The Starry Night

A look into turbulent flow through the eyes of Vincent Van Gogh

Word Count : 2611

Van Gogh captured the world in an utterly unique sense that naturally gave rise to a timeless admiration of his genius. Until recently this has remained solely in the realm of the arts, however physicists have now unearthed some intriguing parallels between turbulent flow and a handful of his paintings. According to Richard Feynman turbulence remains one of the 'biggest unsolved problems in classical physics' [1]; its complex motion has never been fully mathematically modelled and patiently loiters in the waiting room of the unknown universe. This Heisenberg consolidates with allegedly claiming on his deathbed that 'when I die I will ask God two questions; why relativity? Why turbulence? I believe he will have an answer for the first.' [2] Some might say Van Gogh was serving the whims of this God when scientists discovered that the patterns in certain of his paintings closely follows the observed actions of this illusive turbulence. Furthermore, the link between his masterful brushstrokes and the murkiest depths of fluid dynamics only occurred in paintings post his admission into a mental asylum; there is certain beauty in that only at his most turbulent state of mind could he see this unseen.

The physics of Turbulence

The implausibility that Van Gogh could create images of turbulence is emphasised by the current understanding of the physics behind turbulent flow. Turbulence is a certain pattern of fluid motion dictated by chaotic changes in pressure and flow velocity; this opposes laminar flow, which refers to fluid whose motion occurs in parallel layers with no disruption. Both Figures 1 and 2 illustrate the change between these two differing flow motions. Although only contributing a mere 30 pages on the subject, A.N Kolmogorov has probably had the most profound impact on turbulence theory. [3] With great physical intuition he posited theories about the structure of small-scale turbulence, which in concise mathematical form have remained highly applicable since their publication in 1941. Before outlining his input, it is perhaps conducive to begin by mentioning the mathematics behind the motion of all viscous fluid substances. These present themselves in the Navier-Stokes Equations. Their failure to fully predict turbulent flow demonstrate why it is necessary to bridge the gap with other physical models. [4]

Figure 1, This exhibits the change from a laminar to turbulent flow in a candle flame. It arises when the external conditions generate the correct ratios of forces within the fluid. [5]

This set of partial differential equations arise from applying Newton's second law of motion to fluids. Terms such as viscosity are also considered for a vastly more realistic representation of their motion. The hitch of course comes with the fact they remain unsolved. They were actually designated one of the seven millennium problems in 2000 by the Clay Mathematics Institute; finding the solution to one of these is worth a considerable 1 million dollars. With the correct treatment these equations can somewhat capture the physics required to describe turbulence, however as anticipated they become even more impenetrable. Consistent with what one would expect physically, turbulent flow only occurs when the viscosity term is negligible in comparison to the other forces governing the motion. This is problematic in the Navier-Stokes equations as the viscosity appears as the highest derivative term. To envisage why this makes any mathematician cringe, it is perhaps best to imagine multiplying this term with a factor and then think about the limiting behaviour of the equation as this factor approaches zero. It has been coined the turbulence closure problem and ultimately produces more unknowns than equations to be solved. [4] The solutions can of course be computed numerically, although it is near impossible to acquire an accurate picture; this is due to the current limits of our technology and the sheer scale and resolution required to encompass the chaotic motion. [6] It is hopefully now clear why scientists had to look elsewhere for an answer to this natural chaos.

There is a certain point, due to external conditions, where turbulence in fluids begins to form. Again, both Figure 1 and 2 exhibit the conception of this flow as opposed to a previous more linear motion. This can crudely be described by the Reynolds number, which is the ratio of inertial forces to viscous forces within a fluid. The onset of turbulent flow as opposed to laminar flow occurs when this number becomes sufficiently large. Although it is only a guide. it can aid valuable predictions in fluid motion. It also mathematically exhibits the simple idea that the more viscous a fluid, the more resistant it is to turbulent flow.

Figure 2, There is a tradition in Japanese art to illustrate turbulent ocean waves. The above is one of the most famous of this genre, here as the waves begin to break the water edges towards an almost ominous turbulence. [7]

Now back to Kolmogorov and the concepts that lie behind his fairly successful mathematical model. This initially stems from viewing turbulence as a series of eddies that form a sort of hierarchy; the smaller feed on the larger until a certain size, where the viscosity is sufficient to smooth the flow towards a more laminar motion. [3] The existence of eddies solely serves a conceptual purpose and they can be thought of as the circular motion of a patch of fluid separated from the rest of the flow. This multiscale image of turbulence was in fact proposed by Richardson in 1926, though not immediately accepted due to other infatuations at the time. As eddies are not particularly well defined it is difficult to know exactly what he envisaged in this decay, however Figure 3 helps visualise the process.

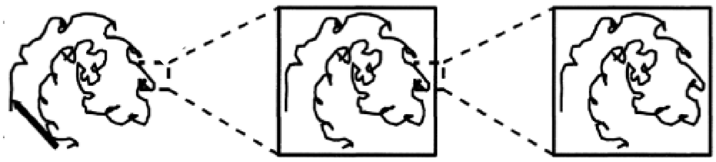


Figure 3, depicting what is thought to be meant by the cascade of eddies. The smaller feed off the larger in size. [3]

Kolmogorov combined this school of thought with another by Taylor and Von Karmen to finally obtain a somewhat successful model. This second concept viewed turbulent flow not as individual particles, but instead considered the velocity as a stochastic field. Stochastic refers to the asset of a random probability distribution whose statistical properties can then be explored and quantified. With this, Kolmogorov had the idea that as this energy cascade perpetuates, the smaller eddies become statistically isotropic. [3] This refers to their independence from the mechanism that maintains turbulence, effectively they have no preference in spatial direction. It arises due to their limited interaction with constraints on geometrical features such as boundaries. This meant that for small scale turbulent motions his equations could be applied universally, which is why his contribution is so valued in this field.

The final logic behind his model now follows. Turbulence only occurs in instances of very high Reynolds numbers where the fluid viscosity is low; this in turn suggests that the fluid in question does not dissipate energy as its viscosity is negligible. Once an eddy deforms it therefore donates all its energy to smaller members of the hierarchy. This process continues until the scale is small enough for viscosity to become significant in the subsequent eddy; here the kinetic energy dissipates into internal energy and so the model collapses. This combined with the statistically isotropic nature of smaller eddies and some elegant dimensional analysis enabled Kolmogorov to come up with his infamous equation. Although it can come in many forms, the simplest states that the energy spectrum of a turbulent fluid varies with the power 5/3 of the length scale.

Even with this longstanding equation and recent developments in technology, a full image of turbulence remains very much absent. A smooth continuous solution to the complete Navier-Stokes equations would objectively relieve the world of this problem and one can perhaps hope for a genius to come along and make this momentous leap forward. However, the likelihood of this remains slim and turbulence remains elusive to all except it seems Van Gogh.

Examples of Turbulence in Nature

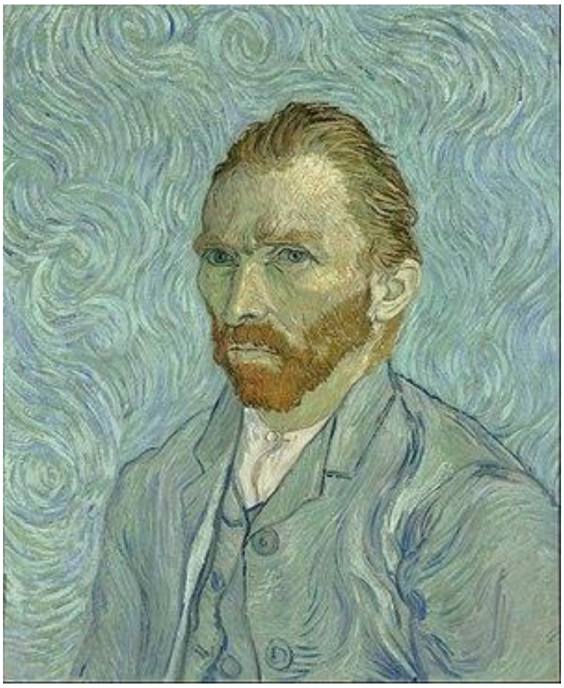
Turbulence is all around us, yet often invisible. These incalculably complex motions can be created by an action as trivial as waving your hand. Nature has allowed us a small window into this world, often with beauty in no need of human imprint. Phenomena such as the plumes of an erupting volcano or a humble stream are examples of this chaotic magnificence. In fact, the actual roar of a volcano arises from the turbulent motion of the hot gases escaping

Figure 4, the turbulent motion of the plumes of a volcano. These menacing clouds carry debris that is later deposited all over the surrounding environment. [10]

through the vents. [8] The frequency of these jets characterises the alarming grumble that signals an imminent attack on the otherwise unassuming world. Turbulence also dictates the height of the eruption column as well as factors concerning the material expelled. The extent of the homogeneous mixing of particles within an umbrella cloud depends on the strength of the turbulent flow. Debris collects at the base of the cloud where there is diminished turbulence; this ultimately leads to its discharge onto the ground below. Figure 4 demonstrates these clouds that have the strength to carry such huge loads and cause such inevitable destruction. [9]

A more mundane example concerns a river, where turbulent flow here goes beyond mere aesthetic pleasure. An understanding of this motion is required now more than ever as it provides key insight in the management of natural resources and protection of aquatic ecosystems. [11] The increase in temperature from climate change has bought about many alterations to the world we're accustomed to, one of these being an increase in flooding. A thorough appreciation of turbulence here could abet confronting these negative consequences, as knowledge of the hydrodynamic state ensures effective protection measures for the surrounding areas.

Turbulence and Van Gogh



Now to return to our protagonist in Van Gogh and how his ethereal ability to depict turbulence was discovered. There was a group of scientists in 2004 that, with the Hubble telescope, were observing the eddies in star dust. The patterns they saw around this distant star reminded them of a certain infamous Van Gogh work, The Starry Night. [12] Such conviction in this link motivated them to study the painting more closely and analyse the patterns made on a computer. This ultimately unearthed a shocking revelation within The Starry Night as well as a multitude of his other paintings; hidden underneath the exquisite artistic cloak, the turbulent fluid structures predicted by Kolmogorov's equation peeked through. This discovery was made even more haunting by the lack of this pattern in any other artist's work. Similar post-impressionist paintings such as The Scream by Munch superficially utilise the same distorted technique, however were found utterly lacking in any depiction of turbulent flow.

The analysis

It is in fact the variation in the probability density function of luminance that is the key stepping stone between these two ostensibly different fields of physics and art. Luminance features in a lot of impressionist art and refers to the perceived brightness of a colour; when used effectively it can blur outlines and suggest motion. This arguably intuitive tool was used by Van Gogh and accounts for the seemingly flickering and swirling movement of the stars in The Starry Night. [13] Motion like this can be fashioned by the use of equiluminance, which refers to areas of a painting with the same luminance. [14] This phenomenon comes from the fact that different aspects of the world around us are analysed by differing regions of the visual system. [15] Although shape is registered in an area that also processes colour information, the part of the brain responsible for motion is completely colour blind. Regions therefore of equiluminance can be differentiated by colour, however there is certain uncertainty in their position. This is due to their lack of luminance contrast, which provides too little information for effective visual analysis on the motion or location in space. Figure 5 below illustrates this illusion of flickering and pulsating movement.

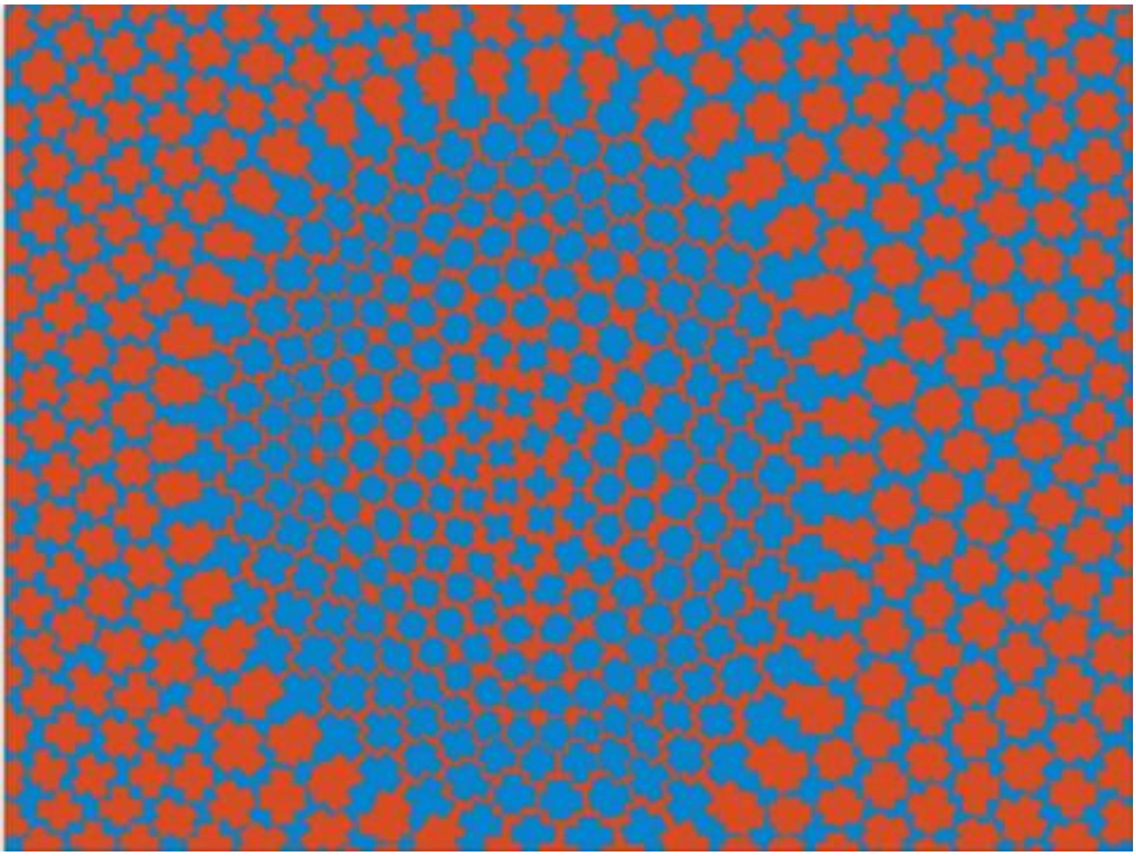


Figure 5, here the two colours are approximately the same luminance. This gives rise to the almost distorted motion you see when you look at the image. [14]

The scientists who originally explored this connection, endeavored to quantify the turbulence in certain Van Gogh paintings with a statistical analysis of luminance. This appeared to compare shockingly well with the probability density of velocity predicted by Kolmogorov's theory. Having digitalised the artworks, they computed the luminance of a pixel by measuring the scalar intensity of red, green and blue. This was further combined with a consideration of how the eye sees certain colours better than others. A PDF of pixel luminance was then obtained and several were plotted for given pixel separations. This, when compared to the predicted PDF of velocity in turbulent motion, followed a very close relationship. [15] The method was repeated for many other of Van Gogh's paintings and perhaps the most fascinating outcome arose from the Self-portrait with pipe and bandaged ear. This was conceived in a relative period of calm in his otherwise tempestuous final years. It can be gleaned from letters he wrote to his siblings that he viewed this painting as an attempt to escape his image of a tortured artist. [16] While attempting to erase his grave reality, he also erased the turbulent motion he had previously managed to create with his brushstrokes.

There is also further evidence of Van Gogh's unique gift in capturing the intrinsic nature of the world. An experiment was conducted recently where the behavior of bumblebees was studied around different paintings. These bees had never seen natural flowers and were observed to be significantly more enticed towards Van Gogh's flowers than with any of the other artists. [15] This seems to suggest that, as with turbulence, he succeeded in reproducing the true essence of his environment.

The Artist

Having looked at the surrounding physics, it seems only right to finish with a look into the life of Van Gogh. This was predominantly governed by his mental illness, which appears to have wrenched open the doorway into a completely inimitable way of viewing of his surroundings. It almost transcends comprehension that he could somehow see this turbulent flow and we are left merely to consider our own inadequacies.

Born in Holland in 1853, he was the son of a pastor and bought up religiously from a young age. [17] A struggle with identity and onslaught of fluctuating emotions meant that it took him until 27 to find his calling as an artist. It is said he was determined to give happiness with his creations of beauty, which just highlights more extensively his lack of confidence and struggle to form personal relationships. His art provides snapshots of this turbulent life and his style of painting evolves with his age and unsettled emotions. The period where he admitted himself to the Saint-Paul asylum in the Saint-Rémy-de-Provence is perhaps most relevant as it contains the most works that illustrate turbulence. [18] This was post the infamous psychotic episode where he cut off part of his ear. In the asylum he produced around 150 paintings and was finally starting to attain recognition as an artist; there is great misfortune in that great genius always seems to have to come from such great torment. In 1890 he took his own life in the sinister scenery of his final painting, Wheatfield with crows. It took only mere decades following his death for his work to rise to international acclaim. It is a shame that he will remain in complete ignorance of the happiness that his paintings did indeed give to so many people.

It is far too difficult to express let alone explain this connection between mathematics and art that Van Gogh seemed to have unified in his mind. He once wrote, “Why, I ask myself, shouldn’t the shining dots of the sky be as accessible as the black dots on the map of France? Just as we take the train to get to Tarascon or Rouen, we take death to reach a star”. [19] He however had no need to trek along the path of death to reach the stars, their true essence seemed already to live within him. Nature presented before us a hugely complex concept and a haunting response has come in a single man able to unconsciously unlock and represent this deep mystery. There is a certain rousing beauty that this only occurred during his most intense period of suffering; articulated by Van Gogh himself in his final words to his brother, ‘The sadness will last forever’.[20]

Bibliography

[1] Mpg.de. (2019). What is behind Einstein’s turbulence?. [online] Available at: https://www.mpg.de/7078493/Einstein-turbulence

[2] Scienceworld.wolfram.com. (2019). Heisenberg, Werner (1901-1976) -- from Eric Weisstein's World of Scientific Biography. [online] Available at: http://scienceworld.wolfram.com/biography/Heisenberg.html

[3] Jiménez, J. (2004). The Contributions of A. N. Kolmogorov to the theory of turbulence. Arbor, CLXXVIII(704), pp.589-606.

[4] Tennekes, H., Lumley, J. (1997). A first course in turbulence. Cambridge, Massachusetts: The MIT Press.

[5] En.wikipedia.org. (2019). Laminar–turbulent transition. [online] Available at: https://en.wikipedia.org/wiki/Laminar–turbulent_transition#/media/File:Laminar-turbulent_transition.jpg

[6] Why global regularity for Navier-Stokes is hard. (2019) [online] Available at: https://terrytao.wordpress.com/2007/03/18/why-global-regularity-for-navier-stokes-is-hard/

[7] Bourne, M. (2019). Math in art - Hokusai’s “The Wave”. [online] Intmath.com. Available at: https://www.intmath.com/blog/mathematics/math-in-art-hokusais-the-wave-595

[8] Volcano.oregonstate.edu. (2019). Volcano Sounds During Eruptions | Volcano World | Oregon State University. [online] Available at: http://volcano.oregonstate.edu/volcano-sounds-during-eruptions

[9] Suzuki, Y. and Koyaguchi, T. (2010). Numerical determination of the efficiency of entrainment in volcanic eruption columns. Geophysical Research Letters, 37(5),

[10] Fearless Flight. (2019). Volcanic Ash Shows Air Travel at Its Best and Worst - Fearless Flight. [online] Available at: https://fearlessflight.com/volcanic-ash-shows-air-travel-at-its-best-and-worst/

[11] Mossa, M., Termini, D. and Davies, P. (2018). Turbulence in River and Maritime Hydraulics. Water, 10(7), p.963.

[12] The Fluid Dynamics of “The Starry Night”: How Vincent Van Gogh’s Masterpiece Explains the Scientific Mysteries of Movement and Light. (2019) [online] Available at: https://www.brainpickings.org/2014/11/13/van-gogh-starry-night-fluid-dynamics-animation/

[13] Workwithcolor.com. (2019). Color Luminance: perceived brightness, luminance and contrast. [online] Available at: http://www.workwithcolor.com/color-luminance-2233.htm

[14] Webexhibits.org. (2019). Luminance Differences Affect Our Perceptions. [online] Available at: http://www.webexhibits.org/colorart/anuszkiewicz.html

[15] Aragón, J., Naumis, G., Bai, M., Torres, M. and Maini, P. (2008). Turbulent Luminance in Impassioned van Gogh Paintings. Journal of Mathematical Imaging and Vision, 30(3), pp.275-283.

[16] Painting-planet.com. (2019). Self-portrait with a bandaged ear by Vincent Van Gogh - Description of the Painting. [online] Available at: https://painting-planet.com/self-portrait-with-a-bandaged-ear-by-vincent-van-gogh/

[17] Vangoghgallery.com. (2019). Van Gogh Biography Overview | Van Gogh Gallery. [online] Available at: https://www.vangoghgallery.com/misc/biography.html

[18] My Modern Met. (2019). Van Gogh: How the Post-Impressionist's Work Evolved During His Short Life. [online] Available at: https://mymodernmet.com/vincent-van-gogh-life/2/

[19] Satyagraha. (2019). Van Gogh: "We take death to go to a star". [online] Available at: https://satyagraha.wordpress.com/2018/01/29/van-gogh-we-go-to-a-star/

[20] Webexhibits.org. (2019). Theo van Gogh to Elisabeth van Gogh : 5 August 1890. [online] Available at: http://www.webexhibits.org/vangogh/letter/21/etc-Theo-Lies.htm

Other Images sourced: https://www.vincentvangogh.org. (2019). Vincent van Gogh: 300 Famous Paintings Analysis & Complete Artworks. [online] Available at: https://www.vincentvangogh.org/