Ghostbusting: Empirical (In)validity of Hidden-Variable Theories

I. Ideal and the Real

In 1927, Heisenberg published a paper arguing that both the position and the momentum of any particle cannot be determined simultaneously - and stating that the product of the uncertainties of the two quantities must be greater than some constant. In 1930, Schrödinger generalised this result and showed that some pairs of physical quantities - be it position, energy, spin, etc - are ”incompatible”: there is a lower bound on the precisions to which we can know them simultaneously, and measuring one will alter the world so that the other has more allowed values. This was a groundbreaking result: following purely from the nature of small particles to behave like waves, it was a fundamental statement about how the universe can and cannot be described [1, Chap. 10].

However, in 1935, Einstein, Podolsky, and Rosen published a paper describing what they thought to be a paradox: suppose we have two particles with two measurable properties (”observables”) that are incompatible, which interact in a way that makes each observable value related for both particles; for example, two entagled particles whose spins must have opposite values along every axis, with spin values for orthogonal axes being incompatible. The three authors argued that if we measure one of the two observables with absolute precision for one of the particles and the other observable for the other particle, this will allow us to know both observables with absolute precision for both particles, even though they should be incompatible according to Heisenberg [2].

The resolution of the paradox stems from the fact that Einstein assumed ”local realism”; that is, that the world possesses both *reality* and *locality*. *Reality* means that particles have well-defined observable values outside of measurement - specific position, momentum, spin... at any time, even when we don’t observe that particle. This is closely related to *determinism*: the notion that the future shape of a system is completely determined by its present state, and if we were to know all particles’ positions, momenta... etc with absolute precision, we could accurately predict where and with what properties all the particles will be at any given time. *Locality* means that change in one particle’s properties cannot instantaneously affect another, distant particle.

The most common way to resolve the paradox is via the Copenhagen interpretation of quantum mechanics. This interpretation rejects *reality* and describes the world via wave-functions, which tell us the probability of measuring a specific value of an observable. This approach is *indeterministic* - before the measurement, we cannot know the value of the observable, and the system will ”choose” a value once the measurment is performed. Moreover, it is meaningless to talk about the value of the observable before the measurement, and the act of measurement actively changes the system in a way that cannot be predicted, even if we know everything there is to know about the system. And while it is true that the measurement on one particle of an entangled pair forces the other particle’s wave-function to collapse via something we call ”steering” and Einstein dubbed ”spooky action at a distance” (Einstein, 1947, in a letter to Max Born), this doesn’t allow for faster-than-light signal transmission and turns out to be compatible with special relativity [3, pp. 426-428]. Hence we can say the Copenhagen interpretation preserves *locality*, depending on which technical definition of *locality* you prefer.

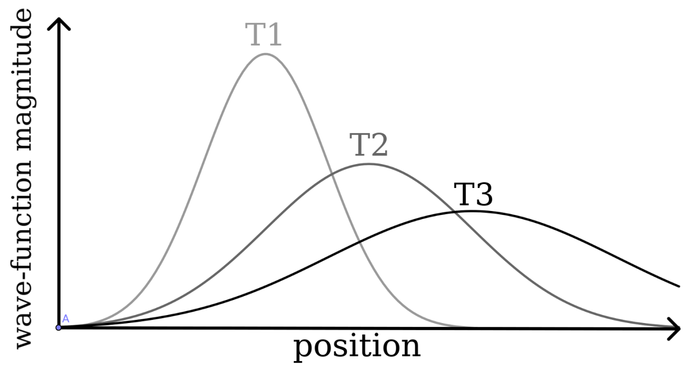

Fig. 1: Wave-function of a particle moving to the right. As time progresses, we get more uncertain about its position.

There are, however, other ways to resolve the paradox; interpretations of quantum mechanics trying to preserve *reality*. The most well-known bulk of these are ”hidden-variable theories”, which allow for scary things lurking in the shadows to get rid of the spooky action at a distance.

II. How Not to Be Seen

The Copenhagen interpretation, as widely accepted as it was when it was developed in the 1920’s, caused controversy in the scientific community. Its inherently indeterministic nature in particular meant a heavy blow to the predictive power of physics, which famously prompted Einstein’s notorious statement that ”[God] is not playing dice” (Einstein, 1926, in a letter to Max Born). By 1927, several physicists have tried to propose alternative interpretations of quantum mechanics that would first and foremost preserve *realism*, and, implicitly, *determinism*.

The first attempt we know of was developed by Einstein in an article based on the idea that the wave-function describes the development of a system in a statistical sense only, and the system does possess real position, momentum etc. Einstein ended up withdrawing the article from publication

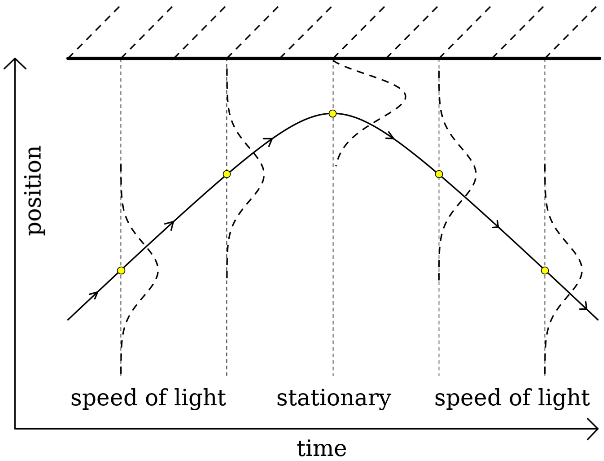

Fig. 2: Light being reflected off a mirror as interpreted by Pilot Wave Theory: the wave-function guides the single photon towards and from the mirror. Around the moment of reflection the wave-function splits into two parts which are superimposed on one another, travelling in opposite direction - their guidance cancels out and the particle is momentarily stationary.

Fig. 3: Light scattering on a diffraction grating. (Courtesy of Nave, 2017.)

because he spotted major problems with the implications of his conclusions.

Later that year, at the 5th Solvay Conference, which was attended by all important contemporary physicists, Louis de Broglie proposed the pilot wave theory, a *realistic* and *deterministic* theory that interprets both the particles and the wave-function as "real": the particles have well-defined positions, which are the hidden variables and therefore are unmeasurable, and constitute all matter. The wave-function "guides" the particles by altering their velocities (think of an ocean wave carrying plastic waste from one place to another) in such a way that the wave-function always represents the particle density in space. This of course means the theory is not *local*, as all particles instantaneously affect each other's positions (as rigirously shown by John Bell [4, pp. 377-379]), however, it can be shown that it's still compatible with relativity [5, Chap. 6].

A. Pauli's objection

When de Broglie presented the Pilot Wave Theory, he was met with an objection by none other than Wolfgang Pauli, a famous pioneer of quantum mechanics. Pauli's objection was based on Fermi's research of inelastic scattering of an electron - i.e. the way a travelling electron changes direction when it collides with a certain object called the rotator. Fermi showed that this situation is mathematically equivalent to an infinitely wide light wave travelling through an infinite "diffraction grating", which is nothing else than thin stripes with spaces between them, which scatter light in an unintuitive fashion which was studied long before quantum mechanics (with first observations dating back to the 17th century and complete mathematical treatment done in the 19th century). Light scatters on diffraction grating in a way that produces bright "fringes" at certain angles. The crux of Pauli's objection is that in Fermi's model, the scattered light waves overlap, as they are infinitely wide; this translates to the electron's wave-function's parts being superimposed on one another. However, in the Pilot Wave Theory, when a particle is caught in the superposition of multiple wave-functions, their guiding effects "average out", which in this case would result in the particle not settling on a constant velocity, unable to "choose" one fringe direction to follow.

This may seem like a sound argument and a way to empirically invalidate a hidden-variable theory: after all, we can measure the way electrons scatter on rigid rotators, and it seems that the Copenhagen interpretation and the Pilot Wave Theory give different predictions for this particular system. And, for a long time, this was in fact considered to be a definitive counterargument against what was subsequently treated as a failed theory, abandoned even by its creator de Broglie. However, it was much later rediscovered by David Bohm, who in 1952 developed it into what we now call the de Broglie-Bohm theory or Bohmian mechanics. His developments did not change the principles of the theory: in fact, it was later shown that even the original de Broglie's formulation was merely under-developed, not flawed, as there was an error in Pauli's objection. An exhaustive description of Pauli's objection and its treatment within the Pilot Wave Theory can be found in [7, Chap. 10].

B. The ESSW objection

For a long time it seemed that the Bohmian interpretation would stay around as an alternative to the Copenhagen interpretation. Since they both have equal predictive power, it would be hard to design an experiment in which their predictions would differ in a tangible way. However, Bohmian mechanics has great *retro*dictive power: since it insists that the positions of particles are real and specific at all times, even before we make a measurement, it forces us to assign a trajectory to a particle once we measure its position. In other words, Bohmian mechanics allows us to say where a particle has been after we find out where it is right now. Contrast this

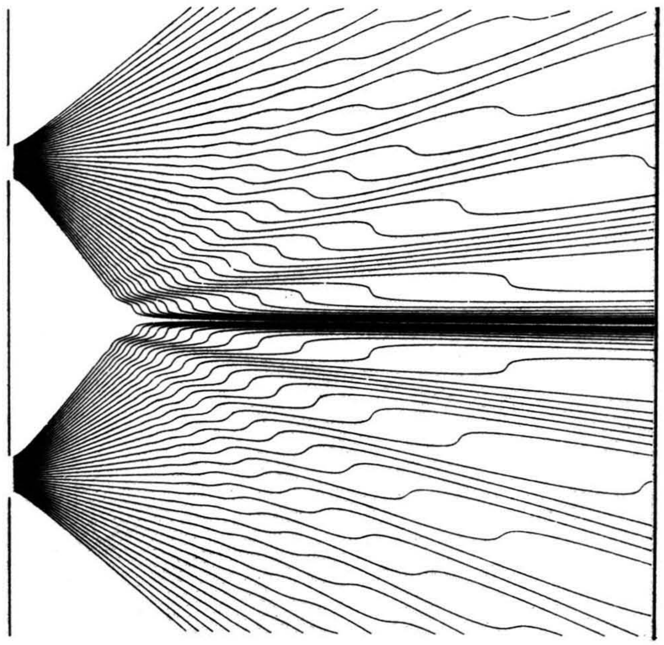

Fig. 4: Bohmian trajectories for the double slit experiment. A particle travels from the left, passes through the slits and terminates at the detector screen, where its position is measured. Note that all the trajectories passing through the upper slit terminate exclusively on the upper half of the screen, and all the trajectories passing through the lower slit on the lower half of the screen. (Figure taken from Englert et al, 1992, p. 1177.)

with the Copenhagen interpretation, which, when confronted with this question, refuses to answer: it is meaningless to talk about the particle's position before measurement, the only thing that was there was the wave-function, and even that is not real in the strict sense.

In 1992, Englert, Scully, Süssmann, and Walther published a paper [8] in which they claimed to have found a fundamental error in Bohmian mechanics based precisely on this retrodictive property. At the core of the problem is what is called the double-slit experiment. This is a famous experiment which demonstrates the wave-like nature of matter at small scales via the effect called "interference", however, this phenomenon is not the subject of the ESSW paradox (as named after the authors' initials). Rather, the problem is quite simple: suppose you're sending particles through a wall with two narrow slits onto a detector screen. Quantum mechanics allows us to calculate how the wave-function of the particles propagates through the system and its shape tells us which parts of the detector screen will have many particles ending up there and which parts won't. Now, Bohmian mechanics makes a *retro*diction regarding the possible trajectories of the particles: if we divide the detector into two symmetrical halves based on the position of the slits, we can show that the pilot wave will carry all particles which pass through one slit onto only the half corresponding to that slit (and the same holds for the other slit as well) - see Fig. 4. Now, if we add a detector right after the wall with the slits which measures which slit a particle passed through, we would want its readings to agree with the Bohmian retrodiction; that is, if we measure a particle on the detector screen in the part corresponding to slit 1, we would expect the detector to say that the particle passed through slit 1.

However, this seemingly contradicts our understanding of quantum mechanics, which predicts that if we measure a particle passing through one of the slits, its wave-function changes in a way that allows it to travel to any of the two halves of the detector screens. This seems as the perfect opportunity to experimentally check the validity of Bohmian mechanics: by this clever trick, we can turn a retrodiction ("For a particle measured at a particular position on the detector screen, we know which slit it travelled through") into a prediction ("For a particle measured travelling through a particular slit, we know which half of the detector screen it must end up in"). The only thing left was to conduct the experiment.

In 2016, more than 20 years after the ESSW paradox was proposed, Dylan Mahler et al published a paper [9] in which they described how they conducted the experiment, observed the contradiction between the which-slit detector reading and the position on the screen measured, and put forward an explanation which seemed to resolve the paradox while preserving the integrity of Bohmian mechanics. Their argument was based on the assertion that the original ESSW paper disregarded the nonlocal nature of Bohmian mechanics - the ability of particles to affect each other instantaneously. The devil was in the detail - particularly in the workings of the which-slit detector. This detector works by entangling the particle traversing the slits with a second particle (think of the EPR paradox), so that when particle 1 passes through one slit, particle 2 has a certain spin, and if particle 1 passes through the other slit, particle 2 has a different spin. Here, the choice of using spin is arbitrary; it is useful because it works as a one-bit storage, or a lever: either it's flipped or it's not, and it's easy to tell which it is with measurement. The crux of the counterargument against ESSW is that, because Bohmian mechanics doesn't have *locality*, the spin of particle 2 continues to be affected by the position of particle 1 even after particle 1 passes through one of the slits and can change so that it seemingly disagrees with the slit which the measured Bohmian trajectory passes through. The paper has many more interesting observations: for example, this "forgetfullness" of particle 2 is proportional to the distance of the detector screen from the slit wall, which is to be expected, as, in the classical view, the particles disperse more on greater distances. It also contains much more technical discussion on the nature of the measurement of the spin of particle 2, and a rigorous mathematical justification for the nonlocal behaviour of the entangled pair of particles within the framework of Bohmian mechanics.

The ESSW objection was the most recent attempt at invalidating Bohmian mechanics experimentally. The debate about it is ongoing and the whole topic of hidden-variable theories is divisive. As of now, Bohmian mechanics is treated as just another one of many interpretations of quantum mechanics which agree on what is experimentally observed, often differing only in technicalities or philosophical context. There are several more hidden-variable theories among them, for example Wheeler-Feynman time-symmetric theory, and for a long time it seemed that showing one interpretation to be superior to others is impossible. However, recent development might have opened a path towards changing the way we look at hidden-variable theories, and maybe dismissing them altogether in one move.

III. Both Directions at Once

Quantum mechanics is currently in a strange, liminal space. On one hand, we focus on solving really specific, technical challenges with very tangible effect on modern chemistry and engineering. On the other, we're still failing to determine how quantum mechanics even works and whether it's *real* and *deterministic* or *local*. Ultimately, our goal in both directions is to have stronger tools to make better predictions. This is the motivation behind studying hidden-variable theories - since they're *real* and *deterministic*, they may hold the secret of transcending the probabilistic nature of the Copenhagen interpretation and allowing us to determine outcomes of experiments with more certainty that we have now. This would require us to understand the mechanism behind why some variables are hidden and if it's possible to infer some information about them which we currently don't have access to.

In 2011, Colbeck and Renner published an article [10] in which they claimed to have proven that it is impossible to create an extension of quantum mechanics which gives us greater predictive power than the current theory, regardless of your choice of the specific interpretation. Their proof is based on principles developed in quantum cryptography and is highly technical. If you accept their conclusion as correct, it would mean that we no longer need to look for conceptual extensions of quantum mechanics, be it hidden-variable theories or something completely different, as the effort is doomed from the beginning, and the only thing left for us is, in the words of David Mermin, to "shut up and calculate".

However, Colbeck and Renner's article is far from universally accepted. A crucial part of their proof is based on the technical definition of what they call the "assumption FR", which refers to the freedom of chosing measurements, and the idea that the choice of what we measure cannot be affected by certain events. In 2013, Ghirardi and Romano published an article [11] in which they've shown that if we formulate assumption FR in a different way, Colbeck and Renner's argument becomes invalid.

To some, this may seem disheartening, as it introduces a new conceptual problem to a theory which is already riddled with those. However, it is also a major opportunity, as further discourse in this area may bring a resolution to the problem; and whichever way the pendulum swings, it will be a major breakthrough. Whether we show that no interpretation of quantum mechanics can be more useful than what we already have or prove that there may be further advancement in the conceptual side of the theory, it will mean a whole new era of research, permanently forwarding our understanding of the world. After all, quantum mechanics is the most fundamental theory developed in the last 100 years, and the problem of its interpretation is inherently linked to the working of the universe. It may very well be that, in the next few years or decades, the way we think about physics will change forever. And, in whichever direction Nature leads us, we shall remember the words of David Hilbert, uttered to mathematicians, but applicable to physics as well: "We must know, we will know!" (Hilbert, 1930.)

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