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Quantum Violations of Macrorealistic Inequalities

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1. What is Macrorealism?

Macrorealism (MR) is the notion that a macroscopic system with time-evolution has a definite trajectory which can be determined without disturbing past or future states.

Leggett-Garg Inequalities (LGIs) are designed to determine if a set of pairwise probabilities of a system, measured at two different times, can be matched to a **joint-probability**, thereby satisfying MR.

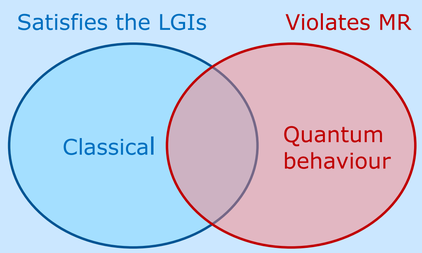

Figure 1: Violations of the LGIs indicate quantum mechanical behaviour. However, alternative notions of MR can still satisfy the usual LGIs while violating MR [1].

Quantum mechanics readily **violates MR**. Determining the degree of violation is important to see if quantum coherence endures on a macroscopic level for the development of new technologies [1].

Project Aim: Determine the inequalities representing alternative notions of MR from extended data sets and their maximum quantum violations.

2. Original Leggett-Garg Formalism

Three-Time Measurements

Classical System

The Leggett-Garg tests involve measurements of a dichotomic variable Q at different times t_i, Q_i, where the expectation values $\langle Q_i \rangle$, and the second order correlators C_{ij}, are measured. For measurements made at three times, the set of twelve **two-time LGIs**,

$$1 + s_i\langle Q_i\rangle + s_i\langle Q_j\rangle + s_i s_j C_{ij} \geq 0, \qquad (1)$$

and four **three-time LGIs,**

$$1 + s_1 s_2 C_{12} + s_2 s_3 C_{23} + s_1 s_3 C_{13} \geq 0, \qquad (2)$$

if satisfied, provide the **necessary** and **sufficient** conditions for MR [2]. $s_i = \pm 1$ are the associated measurement outcomes of Q_i. Then, the joint-probability,

$$p(s_1, s_2, s_3) = \frac{1}{8}\left(1 + \sum_i s_i\langle Q_i\rangle + \sum_{i<j} s_i s_j C_{ij} + s_1 s_2 s_3 D_{123}\right), \qquad (3)$$

exists. D_{123} is a higher order correlator and is not explicitly measured in this measurement protocol.

Quantum Mechanical System

For a quantum system with variable $\hat{Q}$, if the LGIs are satisfied, then there exists a joint-probability $q(s_1, s_2, s_3)$, which is non-negative. However, as quantum mechanics can violate the LGIs, $q(s_1, s_2, s_3)$ can be **negative**, making it a quasi-probability. The correlators of $q(s_1, s_2, s_3)$ differ from those of $p(s_1, s_2, s_3)$.

[1] Majidy, S.S., Halliwell, J.J. and Laflamme, R., 2021, *arXiv:2101.12266*
[2] Halliwell, J.J. and Mawby, C., 2019. *Physical Review A*, *100*(4), p.042103.

3. Notions of Macrorealism

Finer-grained measurements and strength of macrorealistic conditions

Original Leggett-Garg Formalism → N-gon Inequalities Formalism → Higher Order Leggett-Garg Inequalities (Sequential Measurements, Weak Measurements)

Figure 2: Increasing the number of finer-grained measurements results in a stronger notion of MR.

Notions of MR	*Data Set Measured*	*Necessary and Sufficient Conditions for MR*
Original Leggett-Garg formalism	All $\langle \hat{Q}_i \rangle$, cyclic combination of C_{ij}	All two-time LGIs, all three-time LGIs.
N-gon Inequalities formalism	All $\langle \hat{Q}_i \rangle$, all C_{ij}	All three-time LGIs, all N-gon inequalities for N correlators [2].
Higher Order Leggett-Garg Inequalities	All correlators. The form depends on the measurement scheme used.	Corresponding quasi-probability is non-negative. **Sequential Measurements**: All measurements are projective measurements. **Weak Measurements**: Measurements are weak with a final projective measurement. Give imperfect measurements but result in smaller disturbances to the system.

Table 1: Necessary and sufficient conditions for different conditions of MR.

4. Results

N	*Data Set*	*MR Inequalities: Correlators Measured, Lüders Bound* — *Original LGIs*		*Higher order LGIs*	
2	$\langle \hat{Q}_i \rangle$ C_{12}	✓ ✓	$-\frac{1}{2}$	✓ ✓	$-\frac{1}{2}$
3	$\langle \hat{Q}_i \rangle$ C_{ij} D_{123}	✓ ✓	$-\frac{1}{2}$	✓ ✓ ✓	-1
4	$\langle \hat{Q}_i \rangle$ C_{ij} D_{ijk} E_{1234}	✓ ✓ (cycle of 4)	$-2\sqrt{2}$	✓ ✓ ✓ ✓	-2
N	$\langle \hat{Q}_i \rangle$ C_{ij} D_{ijk} ⋮ $Z_{1\ldots N}$	✓ ✓(cycle of N)	$-ncos\left(\frac{\pi}{N}\right)$ N ≥ 4	✓ ✓ ✓ ⋮ ✓	-2^{n-3} $n \geq 2$

Table 2: Maximal quantum violations for different notions of macrorealism. N refers to the number of measurement times.

We have looked at determining the maximal quantum violations of the MR inequalities. This is known as the **Lüders bound**.

We can find initial states of a simple spin-$^1/_2$ system which satisfies the usual LGIs but achieves maximal violation for the higher order LGIs.

Violations of the **Lüders bound** have been discovered when using the degeneracy-breaking von Neumann measurement scheme. Finding an equivalent **von Neumann bound** is our current focus.

5. Implications for Future Research

The **degree of Leggett-Garg violations** give an indication of the system's '**quantumness'.** From these theoretical lower bounds, experimental tests can be performed to determine the maximal Leggett-Garg violations of different systems. Therefore, it is important to understand the limitations of the tests for different notions of MR.