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BOSONIC STRING THEORY, T-DUALITY AND EXTENDED GEOMETRIES

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INTRODUCTION

String theory is a leading candidate for a ***unified theory of physics***, in which all four fundamental forces are unified in a ***quantum mechanical*** framework. The development of string theory has lead to a much deeper understanding of symmetries and dualities within mathematics. Further, the physical implications of 26 spacetime dimensions must be understood topologically - how do these hidden dimensions manifest themselves?

WHAT IS A STRING?

String theory predicts that matter does not consist of point particles, but rather one-dimensional strings. There are two types of string - open or closed. Trajectories of strings are called worldsheets, which are two dimensional surfaces on a spacetime diagram. The worldsheets of open and closed strings are ***strips*** or ***tubes***, respectively [1, 2].

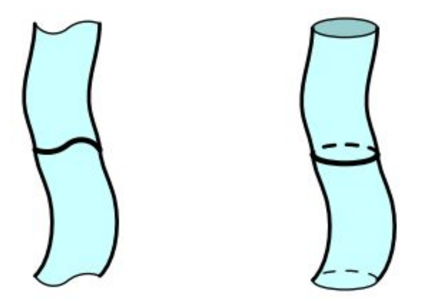

Fig. 1. Comparison of the worldsheets of an open string (left) and a closed string (right) [3]

QUANTISATION OF THE STRING

The string is found to satisfy the 2-dimensional free wave equation and therefore, the general solution of the string coordinates are given by a fourier expansion of the "left-moving" and "right-moving" modes.

$$X^\mu = x_0^\mu + \alpha' p^\mu \tau + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^\mu e^{in\sigma} + \tilde{\alpha}_n^\mu e^{-in\sigma}\right)e^{-in\tau}$$

In order to obtain a quantum mechanical description of the string, the theory must be quantised. In canonical quantisation, the classical poisson bracket relations of the fundamental phase space variables and replaced with commutation relations, in which the variables are promoted to operators. A ***Fock space*** can be built by defining creation and annihilation operators, such that a generic string state is

$$\alpha_{-n_1}^{\mu_1} \dots \alpha_{-n_p}^{\mu_p} \tilde{\alpha}_{-n_1}^{\mu_1} \dots \tilde{\alpha}_{-n_p}^{\mu_p} |0, p\rangle$$

We define the vacuum (or ground) state, $|0, p\rangle$, such that when acted upon with an annihilation operator, it gives zero. In momentum space, this vacuum state is an eigenstate of the momentum operator, with an eigenvalue of momentum.

Every possible state in the Fock space is a different excited state of the string.

Given the Poincaré invariance of the theory, Noether's theorem gives the Virasoro conserved charges, which are given as quantum mechanical operators, by considering normal ordering. The classical Virasoro constraints therefore become quantum mechanical constraints:

$$L_n = \tilde{L}_n = 0 \quad \forall n \quad \rightarrow \quad \begin{aligned} L_m|\varphi\rangle &= \tilde{L}_m|\varphi\rangle = 0 \quad for\ m > 0 \\ L_0|\varphi\rangle &= \tilde{L}_o|\varphi\rangle = a \end{aligned}$$

However, the Fock space may have ***negative norm states***. These states are called ***ghosts*** and aren't physical [1].

26 DIMENSIONS

In order to ensure string theory has a ***ghost-free*** spectrum, we can consider a general, physical excited state of the ***open string***. From this, and in order to ensure a residual gauge symmetry remains [2], we obtain the values:

$$a = 1, D = 26$$

Bosonic string theory predicts exactly 26 spacetime dimensions!

QUANTUM GRAVITY

In considering the spectrum of ***closed strings***, it becomes clear that the ***symmetric*** state of the first excited mode mirrors the equations of gravity.

The first excited state of the closed string can be the massless, spin-2 graviton

The ***antisymmetric*** state of the first excited mode mirrors Maxwell's equations of electromagnetism giving rise to the B and H fields.

TACHYONS AND FERMIONS

One of the main issues with the bosonic string is the ***tachyonic ground state***. Tachyons are hypothetical particles that have a negative mass and therefore travel faster than the speed of light. Bosonic string theory also fails to incorporate ***fermions***. Therefore, superstring theory, which predicts 10 or 11 spacetime dimensions, has been developed to successfully overcome these shortcomings by introducing supersymmetry [2].

CONCLUSION

We have shown bosonic string theory exists quantum mechanically as a ***Fock space.*** Given the similarity between the dynamics of the first excited state of the closed string with the equations of gravity, string theory could explain quantum gravity. However, the predictions of ***tachyons*** and 26-D spacetime are weak points in the bosonic theory. Attempts to compactify the unobserved 22 dimensions reveals ***T-duality*** symmetry and results in new topological conditions such as the quantisation of flux H. The full topological implications of T-duality are not fully known.

T-DUALITY

T-duality is a symmetry in the string dynamics which emerges when strings are compactified onto a ***circular*** spacetime direction. For a simple 1D circle there is a strange equivalence between strings wrapped around a circle of radius ***R*** and a circle of radius ***R***$^{-1}$. Both geometries will exhibit the same physics.

$$R \Leftrightarrow \frac{1}{R}$$

Under this symmetry, the quantised string momentum in one regime is equivalent to the energy with which the string wraps around the circle in the other regime and vice versa.

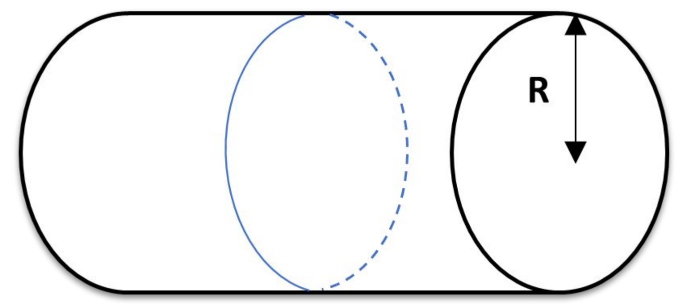

Fig. 2. Representation of circular spacetime. The non-circular dimensions lie in the direction of the axis of the cylinder.

Buscher Rules

T-duality also emerges in more complicated spacetimes which contain ***isometric*** circular directions. The complex spacetime has a metric, $g_{\mu\nu}$, and a B-field, $B_{\mu\nu}$.
The symmetry transformations are determined by the ***Buscher Rules***:

- $\tilde{g_{yy}} = \frac{1}{g_{yy}}$
- $\tilde{g_{yi}} = \frac{B_{yi}}{g_{yy}}$
- $\tilde{B_{yi}} = \frac{g_{yi}}{g_{yy}}$
- $\tilde{B_{ij}} = B_{ij} - \frac{g_{yi} B_{yj} - g_{yj} B_{yi}}{g_{yy}}$
- $\tilde{g_{ij}} = g_{ij} - \frac{g_{yi}\, g_{yj} - B_{yi} B_{yj}}{g_{yy}}$

T^3 + FLUX

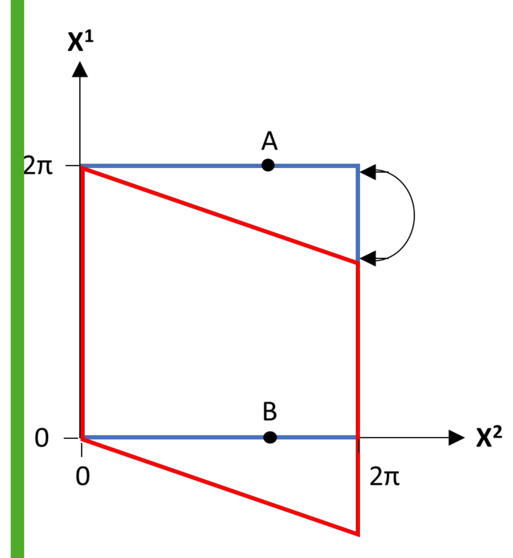

Fig.3. Representation of 3-Torus' as 2D manifolds. Point A is identified with B by periodicity. T-duality converts the square blue Torus to the skewed red Torus.

The topological implications of T-duality can be demonstrated by modelling the periodic spacetime dimensions as a square 3-Torus. Taking a constant B field, acting in one direction of the torus, and performing a T-duality transformation in another, transforms the square 3-Torus to a ***'Twisted Torus'*** [4]. A topological condition on the ***flux*** H_{ijk} arises due to constraints on the ***patching*** of the 'Twisted Torus':

The flux H must be quantised

Paradox?

There remains the freedom to perform another T-duality transformation in one of the two remaining Torus directions. However, the ***isometry*** condition seems to forbid performing T-duality in all 3 Torus directions.

Are there 2 or 3 T-Dualities?

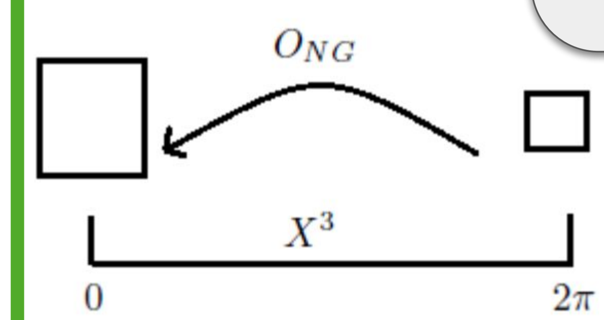

Fig.4.Representation of the patching of the non-geometrical Torus. O_{NG} is a matrix transformation between the two regimes [4].

When there exists a T-duality in two of the three directions of the 3-Torus, the resulting torus is non-geometrical. In order to attempt to patch this and understand this geometry, we must impose another T-duality [4].

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