Geodesics On Directed Acyclic Graphs

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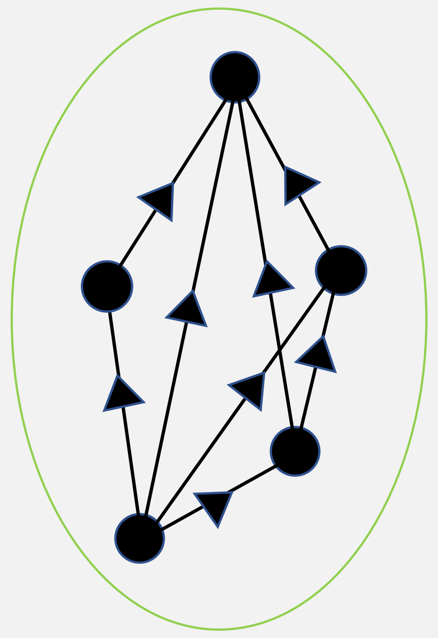
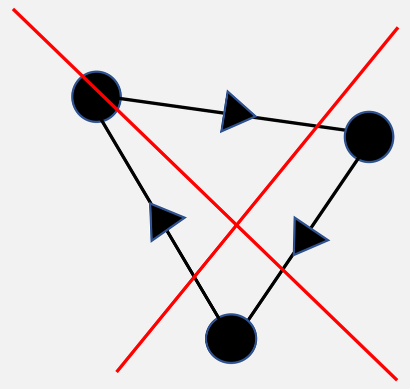
What is A DAG?

Directed – Information flows one way along edges

Acyclic – Paths don't form loops

Graph – Series of nodes connected by edges



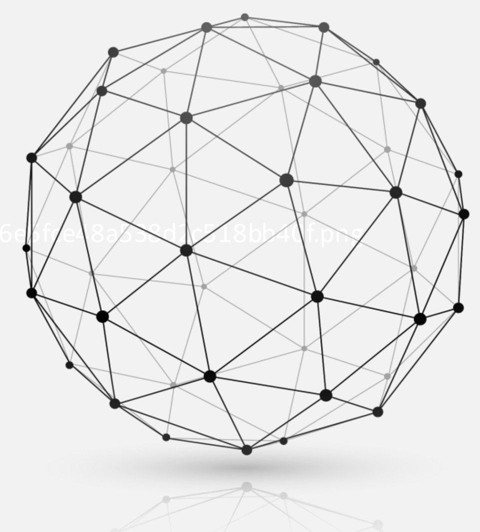


Network Geometry[1]

Certain sufficiently dense DAGs approximate "Lorentzian Manifolds"

Dynamics on manifolds -> straightest paths (geodesics)

Q: which DAGs exhibit Lorentzian geodesics (longest paths)?
Q: which path algorithms exhibit geodesic behaviour?



Generate points on a background space → Connect points according to proximity/causality → Generate paths → Vary background structure and path algorithms → Compare dense limit paths to Lorentzian geodesics

Spatial Variations[2]

Flat Lorentzian background

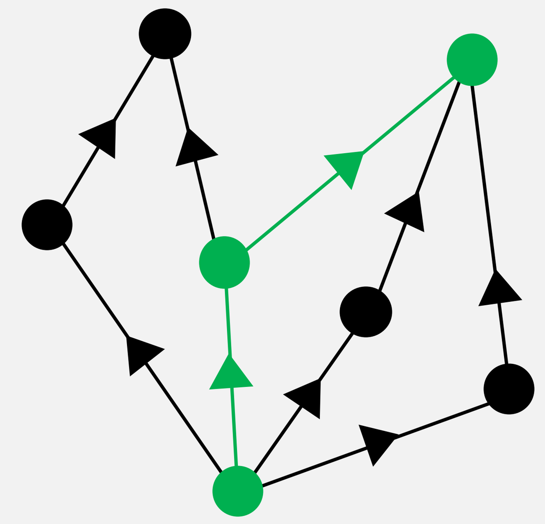
$$s^2 = \eta_{\mu\nu}(x-y)^\mu(x-y)^\nu$$

Minkowski distance model (parameter $p \in \mathbb{R}$)

$$s^p = \sum_{i=0}^{n}(x_i - y_i)^p \ if \ x_i > y_i$$

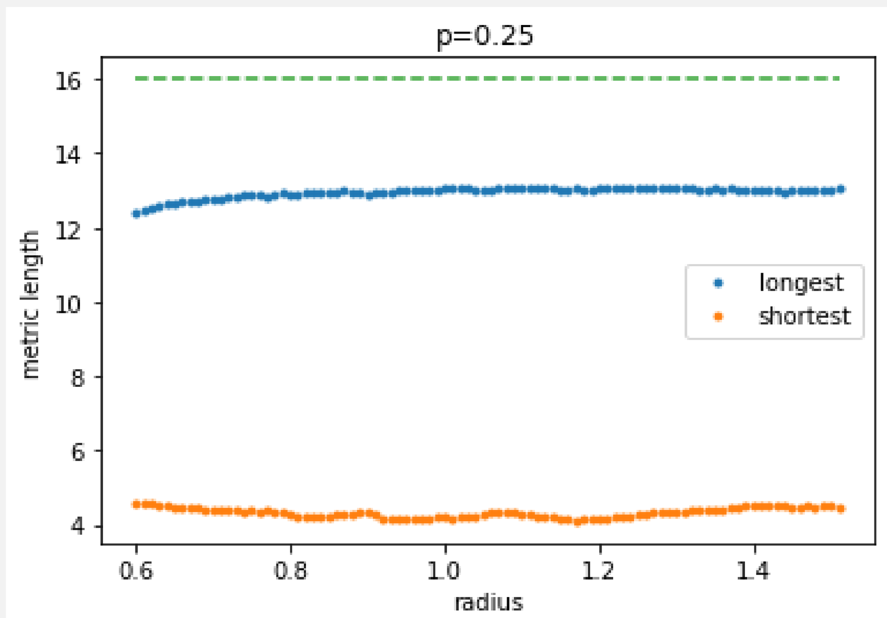
Path Variations

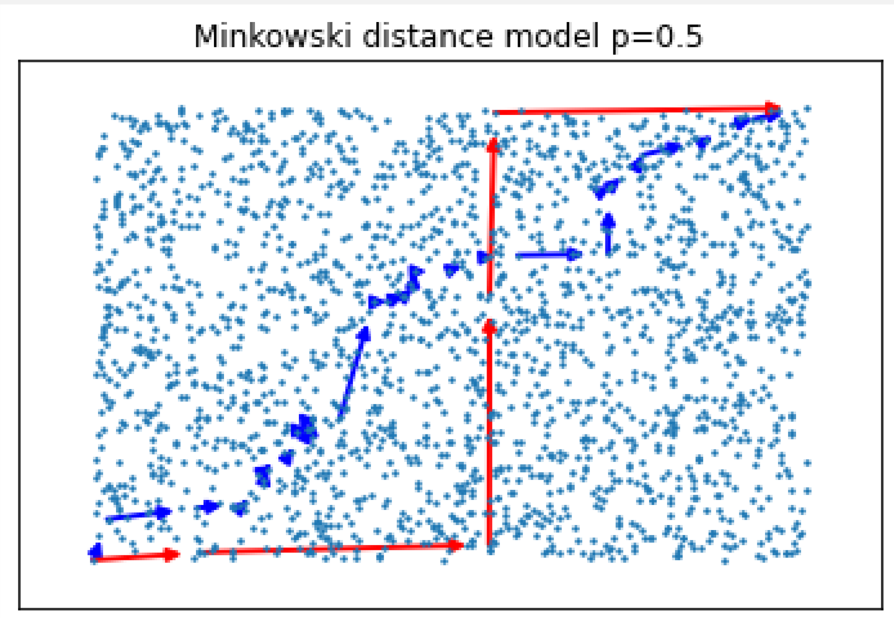
- Shortest Path – Fewest Nodes
- Longest Path – Most nodes
- Greediest Path – Local optimization algorithm. Choose the next closest node to the endpoint
- Largest Diamond Path – Semilocal optimization algorithm. Choose next node in path to maximize number of nodes between new node and one node ago

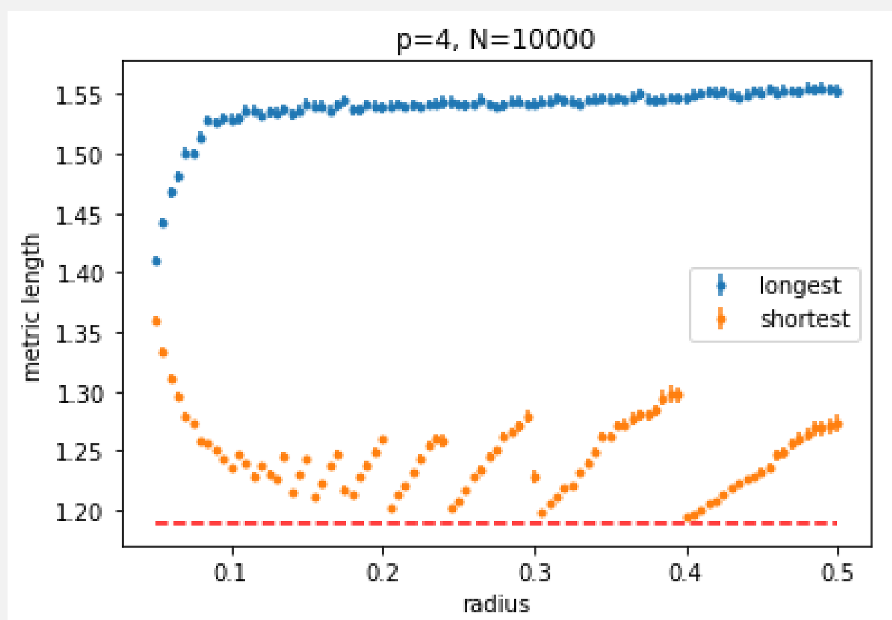


Results (Spatial Var.)

For $p < 1$, the longest path is a better approximation to the geodesic in a network in Minkowski distance model. For $p > 1$, the shortest path is better. The metric length of shortest path across different radius has a characteristic shape, which is absent for $p < 1$.







Results (Path Var.)

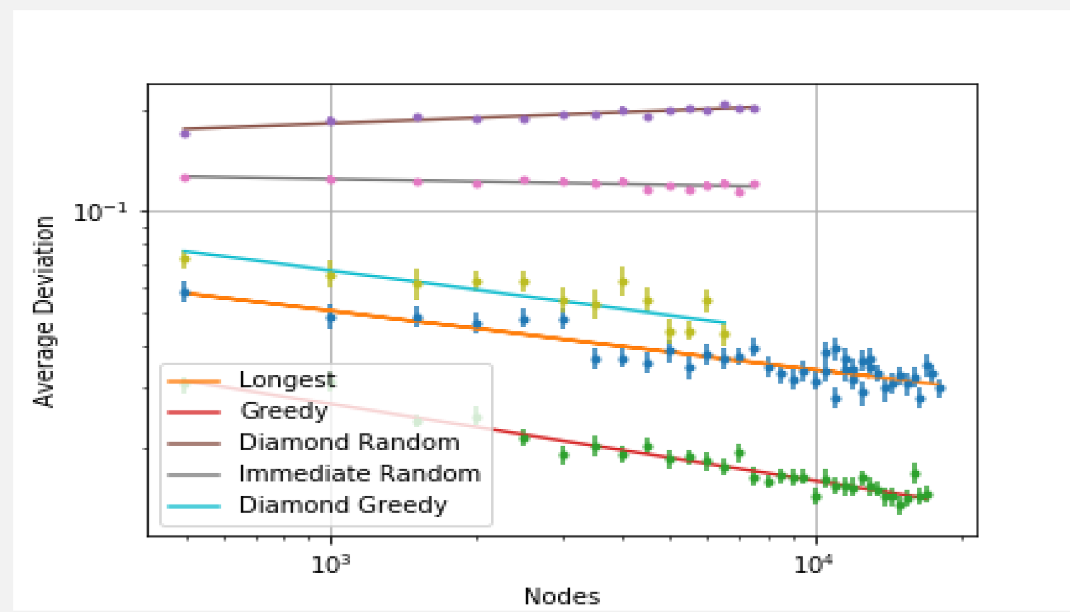
Longest, Greediest, and Random paths behave as expected

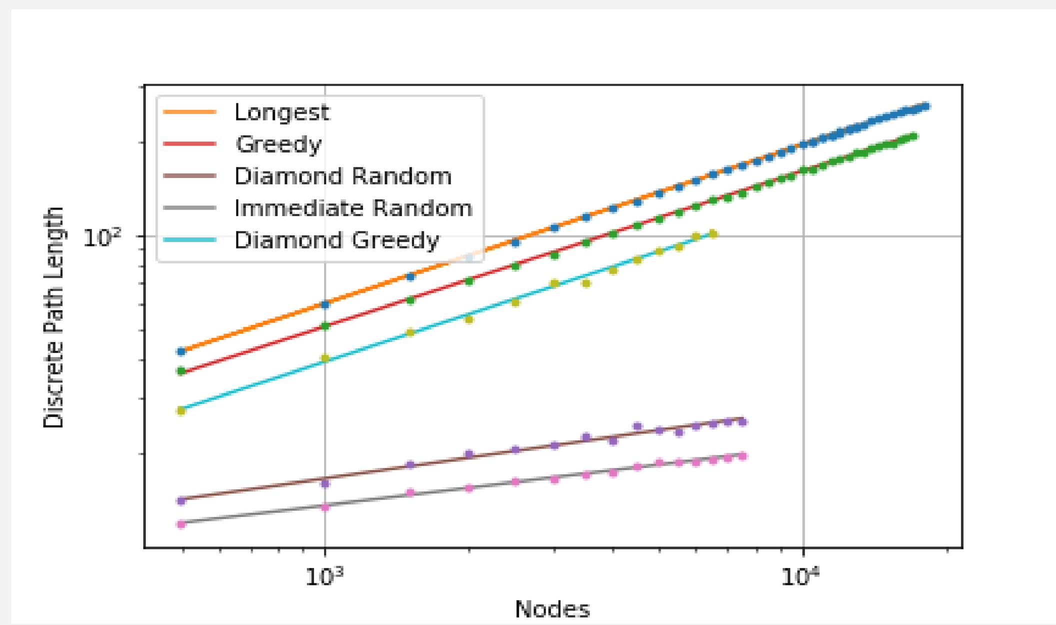
Semilocaly Longest Path ≠ limiting geodesic

Why? Discrete node quantities are unaffected by the limit

Should increase algorithm reach with graph size

Must therefore introduce second length scale





[1] Embedding Graphs in Lorentzian Spacetime James R. Clough, Tim S. Evans, arXiv:1602.03103 [physics.soc-ph] [2] Deza, M. M., & Deza, E. (2016). Encyclopedia of Distances (4th ed. 2016 ed.). Springer.