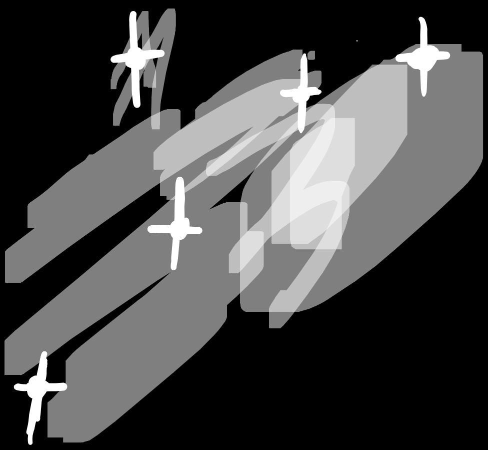
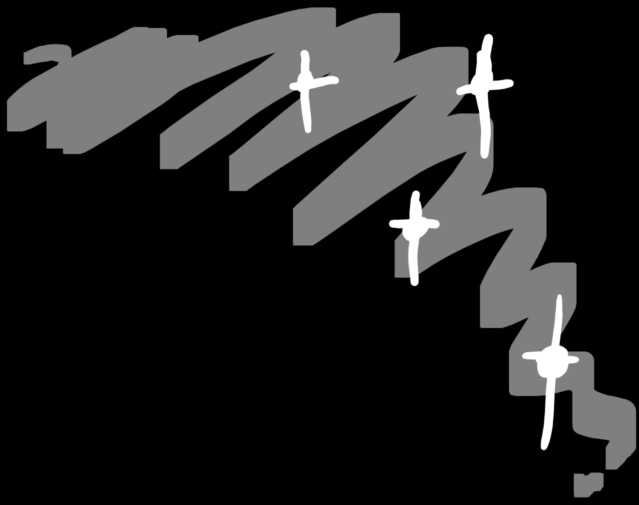
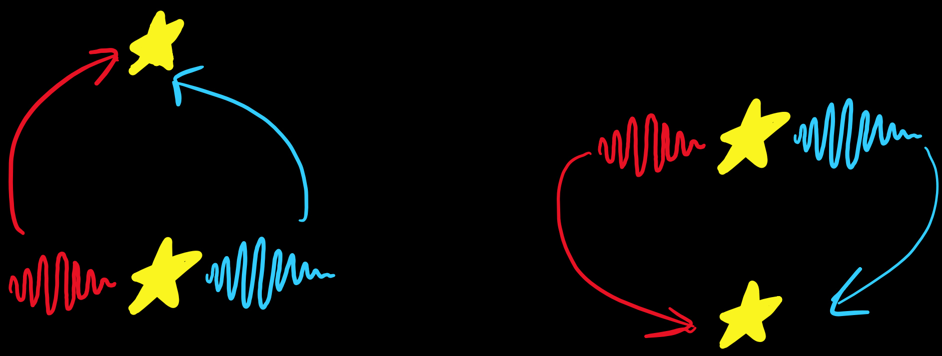






The Faster you Run, the Closer the Horizon Follows:

On the Fulling-Davies-Unruh Effect

Word count: 2746 (Excluding References)

The Faster you Run, The Closer the Horizon Follows:

On the Fulling-Davies-Unruh Effect

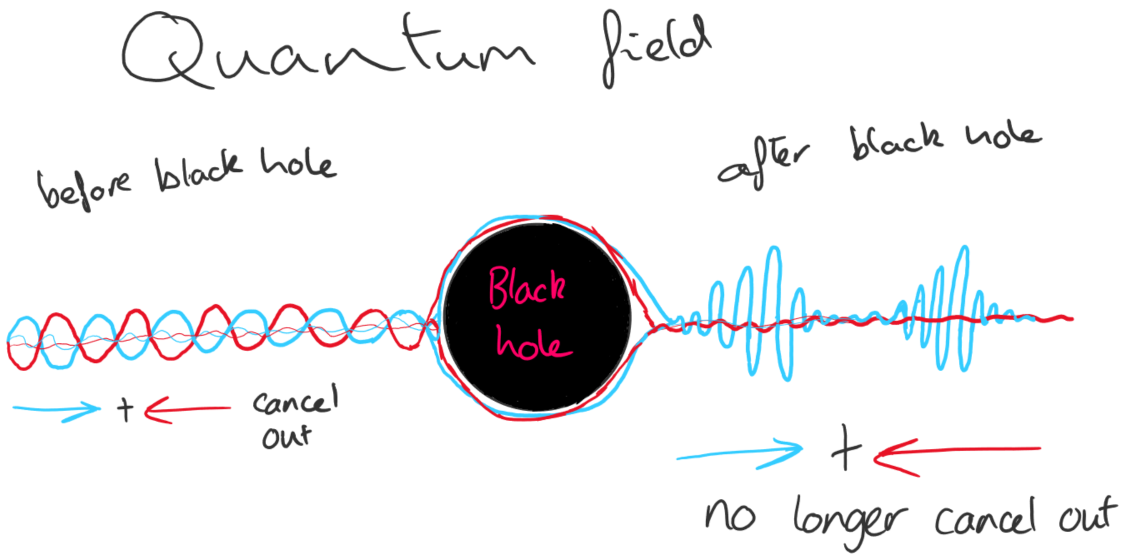
You are surrounded by a sea of particles, buzzing at the temperature of 4×10^{-21}K. And if you tried to run away, you'd only be followed by another horizon. The quicker you accelerate, the closer it chases on your tail. Like a B-movie horror flick monster. It'll never catch up, but the radiation it sends your way can. Why is this? Why does the act of acceleration bring an event horizon chasing after you? In the mid-1970s, three researchers- Stephen Fulling, Paul Davies, and William George Unruh, collectively showed that event horizons do follow accelerating observers, and that these event horizons emit radiation, just like Hawking radiation out of a black hole. This result came at around the same time as Stephen Hawking's derivation of Hawking radiation, and the maths is almost the same between the two.

Pre-requisite

Hawking Radiation

First, for a 'warm up', let's talk about Hawking radiation. This will go a bit beyond the common explanation of virtual particle-antiparticle pairs, but not to the depth of Bogoliubov transformations[4] and the intricacies of Quantum Field Theory (QFT). The common notion is that the vacuum of the universe is replete with virtual particles, that pop in and out of existence, with their antiparticle partners, borrowing energy from the vacuum itself and then annihilating to pay off their debt[7]. And that if these particle pairs appear on either side of an event horizon that they must be separated forever. One of the pair of particles is 'forced' into existence. But that energy debt must be paid. So, the black hole itself pays up, and loses a little mass in the process – black holes "evaporate". But where does this explanation come from?

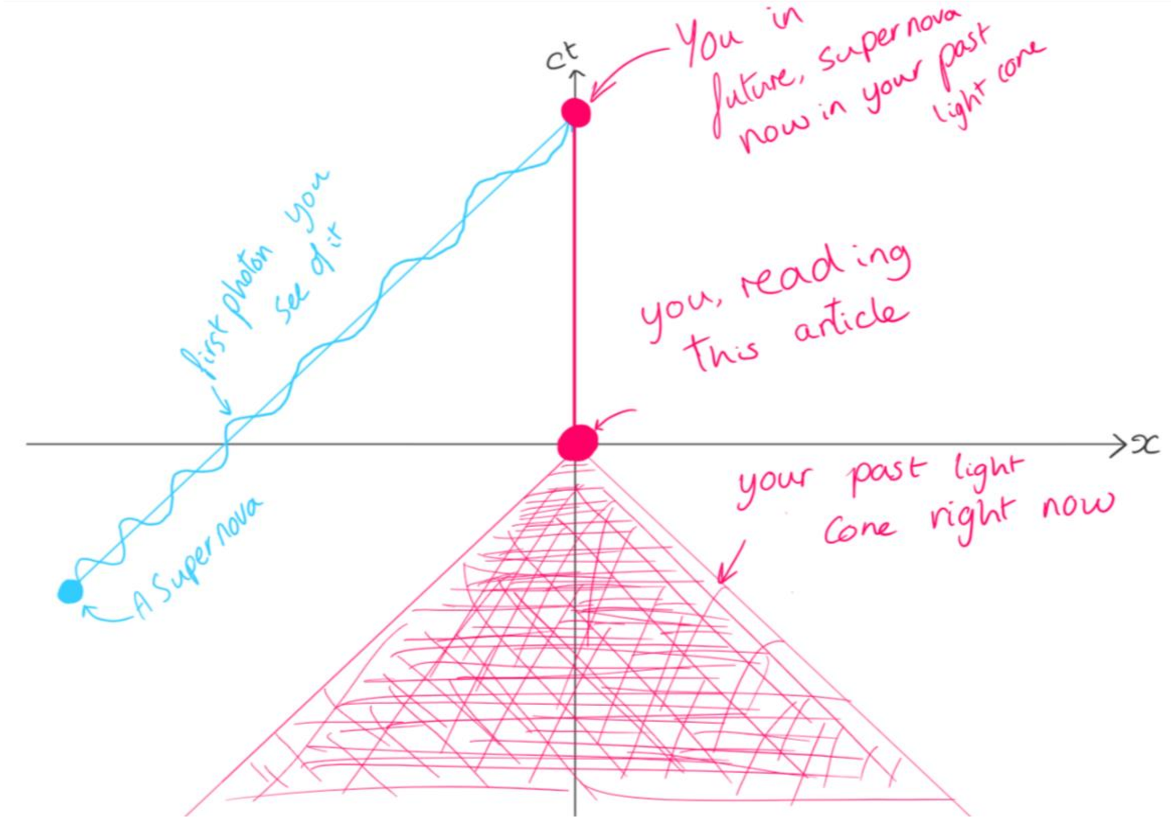
The Universe is made up of constituent quantised fields, at least in Quantum Field Theory. Each field represents each particle- the electron field for the electron, the Higgs field for the Higgs boson etc… But in the absence of a particle – when these fields should be perfectly at a zero-energy state, they keep buzzing, due to the energy-time uncertainty relation $\Delta E \Delta t \geq \frac{\hbar}{2}$. The fields are filled with backwards and forwards propagating waves, that cancel each other out on large timescales.[9] These are the virtual particles, and for reasons that I won't get into, the matter is loosely considered the forwards travelling frequencies and antimatter the backward travelling frequencies. This is what is meant by the matter-antimatter pairs that form and annihilate all the time in the vacuum of space. The next step is to consider one of these fields in a vacuum, in the last instant before a black hole forms. The field before the black hole was in a perfect vacuum state, but the near miss with the black hole for this trajectory changes the field. Some frequencies appear to be scattered, and fall into the event horizon, nudged off their escaping path. On the other hand, some other frequencies pass through safely. Looking at the vacuum field there, the vacuum state of the field now appears to have particles in it. The vacuum appears to now be full of particles. The vacuum now appears to be a warm gas. Indeed, Hawking showed that the radiated heat looks exactly like you would expect from a black body, with size being inversely proportional to temperature. That is why small black holes should "explode" (Hence the name of his paper- ***Black Hole Explosions?***[11]). One thing to note is that these particles appear to be very delocalised, with de Broglie wavelengths on the scale of the radius of the black hole. Smaller black holes have a smaller radius, so the de Broglie radiation emitted has a smaller wavelength (therefore a larger momentum and a larger energy), and thus a higher temperature. No distant observer can see the Hawking radiation as coming from any point on the black hole, just as from the black hole in general. To the distant observer, the vacuum itself has changed.



Spacetime diagrams

Next, let's talk about spacetime diagrams. You all probably know about them, but just for a refresher, here's some information. Spacetime diagrams are how we represent what an observer sees in special relativity. It shows "Minkowski"[10] space (indeed, spacetime diagrams are sometimes called Minkowski diagrams). That is an x-axis (or three of them if you live in the real world) and a time axis, with intervals of ct. Points on this diagram represent events (NOT objects), and each object is described by its world line. This is a line of events that show how the object changes in time.

We give the time axis in terms of the speed of light multiplied by the time, which makes all light speed objects move at 45 degrees on the diagram. When you extend these 45° lines from the origin, we get 2 light cones. One extending to the past and one to the future. The light cone of past shows all the points, all the events, that are causally connected to you and have happened. You have information of these events. The future light cone shows what events the origin event will be causally connected to. All the future you can affect. Clearly, the majority of spacetime at any given time cannot be affected by you or cannot affect you. But you can also see that for a given point not in the past light cone, any object allowed to move at some constant velocity will eventually have that event enter its past light cone. There is no horizon from which you can ***NEVER*** receive light for these observers. Eventually, the whole universe will be in view.



Acceleration in Special Relativity

But what about this scenario? Lets say that you are at the gym, waiting for your workout buddy. He's running towards the gym at near the speed of light, but then realises it's leg day. He just happens to recall he has 'injured knees' and is accelerating uniformly away from you. Just as he's instantaneously stationary (this will be our arbitrarily chosen $ct = 0$) you send him a message asking whether he's going to show up. Does he ever get the message? What does constant acceleration even look like in these spacetime diagrams?

So let's say this guy sees himself moving (or rather, an instantaneous co-stationary inertial frame at each point sees him moving) at a constant acceleration a at any given time, what you see his acceleration as is affected by the Lorentz transformation as[1]:

$$\frac{dv}{dt} = \frac{a}{\gamma^3}$$

Where γ is the Lorentz transform factor from special relativity. Integrating with the boundary condition that at $t = 0, v = 0$ we get the velocity of the accelerating man is:

$$v(t) = \frac{at}{\sqrt{1 + (at/c)^2}}$$

And the equation for t as you see it as a function of the proper time for the accelerated observer, τ is given by this equation (with the correct boundary condition of $t(0) = 0$) :

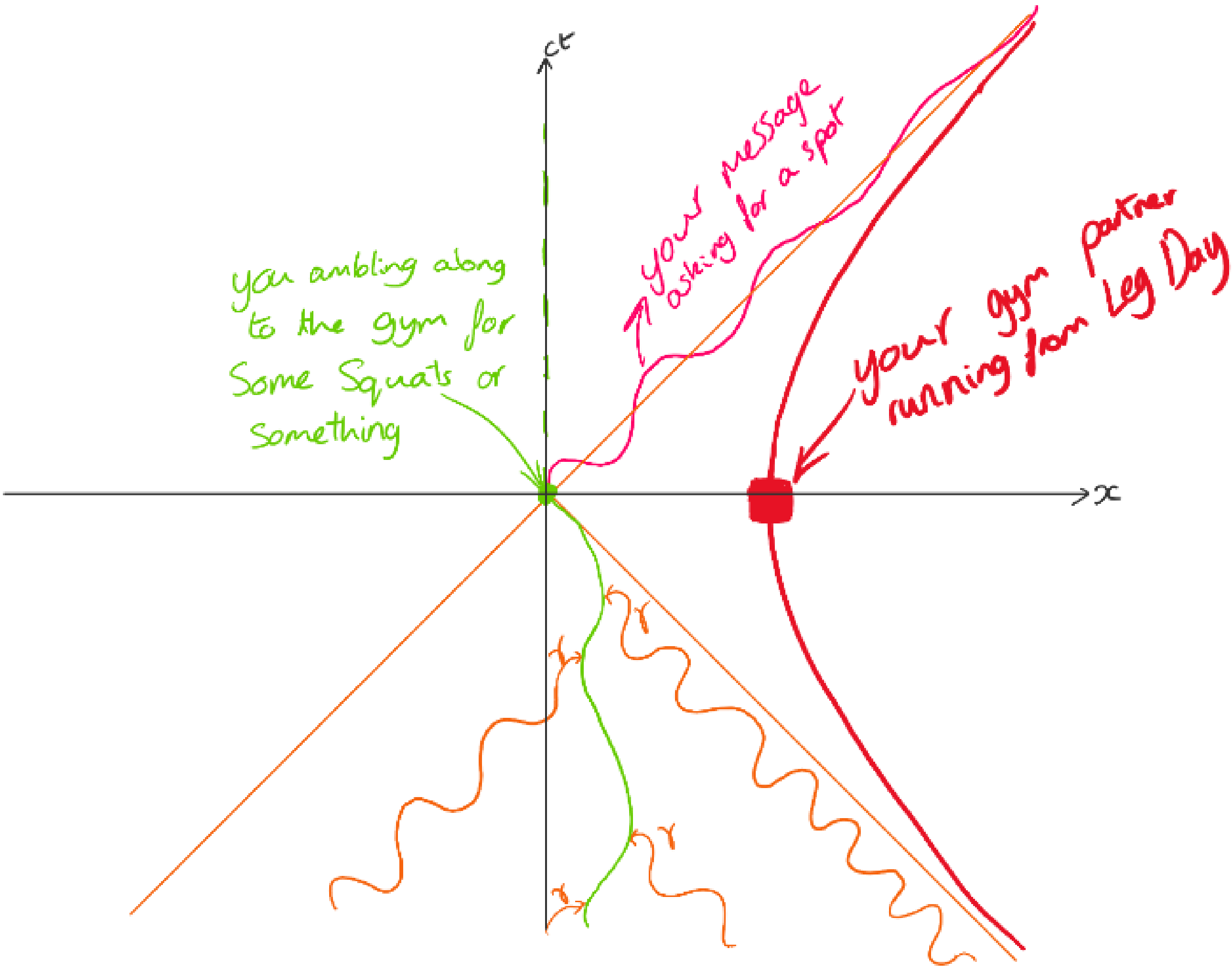
$$t(\tau) = \left(\frac{c}{a}\right) sinh \frac{a\tau}{c}$$

So what does this mean for the trajectory of the person?

$$v(\tau) = c\, tanh \frac{a\tau}{c}$$

Which when integrated gives a hyperbolic orbit:

$$x(\tau) = \frac{c^2}{a} cosh\left(\frac{a\tau}{c}\right) \qquad ct(\tau) = \frac{c^2}{a} sinh\left(\frac{a\tau}{c}\right)$$



This is the motion of a "Rindler Observer", that is your 'constantly' accelerating gym buddy. He moves in this hyperbolic motion, with an oblique asymptote for the $x = ct$ line. Often we replace the terms with $\xi = \frac{c^2}{a}$ and $\eta = \frac{a\tau}{c}$. [2]

The gym partner who's skipping legs is following that hyperbolic path mentioned earlier (see the diagram above). The faster the acceleration, the more pinched the Rindler orbit[8]. See how the message never reaches him? Indeed, there is a whole half (of this diagram) of the universe which, as long as your friend keeps running from doing legs, he will never be able to see- an event horizon. And we know from Hawking radiation what happens at the boundary of an event horizon. This is called a Rindler horizon, as the constant acceleration paths are described by Rindler coordinates.

Another way to look at it all

The other way to see how this horizon comes about is to imagine a large rod, all accelerating at a constant rate. At a given velocity, the rod will in the inertial frame will undergo Lorentz contraction relative to its velocity as:

$$L = \frac{L_{proper}}{\gamma}$$

But you in your inertial frame also observe the rod contracting. Therefore tail end of the rod must accelerate both to keep up with the rod and to keep up with the length contraction. It must accelerate “harder” to keep up. This can be formally written as a differential equation, that shows that for a sufficiently long trailing end, the acceleration would have to be infinite to keep up, and so there is the horizon again, beyond which nothing can move indeed. This is the Rindler Horizon again.

Unruh effect- properties

We can transform the spacetime diagram from the normal coordinate system (Minkowski space), into Rindler space – the coordinate system used to describe the world as seen by these constant acceleration observers). This transformation allows us to talk about the properties that your friend who’s leaving you alone on leg day – a Rindler observer – sees.

What are the properties of this Rindler horizon? The distance between the Rindler (constantly accelerating) observer and the Rindler horizon is proportional to $1/a$[8]. This means that the faster the acceleration, the closer the horizon gets. Another property is that this horizon doesn’t require you to be always accelerating. Momentary acceleration produces a momentary Rindler horizon.

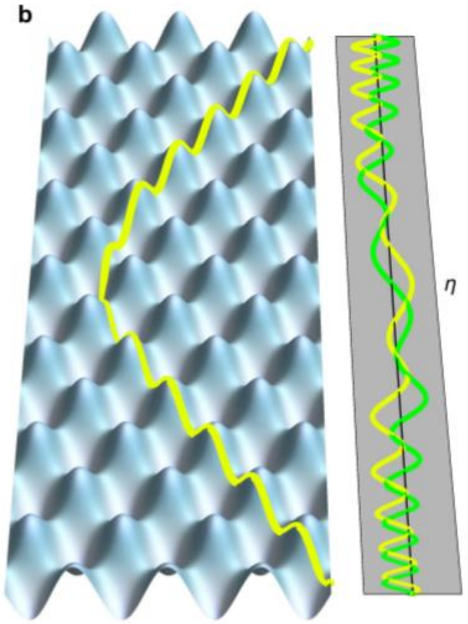
Now what we know from before about these event horizons, is that they supress some frequency nodes in the vacuum state waves of quantum fields. This leads to the creation of particles in the Rindler space. The accelerating observer sees himself in a dilute ‘gas’ of warm particles, with the same temperature relation as for Hawking radiation. This is the Unruh radiation. If when you do the maths to find the equation for the temperature[1] of the Unruh radiation is:

$$T = \frac{\hbar a}{2\pi c k_B}$$

Whereas for the temperature of Hawking radiation that some distant observer sees, we have:

$$T_H = \frac{\hbar g}{2\pi c k_B}$$

Here, g is the gravitational field strength of the black hole at the event horizon. The two have clear links mathematically, sharing the need of the Bogoliubov transforms to transform from one space to another (in black holes, it iss used to transform between the globally flat space of the distant observer and the curved spacetime near the blackhole, whereas for the Rindler horizon, the same transform is used between the normal, flat Minkowski spacetime we are used to, and the Rindler space that the accelerating observer sees). More fundamentally though[4, 8], both share the fact that they are examples of what happens to vibrations in Quantum Field Theory when they hit a boundary or horizon – how the existence of a horizon smothers some frequencies and leaves others unscathed, and how this arises to turn a vacuum into a sea of particles.



Experimental data

But is this what we see?

Plugging in some of the numbers we find that accelerating at a value of 10ms^{-2}, you have a temperature of around 4×10^{-20}K. Clearly at any normal acceleration, this effect will be too small to observe. But here are two experimental examples:

First, there is the relation between Bremsstrahlung radiation and Unruh radiation[6]. Bremsstrahlung radiation is the radiation given off by charged particles accelerating from the electric field of another charged particle, at the loss of the former's kinetic energy. It can be shown that this loss of kinetic energy from the emission of a photon in the inertial frame is mathematically the same as the absorption of Unruh Radiation.

Another experiment demonstrating a classical equivalent of the researchers at the Weizmann Institute, Israel and the Langevin Institute, France, demonstrated a similar effect classically using water waves[2]. They filled a container with water, subject to white noise so that ripples form on the surface. The water surface was scanned by a movable laser, while a camera took a video of the height of the illuminated spot. This spot followed a hyperbolic trajectory, as a Rindler observer would move. The height of the water waves were recorded along this trajectory. This amplitude detector is akin to a particle detector one would use to observe the Unruh radiation. By then committing to some analysis of these waves, the same Planck spectrum of black body radiation that is predicted by the Fulling-Davies-Unruh effect is seen.

The method is shown in the image[2] above, where the accelerating frame sees oscillations more akin to particles (when Fourier transformed) when accelerating over the white noise water ripples.

Paradox

So, we now know what your friend as a Rindler observer sees – a bath of particles that are very real for him. But what about you? Do you see nothing? Afterall, for you there is no event horizon, no boundary to create a sea of particles. So you should see the vacuum as just that – a vacuum.

But then consider this case. Imagine you are sitting around being your standard inertial observer. A friend, accelerating past, is carrying his particle detector. In the Rindler frame, your friend's frame of reference, there are plenty of particles all around. That particle detector will detect them (after all the detector is also accelerating). You, in your inertial frame, also see the detector go off. But there is an issue here. Where are the particles?

These detectors are known as Unruh-DeWitt detectors[4]. Such a detector weakly interacts with a quantum field in a certain vacuum state. It detects when the field is in a 1 particle state, and when it empty. Such a detector if it's kept in an inertial frame in Minkowski space, will not detect anything in the vacuum. No particles, as you would expect. But in a Rindler frame, it will begin detecting particles in the same space, consistent with the Unruh radiation equation seen above.

So again, what do you see? How can the detector be detecting particles where there are none? Simply put, in the Minkowski inertial frame, the detector appears to be emitting the particles, and exciting as it does so. So you both agree the detector beeps consistently, but you disagree on the reason. One sees particles being detected, and another sees particles being emitted. Both at a rate consistent with the Unruh Temperature equation.[5]

You as an inertial frame, see instead that this weakly interacting detector (and by extension, anyone with this constant acceleration) has a sort of drag acting upon them, with the energy coming from the acceleration itself. It's as if the Rindler observer is having energy siphoned off their constant acceleration, as they begin producing particles at that temperature relating to their proper acceleration.

Summary

So, we have talked about what acceleration looks like in special relativity. How to an inertial frame in the normal (Minkowski) space, the accelerating frame's trajectory is a hyperbolic path. How the light cone that follows the accelerating observer cannot reach the whole universe, no matter the time (until of course they stop accelerating). We saw how this inability to ever see a part of the universe creates a horizon, and how that horizon creates a boundary, changing how the vacuum state of quantum fields appears, and making real particles arise from seemingly nothing, in a manner near identical to the more famous Hawking radiation of black holes. And finally we went about resolving the apparent paradox of particles seen by the accelerating, and not by the inertial observers. All that is left is to go back to that number from the beginning. Where did the number of 4×10^{-21}K come from? In General Relativity, what is meant by an acceleration is slightly different. To account for curvature, a "straight" line in spacetime (like in our spacetime diagrams) is defined by the line you would travel in the absence of any input forces[8]. If the spacetime happens to be flat, then we get our usual straight line, as we would expect. So where is our straight line taking us? It's supposed to be headed down, towards the centre of the Earth for us. But the reaction force of the ground on our feet prevents us, makes us deviate from "straight", ***accelerates*** us. Specifically, it accelerates us as $g \approx 9.8$ms^{-2}. If you plug that back into the equation for the Unruh temperature, you get the temperature of 4 sextillionths of a kelvin.

The Fulling-Davies-Unruh effect is certainly a very interesting, and yet generally unknown physical phenomena, overshadowed by the bigger brother of Hawking radiation in popular science. Hopefully this article has succeeded in bringing some much-deserved attention and interest into this union of Quantum Field Theory and Relativity.

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All images are my own except when otherwise stated

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