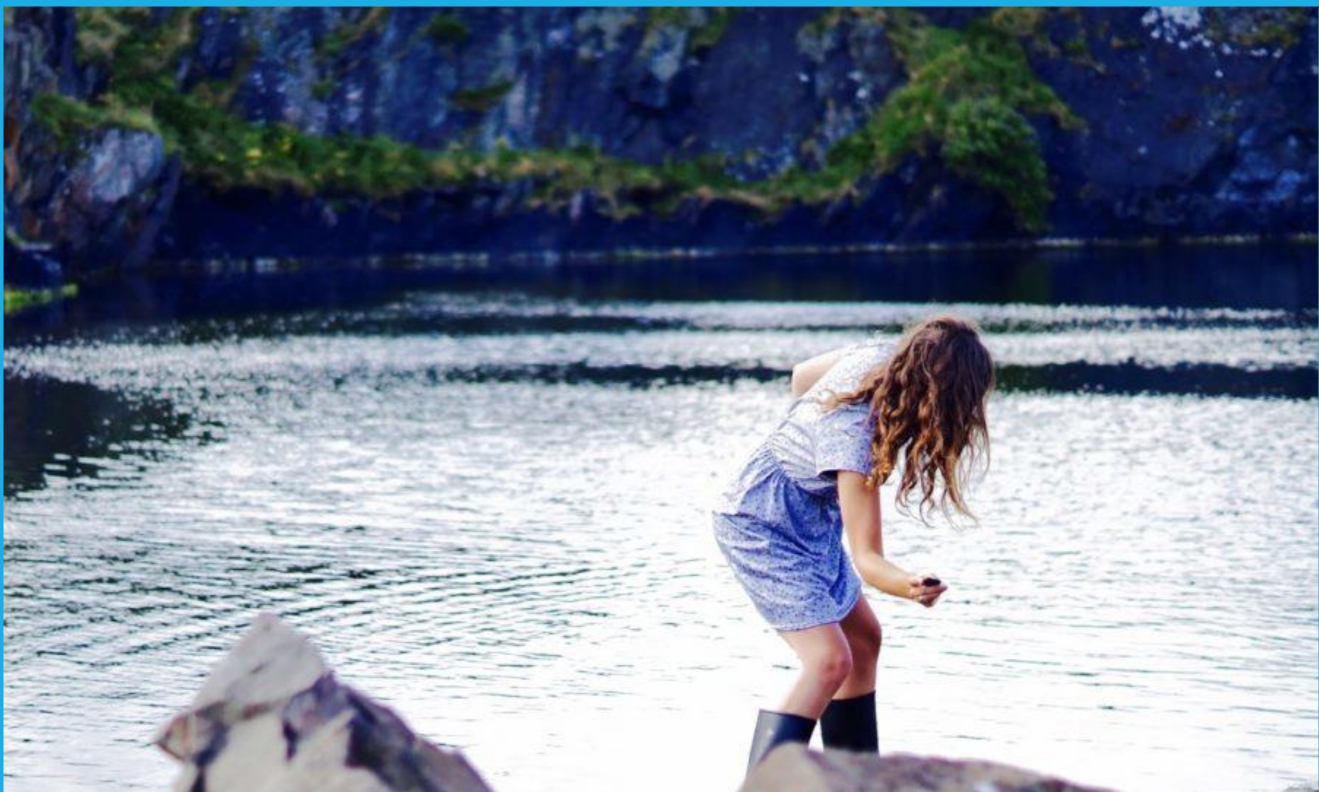
DUCKS AND DRAKES: THE PHYSICS OF STONE SKIMMING



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The Physics of Stone Skimming

It's a warm summer's day and you decide to drive down to the coast. Forgetting to do your research, you find yourself at a shingle beach. Those dreams of feeling the velvety golden sand between your toes are history. Rather than heading home, you decide to make the most of the situation. Walking along the deserted beach (with your shoes on) you reach the shoreline. In an attempt to reconnect with your youth, you pick up a pebble and toss it into the water, hoping to see it skip across the surface. All you hear is a disappointing *plop*. So, you pick up another and decide to refine your technique. Perhaps you alter your throwing angle or the tilt of the pebble upon release. Or maybe you impart some spin on it using your index finger. You may even search for a particular shape of stone you consider to be optimal. Upon further revision of your throw, you see that your stones are travelling further, and they achieve more skips across the water's surface. *Maybe the day isn't ruined after all* you think to yourself as you continue stone-skimming until you either become bored, it gets late, or you rid the beach of all its pebbles.

(J.D. Creaghan Group Inc., 2015)

The art of stone-skimming is one many have endeavoured to master. The current world record for the number of skips is 88, achieved by Kurt 'Mountain Man' Steiner on September 6th, 2013; the record for the greatest distance is held by another man, Dougie Isaacs, at 121 metres (400 ft) who broke his own world record. There is even a World Stone Skimming Championship held every year in which anyone is allowed to compete.

After many skips, it becomes difficult to count the number of bounces since the distance between them gets increasingly smaller; the stone appears to glide across the water near the end of its run. The stone is said to 'pitty-pat' across the water's surface.

At the Franklin Championship in 2011, Eric Henne threw his stone across the water which was announced by three judges to have completed 37 skips. Though many spectators remarked that it seemed as if there were more. The subjective nature of counting the skips may have cost Eric the title, which was awarded to Kurt Steiner; his stone had completed 39 skips (Kennedy, 2014).

(World Stone Skimming Championships, n.d.)

Whilst the stone's trajectory through the air can be modelled as being parabolic, its interaction with the water is more challenging to approximate (Bocquet, 2003). If the stone can be assumed to be perfectly flat and circular, the problem can be broken down into a simpler one to solve with some 'user-friendly' physics; this analysis can be used to determine the optimal parameters of a throw to maximise the number of skips.

Walking on water

Why does a stone 'walk' on water? During the collision, the stone experiences a reaction force from the water, which provides lift. Provided the reaction force generates enough lift to counteract the stone's own weight, it will skip. If during a collision, the entire stone becomes submerged in the water, according to Archimedes' principle it will sink, ending its run.

Deformation of water during collision. The stone does not skip off the surface, rather it becomes partially submerged. The reaction force from the water provides lift on the stone; if large enough to counteract the stone's weight it will skip (Richard, 2016).

How fast must you throw your stone?

Intuitively, it makes sense that if the stone is not thrown with enough speed, it will not skip. So, what is the lower bound on the stone's speed for a successful bounce?

French physicist, Lydéric Bocquet's simplified model of the stone-water collision is shown in Fig. 1. The stone enters the water at a tilt θ, with velocity V at an 'angle of attack' β to the horizontal. The stone penetrates a distance z below the unperturbed surface of the water.

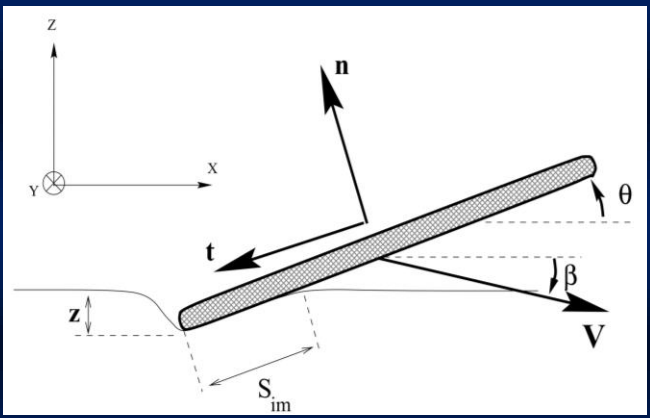

Fig. 1. Simplified model of stone-water collision. Motion occurs in the XZ plane. The reaction force acting through the centre of mass of the immersed section of the stone can be resolved perpendicular and parallel to the stone's surface (Bocquet, 2003).

On collision with the water, the stone experiences a reaction force which can be resolved along $\underline{\boldsymbol{n}}$ and $\underline{\boldsymbol{t}}$. The magnitude of the force experienced is dependent on the type of flow of the water around the stone. A Reynolds number (ratio of the inertial forces to viscous forces in a fluid) of 10^5 suggests turbulent flow around the stone. Therefore, the reaction force $\underline{\boldsymbol{R}}$ is proportional to the square of the stone's velocity, the immersed area, S_{im}, and the mass density of water, ρ.

$$\underline{R} = \frac{1}{2}C_l\rho V^2 S_{im}\underline{\hat{\boldsymbol{n}}} + \frac{1}{2}C_f\rho V^2 S_{im}\underline{\hat{\boldsymbol{t}}}$$

where C_l and C_f are the lift and drag coefficients respectively (Bocquet, 2003).

Motion in the x and z directions are governed by the equations,

$$M\frac{dV_x}{dt} = -\frac{1}{2}\rho V^2 S_{im}(C_l sin\theta + C_f cos\theta)$$

$$M\frac{dV_z}{dt} = -Mg + \frac{1}{2}\rho V^2 S_{im}(C_l cos\theta - C_f sin\theta)$$

where M is the mass of the stone (Bocquet, 2003).

Solving the z equation of motion assuming θ is small and using the constraint that the stone must not be completely immersed in the water, Bocquet (2003) found that the minimum critical velocity, U_{min}, for skipping was,

$$U_{min} = \frac{\sqrt{\frac{16M\cdot g}{\pi\cdot C\cdot\rho\cdot a^2}}}{\sqrt{1-\frac{8M\cdot tan^2\beta}{\pi\cdot a^3\cdot C\cdot\rho\cdot sin\theta}}}$$

where $C \approx C_l \approx C_l cos\theta - C_f sin\theta$ and a is the diameter of the stone. Provided the stone's incident velocity upon collision is greater than U_{min}, it will skip.

The 'Magic' Angle

If thrown at the wrong angle, the stone will not travel very far. So at what angle must you throw your stone for it to achieve the maximum possible number of skips?

Clanet, Hersen and Bocquet (2004) found that for a fixed angle of attack $\beta = 20°$, the optimal tilt angle was about 20°. The relationship between the minimum critical velocity and θ is shown in Fig. 2. This angle allowed the largest domain for β $(15-45)°$ for a successful skip and was also found to minimise the stone-water collision time.

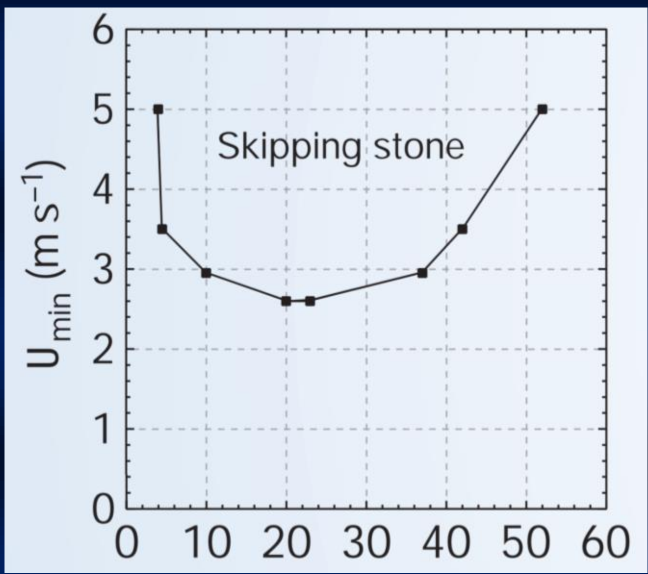

Fig. 2. Minimum critical velocity for varying stone tilt angle θ, for $\beta = 20°$. At $\theta \sim 20°$, U_{min} is at its minimum value (Clanet, Hersen and Bocquet, 2004).

As the stone gets further into its run its velocity will decrease due to drag. But if U_{min} is at its minimum value, it will take longer for the stone's velocity to fall below it. It would therefore be beneficial for the stone to enter at $\theta \sim 20°$ for every collision. But how can we preserve this angle throughout the stone's motion?

Nobody likes rolling stones

Throw a frisbee across an open field without any spin and you observe that it simply flips over and falls to the ground. Spin on a frisbee acts to balance it during its flight (Scodary, 2007). This 'gyroscopic stabilizing effect' is also why it is important to impart spin on the stone upon release.

The stone experiences a reaction force from the water; the line of action of which passes through the centre of mass of the submerged section of the stone. In the absence of spin, the stone will rotate about its centre and in its subsequent motion, θ will change.

The relationship between the minimum critical velocity and θ is shown in Fig. 2. On subsequent collisions with the water, the angle θ may be such that the stone's velocity falls lower than U_{min} and hence cease skipping.

Imparting spin on the stone with its angular momentum vector $\underline{\boldsymbol{L}}$ parallel to $\underline{\boldsymbol{n}}$, generates a stabilizing effect. The torque of the reaction force about the stone's centre is in the **y**-direction as shown in Fig. 1. (from the right-hand rule). Torque is defined as the rate of change of angular momentum; $\underline{\boldsymbol{L}}$ will not process in θ.

Whilst spinning the stone stabilises the value of θ throughout its motion, roll stability is compromised; $\underline{\boldsymbol{L}}$ processes in the YZ plane. The thrower may observe their stone wobbling from left to right during its motion.

Pitty-pat

Poor Eric Henne may have been 'robbed' of the Franklin Championship title in 2011 due to a miscount. At the end of a stone's run it becomes increasingly difficult to judge the individual skips with the naked eye because experimentally, the distance between them decreases.

Successive skips of a stone across a lake. The distance between each successive skip decreases and the stone 'pitty-pats' until it sinks (Anon, n.d.).

Modelling the stone's motion through the air as parabolic between each skip, the distance Δx_N between the N^{th} and $(N+1)^{th}$ bounce can be approximated by,

$$\Delta x_N = \Delta x_o \sqrt{1 - \frac{N}{N_f}}$$

where Δx_o is the distance between the 0^{th} and 1^{st} skip and N_f is the total number of skips (Bocquet, 2003).

Fig. 3. shows the general decrease in distance between 4 consecutive skips where $N_f = 4$.

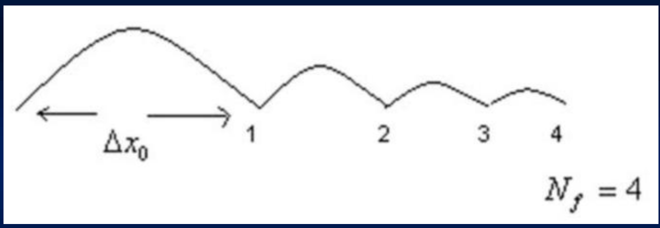

Fig. 3. Decrease in distance between four successive skips (Humble, 2007).

Fig. 4. shows the ratio of the distance between the N^{th} and $(N+1)^{th}$ skip to the distance between 0^{th} and 1^{st} skip for increasing N. As N approaches N_f the distance falls to zero faster.

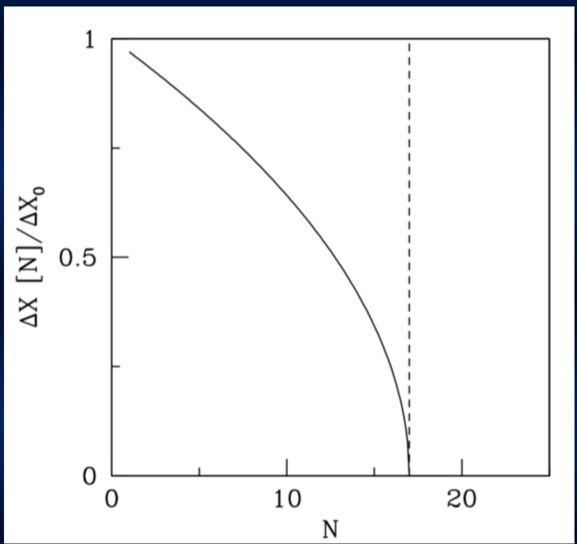

Fig. 4. Ratio of the distance between the N^{th} and $(N+1)^{th}$ and 0^{th} and 1^{st} skips against the number of collisions. The decrease becomes sharper in later collisions in the stone's run producing this 'pitty-pat' effect (Bocquet, 2003).

Hopscotch

Kurt Steiner. Guinness world record holder for the most consecutive skips of a stone on water. But how quickly did the "Mountain Man" need to throw his stone to achieve this near impossible feat of 88 skips?

During collision with the water, the stone's kinetic energy is dissipated as it does work to overcome friction. The number of skips performed by the stone was approximated by Bocquet (2003) to be,

$$N_c \sim \frac{V^2}{2g\mu l}$$

where μ is a dimensionless number related to the lift and drag coefficients and l is the distance traversed by the stone during the collision.

On collision with the water, the 'shock' destabilises the stone and over time, this can cause the stone to sink. The greater the angular velocity, the more stable the stone. Bocquet (2003) derived an approximation for the relationship between the number of skips and the initial angular velocity imparted on the stone.

$$N_c \sim \frac{R\omega^2}{g}$$

Kurt Steiner's world record of 88 skips required (as approximated by Bocquet's model) an initial velocity V of 18 $m \cdot s^{-1}$ and initial rotational speed of 21 $rev \cdot s^{-1}$.

Putting it all together

A good technique requires not only high translational and rotational speeds but also an optimal tilt and angle of attack. Large initial translational speeds are preferable since it takes longer for the stone to lose all its kinetic energy. High rotational speeds preserve an optimal angular tilt on the stone for further successful skips. As the limits of physical strength continue to be stretched, we may see world records continuing to tumble. Based on Bocquet's findings, the theoretical maximum number of skips based on current estimates of physical strength is around 300; this assumes that perfectly circular, flat stones are used (Wired, 2018).

So next time you're at the seaside or the park and decide to skim stones and want to impress someone, remember that you must:

1) Release the stone with as much velocity as you can.
2) Impart lots of spin on the stone.
3) Whilst trying to maximise the velocity and spin, you need to throw the stone downwards at about 20° to the surface of the water with the stone tilted at 20° to the horizontal with the leading edge higher than the trailing edge.

With practice, you may even be able to beat the "Mountain Man"!

References

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