Imperial College London

Hamilton-Jacobi Formalism for Chaplygin Gasses

Theoretical Physics Group
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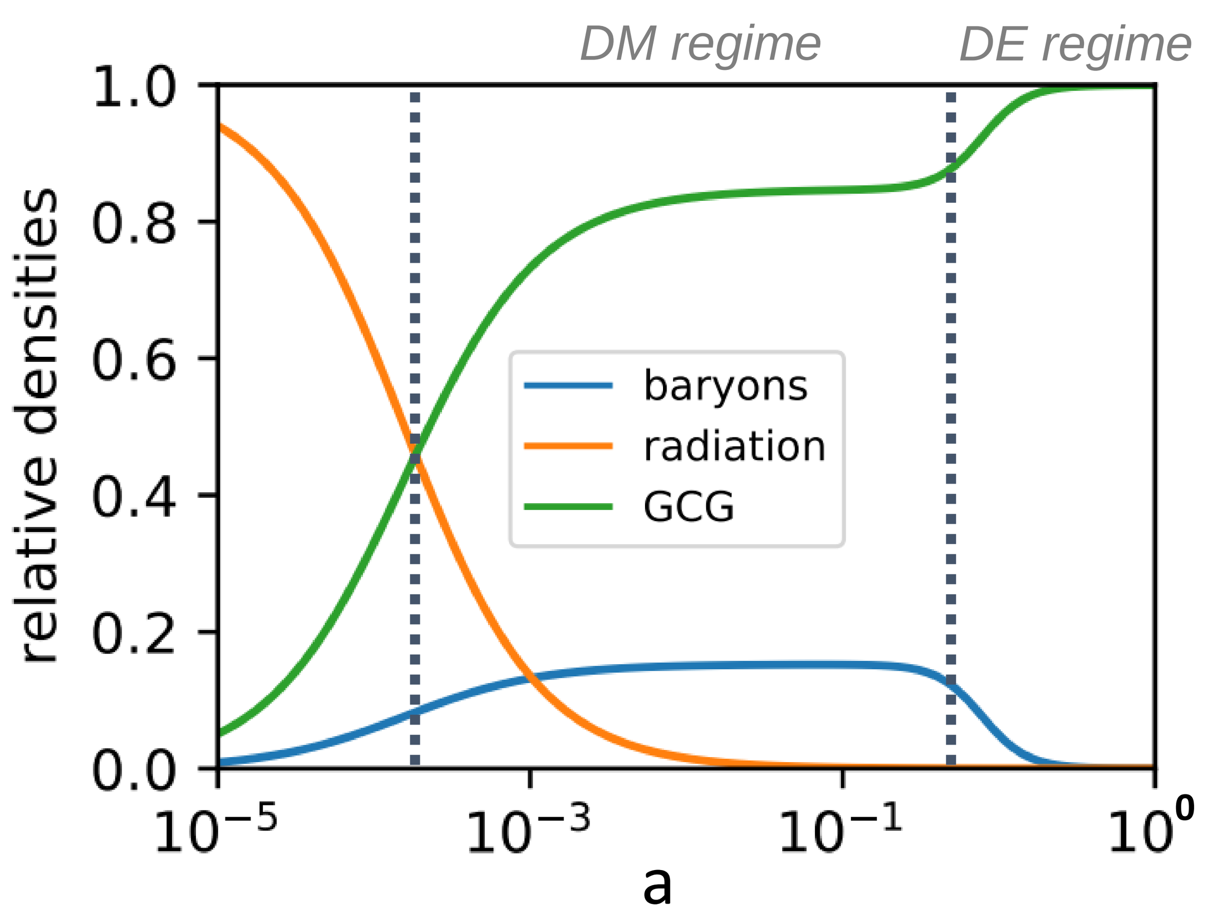
1. Background

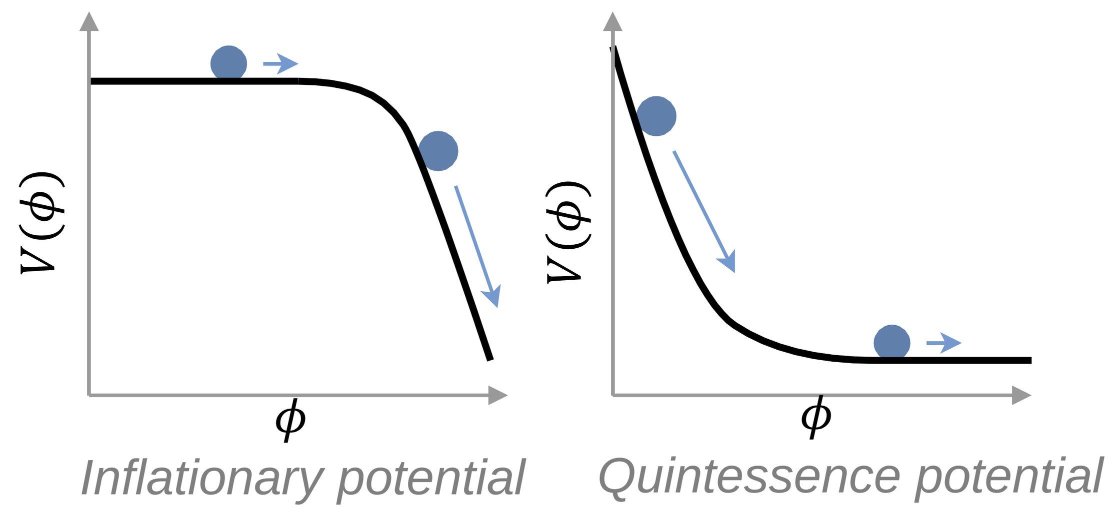
A *Generalised Chaplygin Gas* (GCG) is a cosmological fluid that seeks to unify *Dark Matter* (DM) and *Dark Energy* (DE) [1].
DE domination is characterised by the **exponential expansion** of the universe.

When the universe is small the GCG behaves as DM, switching its behaviour to DE as it grows.



Scalar fields are commonly used to model *inflation* and (more recently) *quintessence* (DM and/or DE). When the field is **moving slowly** in its potential, it causes the universe to undergo **accelerated expansion.**

The field obeys a similar Equation Of Motion (EOM) as a ball on a frictional slope. When rolling slowly, the field acts like the *inflaton*/DE.



The *Hamilton-Jacobi* (HJ) formalism [2] uses the field's value ϕ as the time parameter. This reduces the problem from one second order EOM to two first order equations.

2. Aims/approach

Frame bare GCG as a scalar field model → Solve the bare scalar model in the HJ formalism → Add a nonzero potential → Add baryonic matter (and possibly radiation)

Scalar field Lagrangians with **non-canonical kinetic terms** are motivated by some beyond-Standard Model theories. One of these replicates the behaviour of the standard GCG [3]:

$$\mathcal{L}(\dot{\phi}) = -A^{\frac{1}{1+\alpha}}\left(1 - |\dot{\phi}|^{\frac{1+\alpha}{\alpha}}\right)^{\frac{\alpha}{1+\alpha}}$$

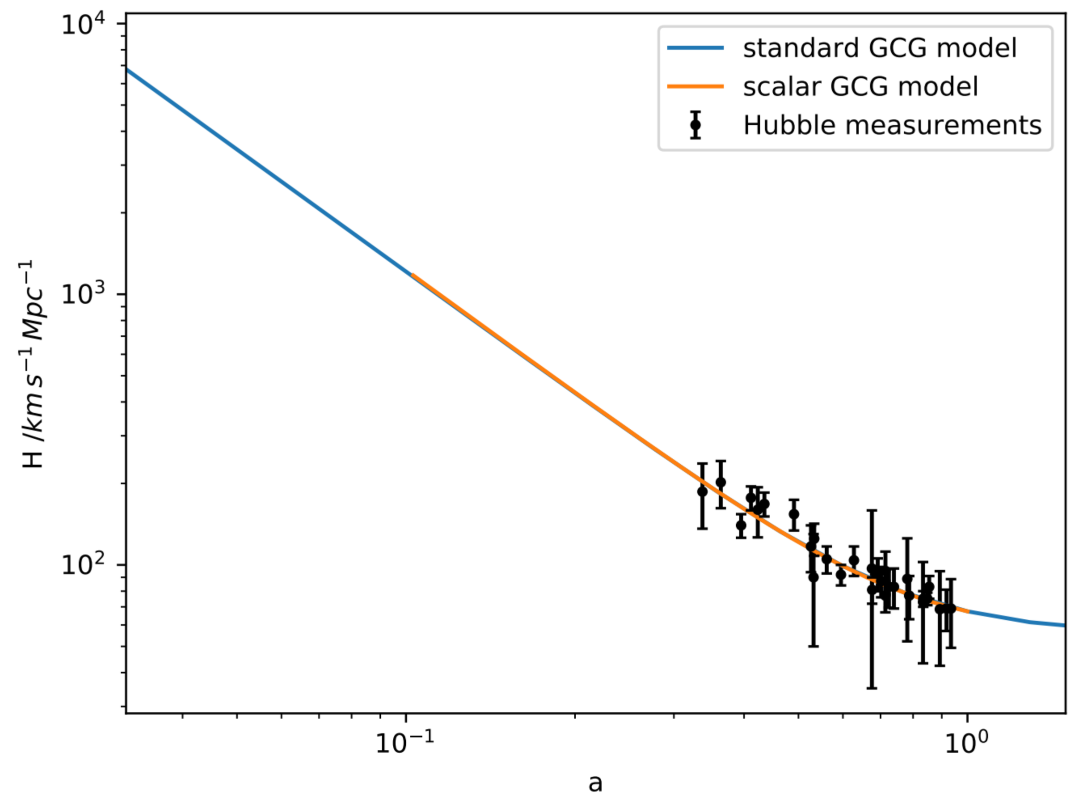
We show this equivalence and derive the **Friedmann equations** for a non-canonical scalar field model in the HJ formalism, following Binétruy et. al. [4].

The model is generalised further by adding a potential term to the Lagrangian, and suitable parameterisations are explored.

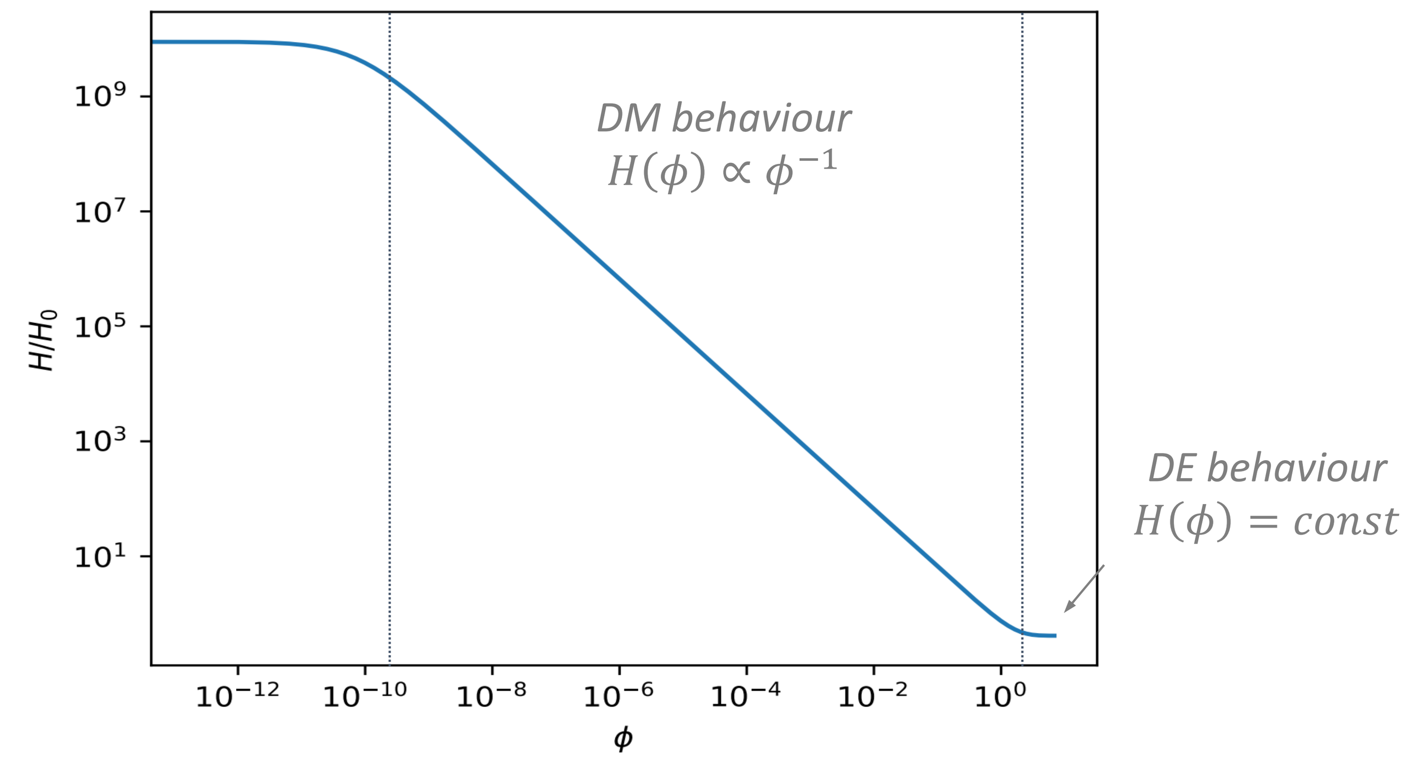
Matter and radiation are added, and the validity of the parameterisations is re-evaluated.

3. Work Undertaken

(a). Framed scalar GCG model in the HJ formalism, solving it numerically. Agrees perfectly with the standard GCG model.



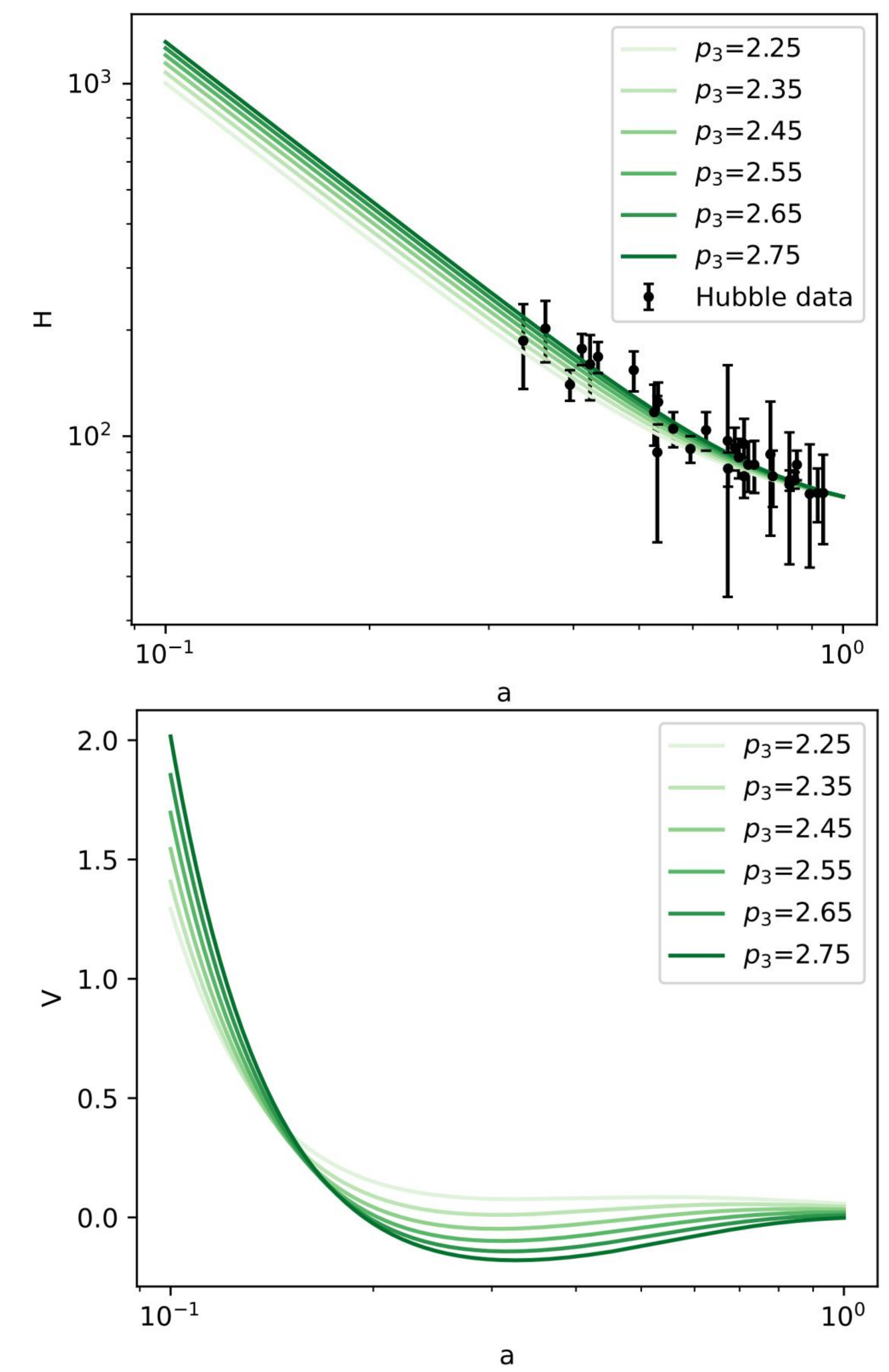
(b). Added a potential, parameterising it indirectly by **parameterising the Hubble parameter**, $H(\phi)$.



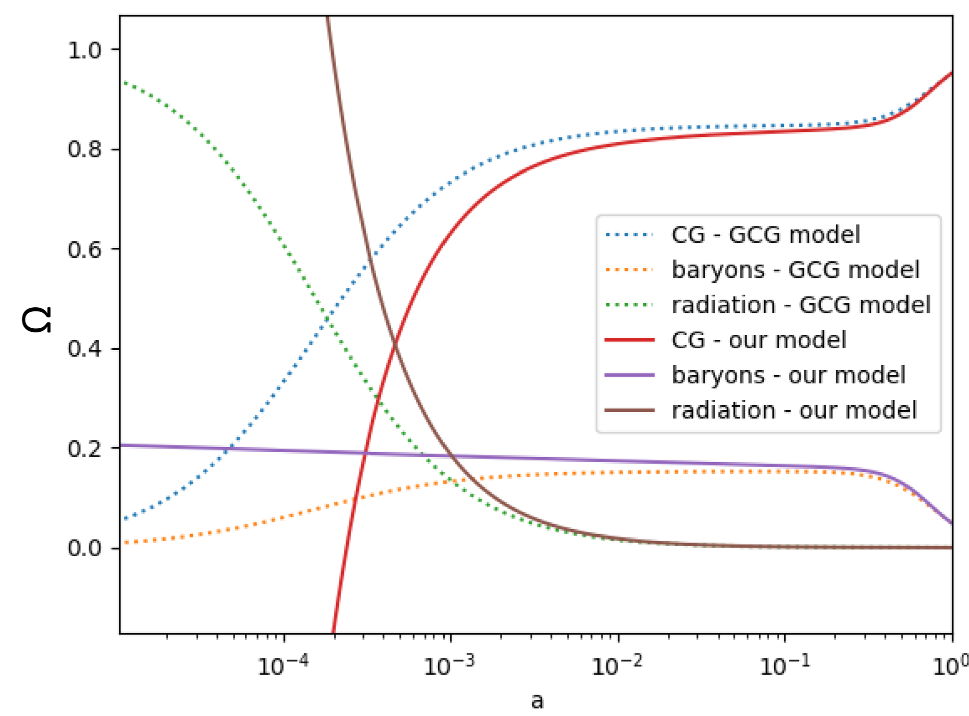
We based the parameterisation on behaviour of $H(\phi)$ in the **zero-potential case** (above):

$$H(\phi) = \frac{p_1}{\phi}\left(1 + \left(\frac{\phi}{p_2}\right)^{p_3}\right)^{\frac{1}{p_3}}$$

(c). Varied the parameters of the potential to visualise their effects on the model.



(d). Added baryonic matter & radiation to the model.



Fits standard GCG model (dotted lines) well for DE and DM. Diverges as expected in radiation regime – parameterisation is not valid there.

References

[1] A. Y. Kamenshchik, U. Moschella and V. Pasquier, An Alternative to quintessence, Phys. Lett. B511(2001) 265 [gr-qc/0103004].
[2] D. S. Salopek and J. R. Bond, Nonlinear evolution of long wavelength metric fluctuations in inflationary models, Phys. Rev. D42(1990) 3936.
[3] M. Bento, O. Bertolami and A. Sen, Generalized Chaplygin gas, accelerated expansion and dark energy matter unification, Phys. Rev. D66(2002) 043507 [gr-qc/0202064]
[4] P. Binétruy, J. Mabillard and M. Pieroni, Universality in generalized models of inflation, Journal of Cosmology and Astroparticle Physics 2017(2017) 060