $\mathcal{S}$.

[4] Theories constructed with this method are often called *effective field theories* (EFT), as they can only effectively describe the phenomenon in question under certain conditions, much like the $|x| \ll 1$ condition in the Taylor expansion analogy. GR itself is (probably) an EFT. Perhaps a better analogy is the heat equation, which effectively describes heat transfer *macroscopically* but fails as soon as we look at the motion of individual particles *microscopically*. Likewise, modified gravity theories are effective up to a certain energy/length scale, so they may fail when we concern regimes of very high energy, such as the beginning of the universe.

GR describes the solar system with extremely high precision [7]. Consequently, any modification to the behavior of gravity that any theory makes must be highly suppressed within the solar system.

This is the bottomline for all modified gravity theories. Going one step further, this is actually where modified gravity *starts*: these theories are *designed*, from the outset, to reduce to GR in the solar system (as opposed to, say, writing down any arbitrary theory and hoping that it *happens* to reduce to GR in the solar system). Meaning, all modified gravity theories have **screening mechanisms** built into them [8], which allow these theories to deviate significantly from GR at large scales, giving them the potential to address the cosmological constant problem, and suppress such deviation to tiny values in the solar system.

The crucial task for these theories is therefore to have sensible screening mechanisms. And it's not actually that hard to imagine how such mechanisms might work. Remember that cosmic acceleration is the behavior of the entire universe, and compared to the cosmic average, the solar system has:

- much greater local gravitational potential, or $|\Phi|$,
- much greater local gravitational acceleration, or $|\nabla\Phi|$,
- and much higher local spacetime curvature, or $|\nabla^2\Phi|$.

Modified gravity theories are thus designed to screen their effect based on one of these three quantities. In other words, their deviation from GR vanishes when *one* of $|\Phi|$, $|\nabla\Phi|$, and $|\nabla^2\Phi|$ is greater than a certain threshold value[5].

We can thus classify modified gravity based on their screening mechanisms, and look at each in detail.

Screening by $|\boldsymbol{\Phi}|$ One of the representative ideas under this class is the *chameleon mechanism*. It introduces an additional scalar field to the action, and the mass of the scalar field is positively related to the local mass density of a region [9, 10]. This scalar field mediates an additional, "fifth" force[6].

How does this work? In regions of high mass density, such as in the solar system, the scalar field has a large mass; consequently it mediates a short-range force (meaning that the force dies off very quickly over a short distance) and thus has little observable effect. In regions of low mass density, such as the majority of the universe, the scalar field has a small mass, and

[5] You might infer, quite reasonably, that we can also use the value of $|\nabla^3\Phi|$, or even higher-order derivatives of Φ, as a screening mechanism. It turns out, however, that all such theories create "ghosts" that cause the entire theory to break down, and are thus forbidden.

[6] A scalar field corresponds to a spin-0 particle, which a boson that can act as a force carrier.

mediates a long-range force[7], and this large-scale behavior may be what is needed to address cosmic acceleration [9, 10].

The chameleon mechanism introduces an effective potential, V_{eff}, felt by the scalar field, comprised of a decreasing potential $V(\phi)$ which is intrinsic to the theory and an increasing potential $A(\phi)\,\rho$, where ρ is the local mass density, which results from the relationship between the scalar field and local mass density, as shown in Figure 1. Here ϕ is the scalar field. The square of the mass of the scalar field is given by the second-order derivative of V_{eff}, and it is obvious from Figure 1 that the mass of the scalar field is larger in regions of higher mass density.

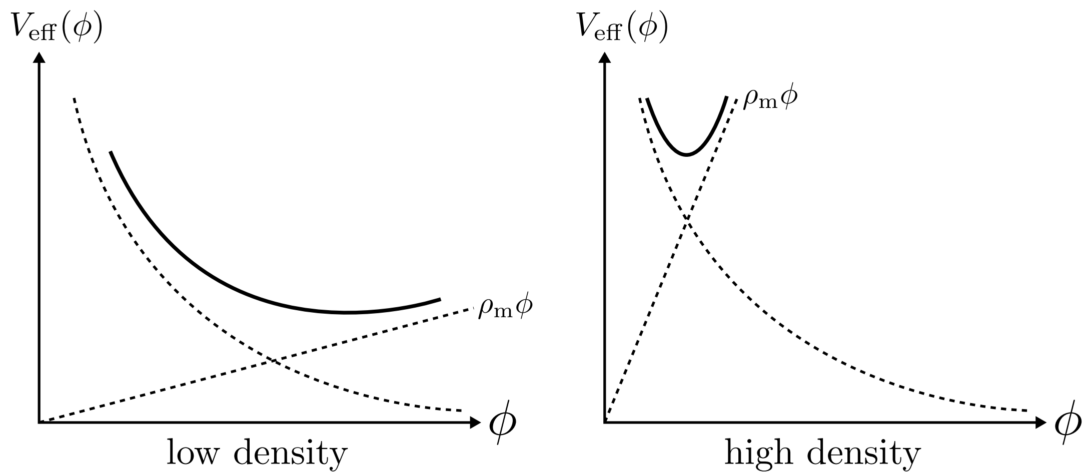

Figure 1: Effective potential for the chameleon mechanism. The decreasing dotted line is $V(\phi)$, and the increasing dotted line is $A(\phi)\,\rho$, which can be well approximated by a linear function of ϕ and is drawn as such. A greater local mass density results in a steeper $A(\phi)\,\rho$, leading to a greater mass of the scalar field. Reproduced from [8] with permission.

One of the most interesting theories to exhibit this mechanism is $f(\mathbf{R})$ **gravity**. Using equation (5), $f(R)$ gravity proposes that f_{extra} is an arbitrary function of the Ricci scalar R: $f_{\text{extra}} = f(R)$, hence the name. We can therefore understand $f(R)$ gravity as modifying GR by adding higher orders of curvature to the theory. The scalar field f_R and its mass $m(f_R)$ are given by [11]

$$f_R = \frac{df(R)}{dR},\; m^2(f_R) = \frac{1}{3}\left(\frac{1+f_R}{\frac{df_R}{dR}} - R\right). \tag{6}$$

We can then introduce the chameleon mechanism in $f(R)$ gravity by carefully choosing the form of $f(R)$. Indeed, $f(R)$ gravity even has the potential to solve other problems, such as inflation [12], by constructing a suitable f.

[7] As another example of the relationship between the range of a force and the mass of the mediating particle, electromagnetic interaction, mediated by massless photons, has infinite range, whereas weak interaction, mediated by heavy W and Z bosons, acts at very small distances.

Other ideas and theories exist under this class, but they all share a similar basic principle. And they also face the same problem: this "fifth" force has limited range, and it turns out that it can't really reach the cosmological scale. One crucial, and perhaps fatal, consequence is that this essentially means this class of modified gravity theories produce predictions regarding cosmic expansion that are *identical* to GR, and they too face the same limitation of needing a Λ artificially added to them in order to address the cosmic acceleration, so they aren't actually valid solutions to the cosmological constant problem [13].

This may sound a bit disappointing, but such is the reality of exploring uncharted territories. More importantly, however, these theories aren't without merit, as they can still prove to be valuable in helping us understand other unsolved problems of the universe, as is already the case with $f(R)$ gravity.

Of course, other modified gravity theories exist, and they also bring their own exciting stories and reality-checks.

***Screening by higher-order terms:* $|\nabla\Phi|$ and $|\nabla^2\Phi|$** Higher-order screening mechanisms are much more complicated in terms of both their principle ideas and the actual mathematics. Broadly speaking, there are two classes of such mechanisms: *kinetic screening*, and the *Vainshtein mechanism*. To give a simple example of how they work, under these mechanisms, a point mass source produces a "fifth" force through a scalar field that behaves just like gravity sufficiently far away from the source, exhibiting the familiar inverse-square law, but is suppressed near the source, as shown in Figure 2. This ensures that theories employing one of these mechanisms conform to solar system observations.

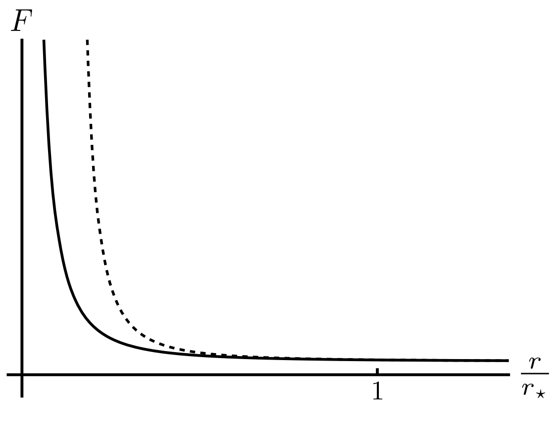

Figure 2: Comparison between the "fifth" force mediated by the scalar field and gravity from a point source when higher-order screening is in effect. Here the dotted line is gravity, the solid line is the fifth force, r is the distance away from the point source, and $r_\star$ is the characteristic length scale. Reproduced from [8] with permission.

One particular theory, called **massive gravity**, stands out in this class. It has a very intriguing proposal: gravitons (the spin-2 boson that mediates gravity) have non-zero mass (hence the name), or equivalently, gravitational waves travel at *less than the speed of light*, which is in direct contradiction with GR. Thus in massive gravity, the scalar field that produces the "fifth" force is effectively absorbed into gravity itself, manifesting as the mass of graviton. Massive gravity exhibits the Vainshtein mechanism.

Traditionally massive gravity has suffered from "ghost", as a massive graviton has a total of six degrees of freedom while the theory only permits five, and the additional degree of freedom, which is the ghost in question, always causes disaster [14]. In 2010, however, a fully ghost-free massive gravity theory, now known as **dRGT massive gravity**[8], was constructed [15, 16] which allows us to have a better look at its cosmological effect. It turns out that massive gravity has enormous potential, as it can address the cosmological constant problem by accomplishing these two things at once:

- **degravitating Λ**, meaning that vacuum energy doesn't contribute to cosmic acceleration at all,
- and producing **self-accelerating** solutions [17] (at least under a suitable approximation[9]), meaning that there is an *alternative* source that is inherent to the theory, which in this case is the mass of graviton, that causes cosmic acceleration.

The advantage of tackling the cosmological constant problem this way is that it is much more *natural* approach in a physical sense. The conventional approach is to try to figure out how vacuum energy is 120 orders of magnitude smaller than what we anticipate from QFT, and this can end up being an enormous degree of fine-tuning, whereas this approach accepts that vacuum energy is large and instead argues that it simply doesn't gravitate. The fact that massive gravity gives self-accelerating solutions also avoids the problem of having to manually add a Λ term into the theory, as cosmic acceleration in this case is naturally generated from the fundamental principle of the theory itself.

Massive gravity thus has a very ambitious and exciting prospect; the catch, of course, lies in its underlying assumption that graviton is actually massive. LIGO has already successfully detected gravitational waves [18] and measurements of the speed of gravitational wave put an upper bound to the mass of graviton [19], which is

$$m \leqslant 7.7 \times 10^{-23}\ \mathrm{eV}/c^2, \tag{7}$$

an extremely small value. Although technically this doesn't disprove massive gravity, it still puts

[8]One of the authors, Professor Claudia de Rham, works in the Department of Physics at Imperial College London. She is one of the main contributors to the recent development of massive gravity.

[9]We need $M_{\mathrm{Pl}} \to \infty$, $m \to 0$, and $(M_{\mathrm{Pl}} m^2)^{1/3}$ fixed. Here M_{Pl} is the Planck mass and m is the graviton mass.

it under a very tight constraint.

And this really brings us to the other side of the story of modified gravity: as we've stated, these theories are designed to agree with solar system observations, which are of course significant and provide a great starting point, but this by no means give us the complete picture. Indeed, if any of them is to replace GR and be accepted as a better theory of gravity that can answer, or at least provide insight to the cosmological constant problem, it needs to pass tests at *all* scales, from laboratory tests to cosmological observations, and everything in between.

It is the best of times

On that note, the limit on grativon mass given by gravitational waves is very much representative of the current state of observational verification of modified gravity: all these theories are subjected to very tight constraints. Many other existing observations, including measurements of the cosmic microwave background by the Planck telescope, characteristics of Cepheids[10] and red giants, observations of dwarf galaxies and supernovae, testing of the Weak Equivalence Principle[11] in satellite orbit, among many others, have either constrained parameters of modified gravity theories down to narrow regions, or concluded that there is insufficient evidence to favor any modified gravity theory over general relativity, which therefore remains our best model for the universe [3, 8, 20].

This is far from the end of the story, however. Current cosmological tests, despite in good agreement with GR and suggesting no evidence for modified gravity, introduce additional assumptions about the characteristics of the universe that may yet still be proven wrong [5]. Additionally, future tests of gravity, such as the LISA mission for observing black holes [21] and the BepiColombo mission for studying Mercury [22], will test gravity at unprecedented accuracy and thus have a chance of revealing small deviations from GR that have evaded current observations. And above all these, the cosmological constant problem remains unsolved, and modified gravity is still one of the few roads ahead.

All eyes on the future then. Indeed, modified gravity has always been an exciting field of research, and there can only be even more enthusiasm looking at the progress we can have. Of course, no one knows where the future leads. Maybe one of the observations yields a brilliant piece of evidence that validates one modified gravity theory, opening up a brand new world of discovery; maybe all the observations end up supporting GR even more, and we have to give up on modified gravity and look for other alternatives. One thing is certain, however: whether it is success or

[10] A

[11] Imagine you are trapped in a closed elevator floating in space and you feel you are pressed against the floor. This principle states that there is no way of telling whether that is due to the elevator being in a gravitational field or the elevator accelerating.

failure that awaits us, it will be a good thing. After all, at the frontier of physics, we love failure as much as success, and regardless of the eventual outcome, modified gravity will point us to a better path toward the eventual resolution of the cosmological constant problem and other unsolved mysteries and continue challenging and advancing our understanding of the universe.

References

[1] R. F. C. Vessot et al. "Test of Relativistic Gravitation with a Space-Borne Hydrogen Maser". In: *Physical Review Letters* 45.26 (Dec. 29, 1980), pp. 2081–2084. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.45.2081.

[2] G. Gabrielse et al. "New Determination of the Fine Structure Constant from the Electron g Value and QED". In: *Physical Review Letters* 97.3 (July 17, 2006), p. 030802. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.97.030802.

[3] Planck Collaboration et al. "Planck 2018 Results. VI. Cosmological Parameters". In: (Sept. 20, 2019). arXiv: 1807.06209 [astro-ph]. URL: http://arxiv.org/abs/1807.06209 (visited on 12/28/2019).

[4] Steven Weinberg. "The Cosmological Constant Problem". In: *Reviews of Modern Physics* 61.1 (Jan. 1, 1989), pp. 1–23. ISSN: 0034-6861. DOI: 10.1103/RevModPhys.61.1.

[5] Philip Bull et al. "Beyond ΛCDM: Problems, Solutions, and the Road Ahead". In: *Physics of the Dark Universe* 12 (June 2016), pp. 56–99. ISSN: 22126864. DOI: 10.1016/j.dark.2016.02.001.

[6] Bhuvnesh Jain and Justin Khoury. "Cosmological Tests of Gravity". In: *Annals of Physics* 325.7 (July 2010), pp. 1479–1516. ISSN: 00034916. DOI: 10.1016/j.aop.2010.04.002.

[7] B. Bertotti, L. Iess, and P. Tortora. "A Test of General Relativity Using Radio Links with the Cassini Spacecraft". In: *Nature* 425.6956 (Sept. 2003), pp. 374–376. ISSN: 0028-0836, 1476-4687. DOI: 10.1038/nature01997.

[8] Austin Joyce et al. "Beyond the Cosmological Standard Model". In: *Physics Reports* 568 (Mar. 2015), pp. 1–98. ISSN: 03701573. DOI: 10.1016/j.physrep.2014.12.002.

[9] Philippe Brax et al. "Detecting Dark Energy in Orbit: The Cosmological Chameleon". In: *Physical Review D* 70.12 (Dec. 10, 2004), p. 123518. ISSN: 1550-7998, 1550-2368. DOI: 10.1103/PhysRevD.70.123518.

[10] Philippe Brax et al. "F (R) Gravity and Chameleon Theories". In: *Physical Review D* 78.10 (Nov. 19, 2008), p. 104021. ISSN: 1550-7998, 1550-2368. DOI: 10.1103/PhysRevD.78.104021.

[11] Wayne Hu and Ignacy Sawicki. “Models of f (R) Cosmic Acceleration That Evade Solar System Tests”. In: *Physical Review D* 76.6 (Sept. 10, 2007), p. 064004. ISSN: 1550-7998, 1550-2368. DOI: 10.1103/PhysRevD.76.064004.

[12] Shin’ichi Nojiri and Sergei D. Odintsov. “Unifying Inflation with ΛCDM Epoch in Modified f(R) Gravity Consistent with Solar System Tests”. In: *Physics Letters B* 657.4-5 (Dec. 2007), pp. 238–245. ISSN: 03702693. DOI: 10.1016/j.physletb.2007.10.027.

[13] Junpu Wang, Lam Hui, and Justin Khoury. “No-Go Theorems for Generalized Chameleon Field Theories”. In: *Physical Review Letters* 109.24 (Dec. 14, 2012), p. 241301. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.109.241301.

[14] Eleftherios Papantonopoulos, ed. *Modifications of Einstein’s Theory of Gravity at Large Distances*. Vol. 892. Lecture Notes in Physics. Cham: Springer International Publishing, 2015. ISBN: 978-3-319-10069-2 978-3-319-10070-8. DOI: 10.1007/978-3-319-10070-8.

[15] Claudia de Rham and Gregory Gabadadze. “Generalization of the Fierz-Pauli Action”. In: *Physical Review D* 82.4 (Aug. 10, 2010), p. 044020. ISSN: 1550-7998, 1550-2368. DOI: 10.1103/PhysRevD.82.044020.

[16] Claudia de Rham, Gregory Gabadadze, and Andrew J. Tolley. “Resummation of Massive Gravity”. In: *Physical Review Letters* 106.23 (June 10, 2011), p. 231101. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.106.231101.

[17] Claudia de Rham et al. “Cosmic Acceleration and the Helicity-0 Graviton”. In: *Physical Review D* 83.10 (May 11, 2011), p. 103516. ISSN: 1550-7998, 1550-2368. DOI: 10.1103/PhysRevD.83.103516.

[18] B. P. Abbott et al. “Observation of Gravitational Waves from a Binary Black Hole Merger”. In: *Physical Review Letters* 116.6 (Feb. 11, 2016), p. 061102. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.116.061102.

[19] B. P. Abbott et al. “GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2”. In: *Physical Review Letters* 118.22 (June 1, 2017), p. 221101. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.118.221101.

[20] Pierre Touboul et al. “*MICROSCOPE* Mission: First Results of a Space Test of the Equivalence Principle”. In: *Physical Review Letters* 119.23 (Dec. 4, 2017), p. 231101. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.119.231101.

[21] *The Laws of Nature | Lisamission.Org*. URL: https://www.lisamission.org/articles/lisa-mission/gravitational-universe-science-case-lisa/laws-nature (visited on 01/05/2020).

[22] Andrea Milani et al. “Testing General Relativity with the BepiColombo Radio Science Experiment”. In: *Physical Review D* 66.8 (Oct. 31, 2002), p. 082001. ISSN: 0556-2821, 1089-4918. DOI: 10.1103/PhysRevD.66.082001.