Sampling the CMB with Advanced Sampling Techniques

Oscar Hickman, supervised by Prof. Alan Heavens and Dr. Boris Leistedt
(Astrophysics Group)

Imperial College London

1) Cosmology and the CMB:

- The Cosmic Microwave Background (CMB) is crucial to building cosmological models of the universe (such as the ΛCDM model), with fluctuations measured to estimate the cosmological parameters of the early universe.
- CMB measurements have improved over time (Figure 1) but still hold great uncertainty because the anisotropies' signal-to-noise magnitude is substantial.
- These anisotropies are harmonically transformed into a power spectrum which for the Planck data holds significant error at small multiple numbers (Figure 2).

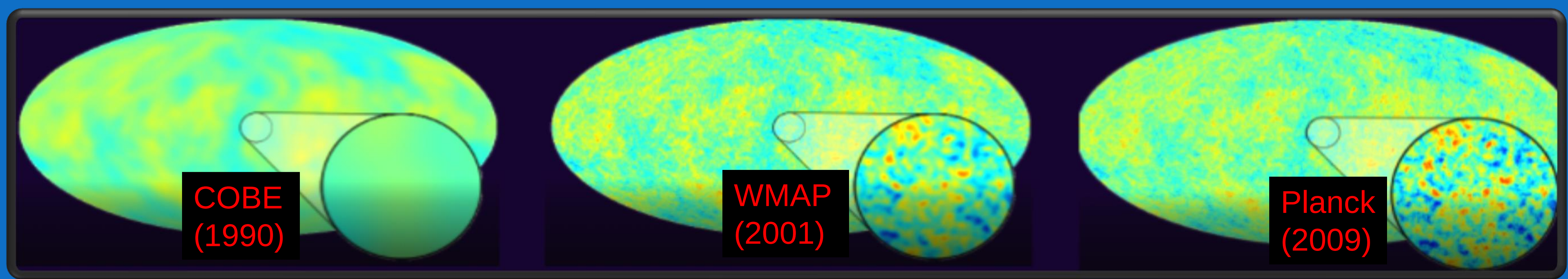

Figure 1: CMB temperature angular power spectrum predicted by the ΛCDM model compared to that of the Planck satellite [1].

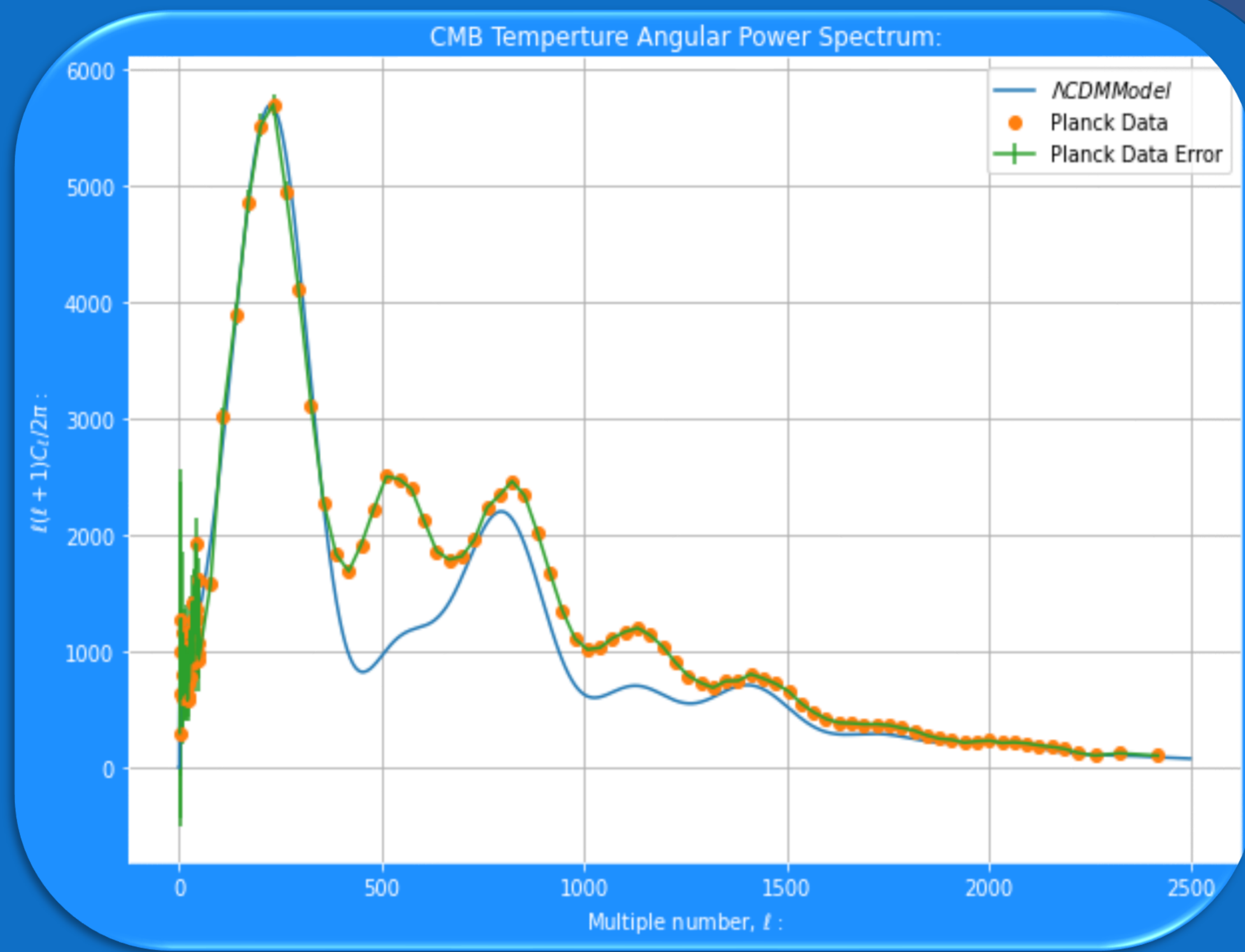

Figure 2: CMB temperature angular power spectrum predicted by the ΛCDM model compared to that of the Planck satellite (Planck Data sourced from [2]).

2) Bayesian Statistics and Sampling:

- Bayesian statistics can be used to estimate parameters which make up a given data set. By creating a 'posterior' distribution using Bayes Theorem.
- This is then updated by sampling the parameter space using the posterior distribution function.
- Hamiltonian Monte Carlo sampling (HMC) is efficient in high parameter space and combined with Tensorflow's optimised functions with automatic distribution, this allows efficient high–parameter sampling.

3) Aims:

- Build a model which can efficiently sample the posterior of the Planck data power spectrum by exploring the posterior using the Tensorflow No U-Turn Sampler (NUTS).
- These samples (after a burn-in phase) can then be averaged to find a better estimate of the CMB angular temperature power spectrum.

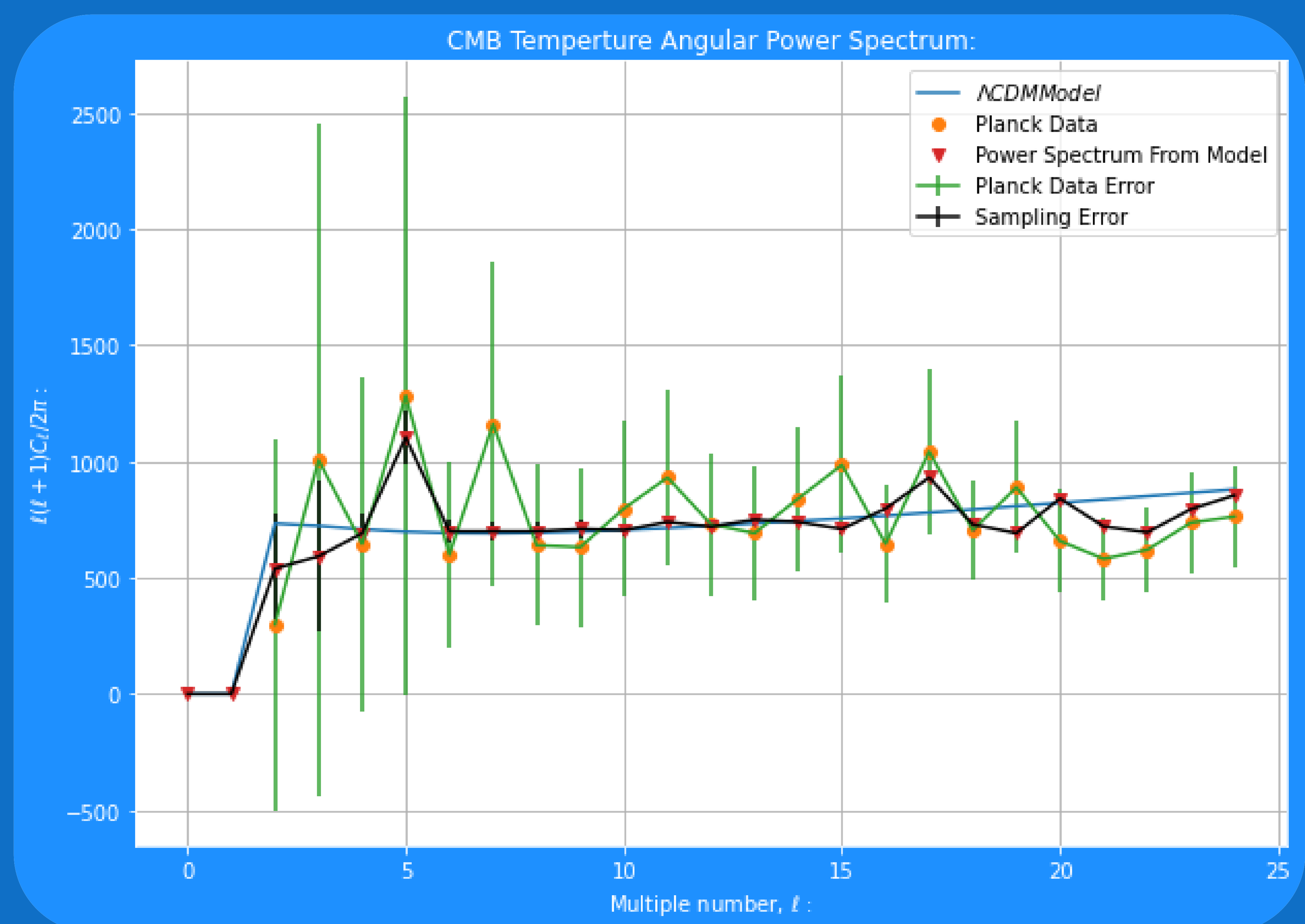

Figure 3: CMB temperature angular power spectrum predicted by the ΛCDM model compared to that of the Planck satellite (Planck Data sourced from [2]) compared to the models returned values (with a map NSIDE = 128) – at low multiple number.

4) Results:

i) 8 different initial positions in parameter space (priors) were sampled 2000 times with the NUTS (with the burn in phase removed and the remaining samples mean calculated.

ii) The data fit to the ΛCDM cosmological model's predicted power spectrum than all recorded satellite data (with the comparison to Planck in Figure 3).

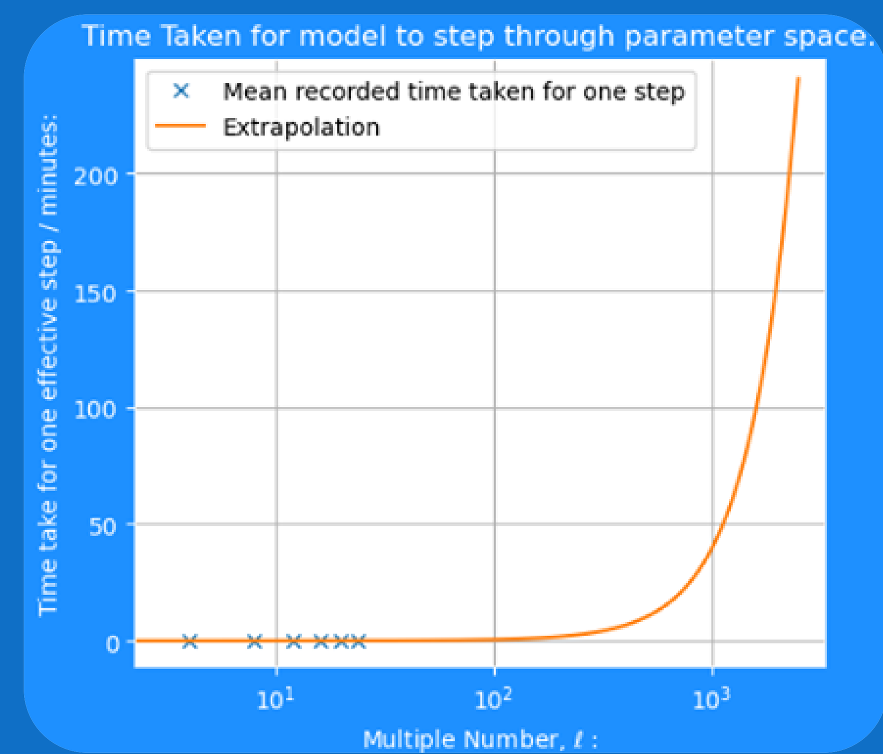

Figure 4: The time taken for an effective sample/minutes as a function of the largest multiple number used (extrapolated using a polynomial with power 3/2 through the origin).

iii) The time taken for an effective step was recorded as a function of the maximum multiple number (Figure 4) used to predict whether the model would be expandable to the Planck data. Since this was run on a standard desktop computer, this certainly seems feasible.

5) Conclusion and Future Outlook:

- A model was built that predicts CMB power spectrum samples more aligned with the ΛCDM cosmological model's predicted power spectrum than all recorded satellite data in the region of low multiple number.
- The convergence of the sampling and the time taken for each sample is predicted to be sufficient at multiple numbers of order of the Planck data.
- Future advancements could expand the model to greater 'l' and greater map resolutions to predict the CMB's power spectrum better.

References:

[1] – Planck Satellite, Cosmic Microwave Background, Available at: https://plancksatellite.org.uk/science/cmb/ (Accessed: 25/02/2021).
[2] – NASA/IPAC (22/02/2021) COM_PowerSpect_CMB_R1.10, Available at: https://irsa.ipac.caltech.edu/data/Planck/release_1/ancillary-data/previews/COM_PowerSpect_CMB_R1.10/index.html (Accessed: 22/02/2021).
[3] - J. F. Taylor, M. A. J. Ashdown, M. P. Hobson (2007) Fast optimal CMB power spectrum estimation with Hamiltonian sampling,. Available at: https://arxiv.org/abs/0708.2989