The Unpredictable Future of General Relativity

Word count: 2644

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Gravity and the Predictability

The general theory of relativity has been the best physical description of gravity since its first publication by Einstein in 1915. General relativity (GR), which describes gravity not as a Newtonian force but as a result of the dynamical geometry of space and time, has seen extraordinary success in predicting new phenomena and withstanding the many experimental tests to which it has been subjected in the last hundred years. The prediction and observation of phenomena from gravitational waves [1] to frame dragging [2], gravitational lensing, and even the expansion of the universe itself has solidified the position of GR as a pillar of modern physics. However, exploring further into the underpinning mathematics, there are still unanswered questions with potentially far reaching consequences for the future of the theory. In particular, when it comes to predicting the future, does general relativity actually make sense?

Firstly, what is meant by predicting the future? GR is a classical theory (a theory with no quantum-ness) and generally in classical physics theories, if you have enough information about a system at some moment in time, then it is in principle possible to know what will happen or what has happened at any time in the future of past - they are deterministic. For example, imagine you have an isolated box containing some classical particles of known masses. If you know the position and momentum of each particle at some point in time, then using Newton's laws of motion it is possible to calculate the position and momentum of every particle at any earlier or later time. The same is true for field theories, described by field equations, such as Maxwell's equations for the electromagnetic field - prediction is possible using these equations given some known initial state of the field. It is often this power of prediction that makes these theories so useful in physics, and it is known to be true for all that are known – except for general relativity.

In the case of GR, whether it is always possible to predict the future given sufficient initial data is unknown and remains an open question. This problem began with the study of the cases of charged and spinning black holes where it was noticed that there is a region of spacetime inside the black hole within which the field equations seem to be unable to uniquely predict the future. Some have attempted to provide possible ways out of this problem, such as Roger Penrose who, in 1979, put forward the strong cosmic censorship conjecture (SCC) [3]. The SCC seeks to protect GR's predictability, and as such has faced much scrutiny. A proof or disproof remains elusive and, even though many work with the assumption that it holds, there is doubt as to whether it will be found to be true. The future of general relativity, and the future in general relativity, remain uncertain…

How is Prediction *Supposed* to Work in General Relativity?

Before going into detail with where the problem with predictability seems to arise, it is a good idea to get a sense of what it means to give initial data in GR and how this is used to determine the past and future.

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Figure 1: *The Einstein field equations. It looks like one equation, but really its ten equations in disguise. All terms on the left-hand side are purely geometrical and describe the geometry and curvature of spacetime, whereas the term on the right-hand side describes the matter content (in field form) in the spacetime.*

The equations of GR, shown in *Figure 1*, known as the Einstein field equations (EFE), are at the heart of the theory. The EFE are essentially a set of ten coupled non-linear partial differential equations that describe how the matter in a region of spacetime (the 4-dimensional joining of three spatial dimensions with one time dimension) affects the geometry of spacetime itself, and how the geometry of the spacetime affects the motion of the matter within it. As put by the physicist John Wheeler: "Spacetime tells matter how to move; matter tells spacetime how to curve" [4]. As with all differential equations, solving them for a particular physical scenario requires the specification of initial conditions, and then the equations may be solved.

There are of course many ways in which initial conditions may be specified, but in general they are given on some surface in spacetime known as a Cauchy surface [5]. A Cauchy surface is like a slice of space at a moment in time. However, in GR, there is not a physical 'global time' that all clocks and observers adhere to, so there is not a unique choice for such a surface that intersects a particular position and time in spacetime. The only restriction when defining this surface is that every possible path through spacetime moving at or below the speed of light must intersect this surface exactly once. Specifying initial data on this kind of surface, such as giving the configuration of spacetime and all the matter fields being used (for example the electromagnetic field, or simply a scalar field obeying a relativistic wave equation), then allows for the EFE to be solved uniquely for the configuration of matter and spacetime at some later or earlier slice of spacetime, giving a full solution – the predictability that is desired of a theory.

Then where's the problem?

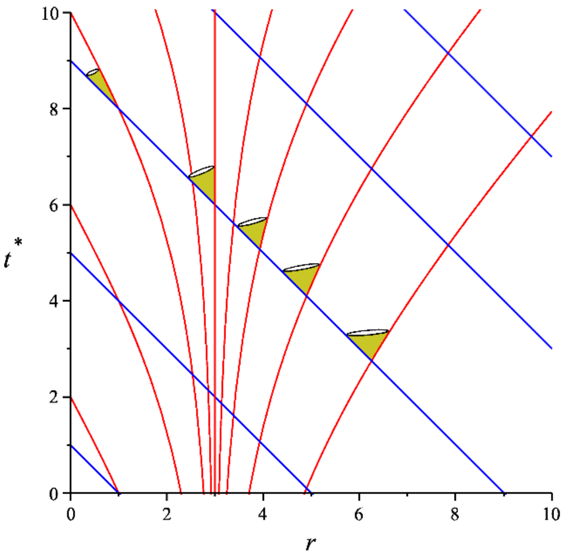

Figure 2: *This diagram [7] of time t* against radius r away from the centre of a Schwarzschild black hole shows ingoing (straight diagonal) and outgoing (curved) light rays. The yellow cones depict the causal future of an observer positioned at the tip of the cone – the regions of spacetime that they may reach in their future. The event horizon is at r = 3, and for any cones that begin for r < 3, their entire future lies within the black hole.*

The problem begins with certain types of black holes. The most famous example of a full solution to the EFE was found in 1916 by Karl Schwarzschild, known as the Schwarzschild solution [5][6], which describes a spherically symmetric empty spacetime around a point mass. This is now understood to describe a black hole, one of the most unusual and studied objects in the universe. A black hole is a region of spacetime from which nothing can escape – light, matter, and information of any kind is forbidden from leaving a black hole once behind what is known as its event horizon. The event horizon is named so for any event that happens within this surface is totally causally disconnected from any event that occurs outside of it – those outside the black hole can never have any knowledge of these events, as illustrated in *Figure 2*.

Similar solutions to the EFE have been discovered since then, in particular those describing charged and/or spinning black holes, known generally as the Kerr-Newman solution. On the surface, these kinds of solutions appear to be very similar to the Schwarzschild case – they are of similar mathematical form with extra terms added to account for the charge and angular momentum of the black hole. However, when examining their causal structure – the regions of spacetime that may affect others and those which are disconnected – they reveal themselves to be quite different.

These types of solutions to the field equations feature an event horizon, like the Schwarzschild solution, but also another kind of horizon – a Cauchy horizon [5]. The Cauchy horizon described by these solutions lies hidden behind the event horizon, closer to the centre of the black hole, and is of a different nature to an event horizon. Formally, the Cauchy horizon marks the boundary of the domain of dependence of a Cauchy surface as previously described. What does this mean? If you choose a slice of space at a moment in time – a Cauchy surface – then due to the finite speed of light, it is possible to know which positions in spacetime – events – are able to be affected by anything that happens on that Cauchy surface. Together, all such events which may be affected are the domain of dependence of that surface, as they all depend on and only on what happened on that surface in spacetime. So, what happens beyond the boundary of such a domain is not totally determined by what happened on that Cauchy surface.

For the Kerr-Newman solution, this seems like bad news. One of the key principles in using the Einstein field equations to calculate the future is that they can only determine what happens for the region of spacetime that is only dependent on the initial data – the initials conditions of the differential equations. In the case of the Kerr-Newman black holes this means that if you knew, in principle, the configuration of spacetime and all the matter in it on a Cauchy surface, the EFE cannot uniquely predict what happens in the future beyond this Cauchy horizon within the black hole. There are many solutions that satisfy the field equations, but there is no way to distinguish which, if any, is the 'correct' solution. Despite giving complete initial data, there isn't enough initial data. Even more troubling, the field equations themselves do not break down at this horizon. This seems bad because a situation in which the field equations break down signals that GR cannot be used and a more complete theory (such as the elusive quantum theory of gravity) may be required. For example, this happens inside the Schwarzschild black hole at the very centre – the singularity – where the physical curvature of the spacetime itself seems to diverge to infinity – an infinity that the field equations cannot handle. Part of the problem then is that this does not happen at the inner Cauchy horizon of the Kerr-Newman black hole – the equations do not cease to make sense, there do not seem to be any infinities, and yet the field equations totally fail to be able to predict what happens past this boundary.

To illustrate the seriousness of this problem, one can find reasonable paths through spacetime beyond this horizon that do not intersect the Cauchy surface outside the black hole on which initial data is given – the surface through which all such paths were meant to have intersected. Furthermore, this region of spacetime begins to admit 'closed timelike curves' – seemingly reasonable paths through spacetime which are closed loops that return to their starting point in time and space. The 'physics' admitted beyond this horizon is more likely the figment of mathematical imagination, so without dwelling too long, we move to discuss attempts at solutions to such problems.

Do these Cauchy horizons really exist?

The discovery of these horizons in the charged and spinning black hole solutions prompted much work from physicists and mathematicians to determine whether these horizons can really exist in nature as described by GR, or whether they lie firmly in the realm of unphysical mathematical ideas – a question often asked in physics (such as the existence of magnetic monopoles, to name one of many).

One way to analyse whether such horizons could actually exist in nature is to look at their stability. In other words, to give the black hole a slight perturbation or 'shake' and see what happens. This is done by examining the quasinormal modes (QNM) [8] of the black hole, much like analysing those of a bell when it rings, and how they behave with time. Quasinormal modes are similar to normal modes, where the oscillation of an object may be decomposed into oscillations of constituent frequencies, except that they also include the decay of the signal with time as the oscillations fade away (the mode frequencies become complex for QNM to allow for changes in amplitude of the signal with time).

It was found that such slight perturbations do not decay away with time, and the solution does not settle back to the normal Kerr-Newman black hole. In fact, it was found that for any perturbations, they become infinitely blue-shifted when they reach the Cauchy horizon – the frequencies became infinitely large, and the energy too grows and diverges [9]. This would cause the curvature of the spacetime too to become intractable for the field equations, as in the case of the Schwarzschild singularity, and the EFE break down at the Cauchy horizon. This implies that the Cauchy horizon is very unstable and would not form in any real physical setting, in particular the formation of any real black hole from a very slightly asymmetric collapsing star or other astronomical object would be subject to perturbations as described here, thus protecting predictability in the theory.

The Strong Cosmic Censorship Conjecture

In light of the dangers posed to determinism in GR by the Cauchy horizons of the Kerr-Newman solutions, Roger Penrose formulated in 1979 the strong cosmic censorship conjecture (SCC) (not to be confused with the weak cosmic censorship conjecture!). This is essentially a hypothesis that predictability is always protected within the theory, and general relativity remains a deterministic theory. This includes phenomena such as Cauchy horizons being disallowed within this conjecture, and the inability of the EFE to uniquely predict past such points being protected. The instability of these inner horizons in the known black hole solutions gives some credibility to the conjecture, however a proof has not yet been found (if it is true at all) and so it remains an outstanding open problem in the study general relativity.

The Present

Until recently, all seemed well with the SCC. It was generally assumed to be true by those working in the field - a problem that would at some point be resolved by various mathematical means, but resolved nonetheless. That is until recently, when once again belief in the conjecture was shaken amongst the community and remains shaken today.

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Figure 3: *The Einstein field equations are exactly as before except now including the Λ term – the cosmological constant. Λ is usually taken to be very small and so is normally neglected, however it may play a key role in the fate of the strong cosmic censorship conjecture.*

So far, we have considered the Einstein field equations in their most commonly used form (as shown in *Figure 1*), however there exists an extended form, as in *Figure 3* that accounts for what is known as the cosmological constant, Λ. As the name suggests, the cosmological constant is useful in some cosmological models of the universe as it characterises the expansion of the universe. Astronomical observations have shown that our own universe does seem to expand in a manner well-described by a cosmological constant – but only a very small one. In particular, the value that best describes our universe has been measured to be of the order $\Lambda \approx 10^{-52} m^{-2}$ [12] – small may be an understatement. As such, given that black holes have monumentally strong gravitational effects in comparison, the cosmological constant is normally neglected and treated to be zero.

However, a recent numerical analysis [10] of the quasinormal modes of a Reissner-Nordström black hole (a non-rotating, charged black hole) in the presence of a positive cosmological constant, found that the slight perturbations may not result in the breakdown of the EFE at the Cauchy horizon as found previously. Instead, for cases where the black hole has a particularly large charge, the perturbation results in more manageable divergences [11] – divergences that do not signal the breakdown of the field equations. This leads to solutions which may be extended past the Cauchy horizon that are non-unique and not completely specified by the initial data, once again a potential disaster for predictability. Whilst this new case may seem like a counterexample, the methods used in analysing the quasinormal modes are a linear approximation to the full non-linear field equations around the background black hole solution. As such there is still hope that non-linear effects of the full system will lend the SCC some protection.

The Future

General relativity is the best theory of gravity to date and is a pillar of modern physics. However, as explored here, there are still unanswered and potentially troubling questions hiding beneath the otherwise elegant mathematics of the theory. In particular, the theory's ability to predict the future within the interiors of certain black holes has brought the determinism of the theory into question – although the details of such cases remain unclear. Whether in any real black hole such exotic behaviour as a Cauchy horizon could really form and lead to an expanded spacetime beyond that which existed before the black hole's formation, in which complete initial data from our own universe would fail to predict the future, or whether somehow the determinism of general relativity is protected and Penrose's strong cosmic censorship conjecture holds, is unknown. This problem remains unresolved, and faith in the conjecture is certainly more shaken now than it once was since the cosmological constant has been brought into consideration. Either way, the future in general relativity, and the future of general relativity, remain uncertain.

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Plan

Article Plan: In this article, I'll discuss the background, motivations, and implications of the "Strong Cosmic Censorship" conjecture (SCC) [1].

Introduction: In the introduction, I'll discuss determinism in a good theory in physics. i.e. how, given sufficient initial data about a system, it should be possible to predict the future at all future events that may be affected by the initial data. I will then introduce how for general relativity (GR), the modern theory of gravity, it is not yet known whether this is true or false. I will also mention that if this is then false, it would be a serious flaw in the theory, and this is why the SCC was conjectured as a way to save the theory (i.e. to protect the future in the theory, and the future of the theory).

What's the problem? In this section, I will discuss what it means to give initial data and predict the future in GR. I will do this by describing as an overview the nature of spacetime in the theory (without anything mathematical), and the classification of timelike, null, and spacelike curves (this will be very brief, it is not the focus). Then I will use these to define a Cauchy surface in spacetime, on which good initial data may be given and how this may be used with the Einstein field equations (EFE) to predict the future for certain curves and events. I will then introduce that it is not known whether it is true or not that the future at all such events may be predicted by the theory, hence the problem. I will then discuss why it is not known whether this is true, in particular looking at the Kerr-Newman solution for charged and/or spinning black holes (again, not mathematically). These kinds of solutions have inner Cauchy horizons behind the event horizon, where the EFE do not break down but also cannot predict the future beyond these horizons, even given complete initial data on some past Cauchy surface. i.e. how there exist many different solutions to the EFE past these horizons, but there is not one unique solution, and so all power of prediction is lost. These Cauchy horizons are the heart of the problem.

How does the SCC address this? In this section I will explain how the SCC attempts to save prediction in the theory by conjecturing that it is in fact possible to predict the future these kinds of curves (timelike, inextensible), and Cauchy horizons are not physical and do not occur in physical reasonable scenarios.

But isn't the Kerr-Newman solution still a violation of the SCC? In this section I will describe how the Kerr-Newman solution does not violate the SCC, given a physically reasonable setting. I'll do this by describing the examination of the quasi-normal modes (QNM) of the black holes [2] (I will not go into mathematical detail on these), in analogy to the normal modes of a bell (and how any oscillations decay away with time). If one gives one of these black holes a slight "shake" (perturbation), which may be decomposed into its QNM, one finds that they are infinitely blue-shifted at the inner Cauchy horizon. When they reach this horizon, the energy of the perturbation grows to infinity (an effect called mass inflation), which leads to a curvature singularity. This means that these solutions are very unstable and would not occur in nature (the EFE break down at curvature singularities and cannot predict past them, and the SCC is saved).

So, the problem is solved? Not quite. In this section I will discuss recent work [3] where it is found that analysing QNM for these black holes in spacetimes with a positive cosmological constant (de Sitter), mass inflation does not always occur, meaning curvature singularities do not necessarily form at the Cauchy horizon, the EFE do