Neutrino Oscillation Measurements with Model-Assisted Generative Adversarial Nets

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Introduction

Deep Underground Neutrino Experiment

- The Deep Underground Neutrino Experiment (DUNE) is a long baseline experiment which will study neutrino oscillations and measure its oscillation parameters.
- It will also measure the CP violation in the neutrino sector, possibly explaining the matter-antimatter imbalance in our universe.

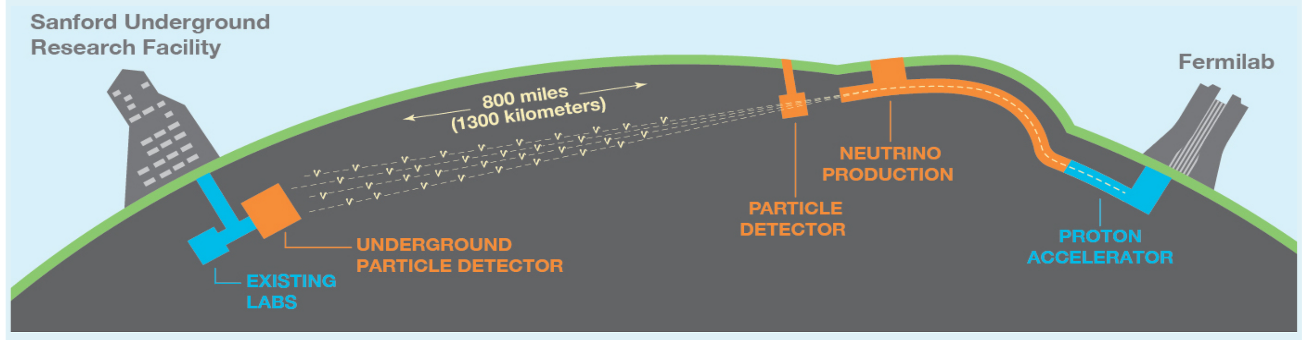

Figure 1. The DUNE long baseline set-up, with an accelerator at Fermilab and the far detector 1300km away. This schematic is taken courtesy of the referenced literature [1].

The Project

- We train a model-assisted **Generative Adversarial Network** to emulate existing Monte Carlo based neutrino simulations, from which we are able to draw samples for any given oscillation parameter-set more rapidly.
- Due to the **differentiability** and potential **invertibility** of the neural network, analytical extraction of the oscillation parameters for experimentally observed events, along with their statistical uncertainties, is achievable.

Theory

Neutrino Oscillations

The neutrino mass eigenstates ν_j for $j = 1, 2, 3$ are superposed, subject to the Pontecorvo-Maki-Nakagawa-Sakata matrix [2], to create the neutrino flavour eigenstates ν_α for $\alpha = e,\ \mu,\ \tau$ [3].

A phase difference is created during a neutrino flavour state evolution due to a mass difference Δm_{ij} in ν_j, resulting in **neutrino oscillations**.

The oscillation probability for the $\nu_\mu \to \nu_e$ regime, is given by:

$$P(\nu_\mu \to \nu_e) \cong \sin^2(2\boldsymbol{\theta_{13}}) \sin^2(\theta_{23}) \sin^2\left(\frac{1.27\Delta m_{32}^2 L}{E}\right)$$

$$\mp \frac{1.27\Delta m_{21}^2 L}{E} 8 J_{CP} \sin^2\left(\frac{1.27\Delta m_{32}^2 L}{E}\right),$$

where $J_{CP} = \frac{1}{8}\cos\boldsymbol{\theta_{13}} \sin(2\theta_{12}) \sin(2\theta_{23}) \sin(2\boldsymbol{\theta_{13}}) \sin\boldsymbol{\delta_{CP}}$, E is the neutrino energy and L is the baseline distance [4]. This probability is parameterised by the mixing angle θ_{ij} and the charge-parity (CP) violation phase δ_{CP} [5].

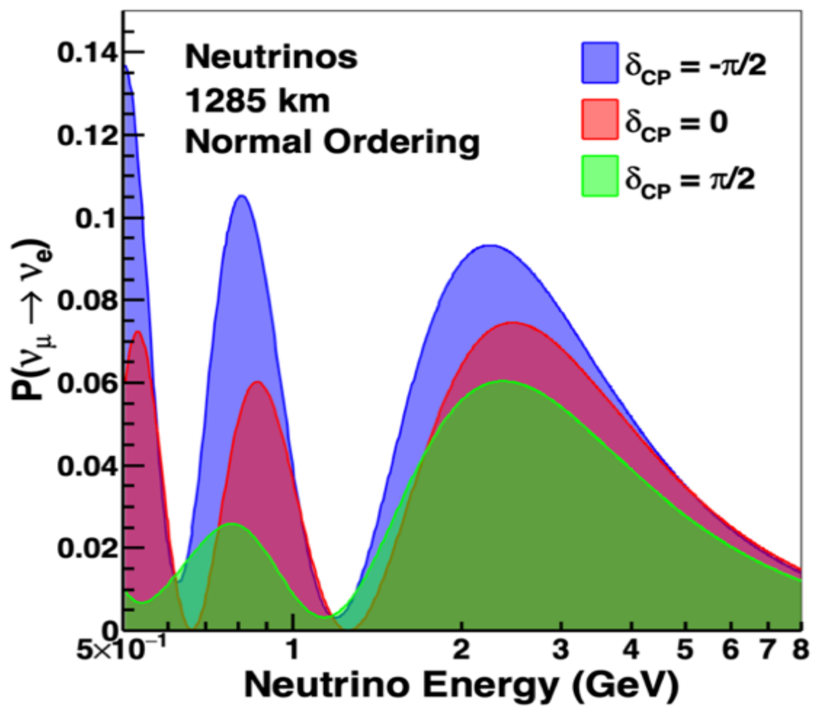

Figure 1. Example neutrino oscillation probability event spectrum, reconstructed for DUNE, as a function of neutrino energy for varying δ_{CP}. Image taken courtesy of referenced literature [6].

Generative Adversarial Networks (GAN)

Using a **GAN**, a generator can be trained to replicate a real image for a given set of input parameters, $\boldsymbol{p}$.

Here, the real image is a 1D histogram of the simulated neutrino oscillation probability event spectrum for the $\nu_\mu \to \nu_e$ regime, which are dependent on $\boldsymbol{p}$: $\sin^2(2\theta_{13})$ and δ_{CP}.

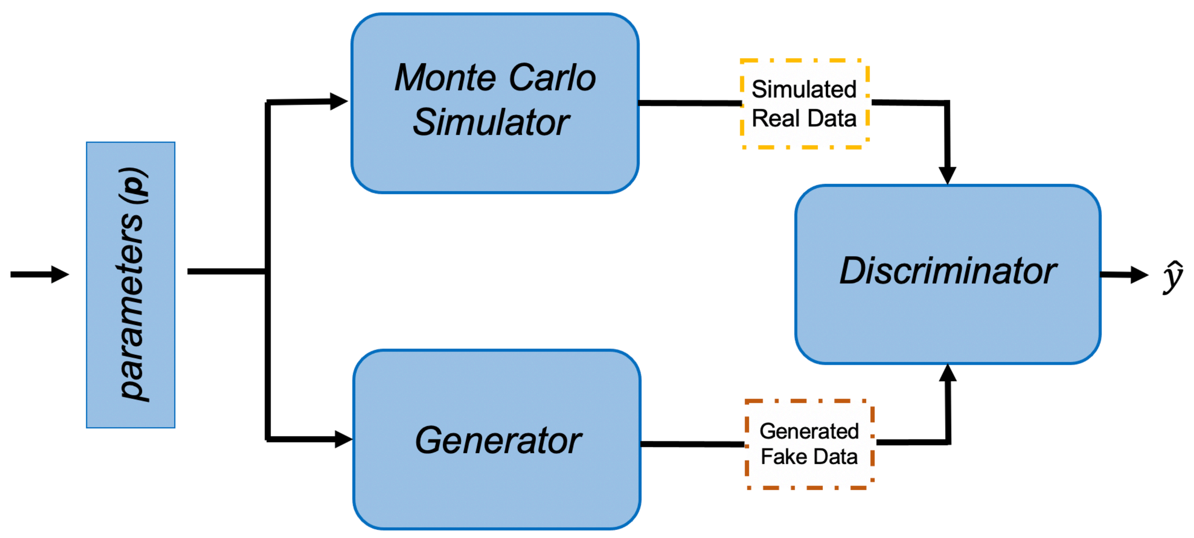

Figure 2. The architecture of the GAN, demonstrating how the generator and discriminator are connected during training. Both the simulated real data and the generated fake data are given misleading labels and are inputted into the discriminator.

- The **generator** is a neural network that takes $\boldsymbol{p}$ and outputs a 1D histogram representative of the neutrino event spectra, creating the fake dataset.
- The **discriminator** is a neural network devoted to differentiating between the real simulated data from DUNE, and the fake histograms. For an unknown histogram input, a decision value, $\hat{y}$ between 0 (fake) and 1 (real) is outputted.

During training, random $\boldsymbol{p}$ are inputted into the GAN and the discriminator and generator play a two-player minmax game - set up shown in Figure 2. The $\hat{y}$ output is exploited to improve the generator histogram replication via backpropagation.

Method

GAN Training

We use the analysis framework CAFAna [7] to create the $\nu_\mu \to \nu_e$ oscillation event spectrums for given sets of $\sin^2(2\theta_{13})$ and δ_{CP}.

Training loop

For generator(θ_{13}, δ_{CP}) $\not\cong$ CAFAna(θ_{13}, δ_{CP}) **do:**

  sample θ_{13}, δ_{CP} to get m pairs of parameters $\{x_1 \ldots x_m\}$
  use generator to **generate** m histograms $\{G(x_m)\}$
  use CAFAna to **generate** m histograms $\{C(x_m)\}$

  calculate discriminator loss:

$$E_D = -\frac{1}{2m}\sum_{i=0}^{m} \log D\big(C(x_m)\big) + \sum_{i=0}^{m} \log(1 - D(G(x_m)))$$

  update discriminator with E_D using Adam

  calculate generator loss:

$$E_G = -\frac{1}{m}\log D(G(x_m))$$

  update generator with E_G using Adam

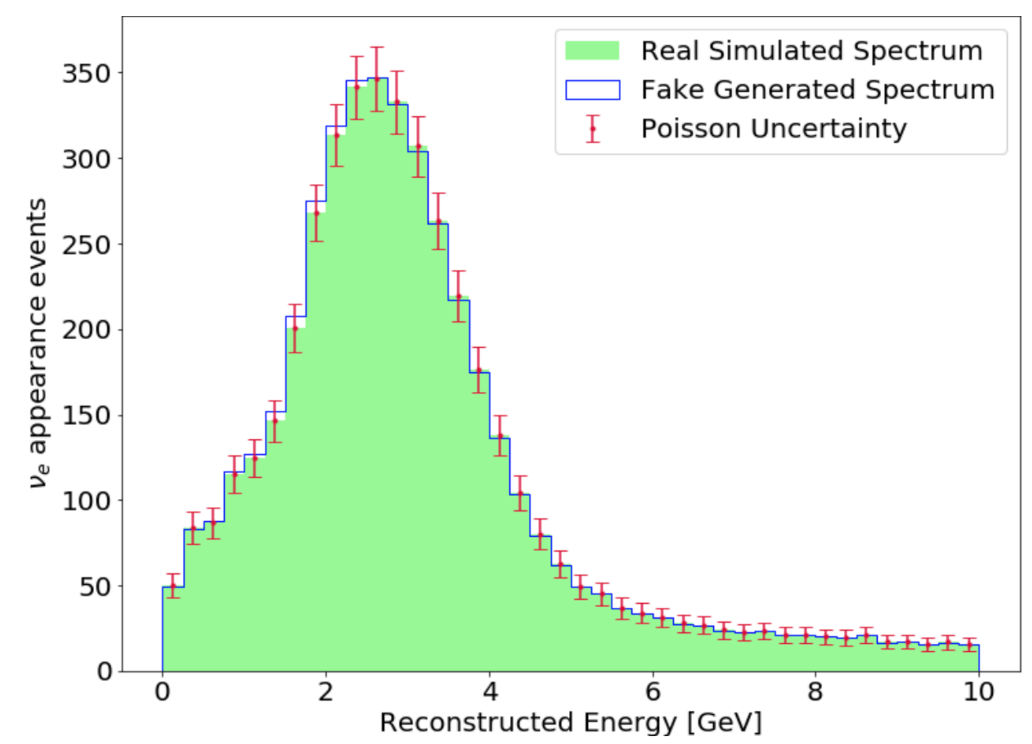

Figure 3. An example $\nu_\mu \to \nu_e$ oscillation event spectrum for a given $\sin^2(2\theta_{13})$ and δ_{CP} parameter set. The generated spectrum is correct to within the Poisson error of the real spectrum. The Poisson error is reflective of the inherent error in DUNE measurements.

Statistical Inference

Parameter & Confidence Interval Estimation

A log-likelihood surface in the parameter space is created using the generator by comparing it's outputs to a toy experiment.

The maximum likelihood estimators may be computed by inverting the GAN, determining the set of $\sin^2(2\theta_{13})$ and δ_{CP} values which correspond to the toy experiment.

Adopting the gradients in the log-likelihood surface and a gradient descent technique, the confidence interval (contour line) of our estimator can be determined. This method is achievable due to the fact that the generator is differentiable.

Results and Future Work

GAN Results

The generator is tested on unseen sets of oscillation parameters within the training $\boldsymbol{p}$ range, as seen in Figure 3.

- The generator replicates the CAFAna simulated data within the Poisson error of each bin in the histogram. This is tested across the entire $\sin^2(2\theta_{13})$ and δ_{CP} space.
- The performance of the generator is evaluated based on: the percentage of predicted spectra within the Poisson error and the cumulative error for all parameter combinations.
- Exploiting the trained generator, the likelihood surface is created by sampling uniformly in parameter space. The maximum likelihood parameter pair is found analytically and a 1σ contour is determined using gradient descent – shown in Figure 4.

Significance of our work

- Ability to draw samples of simulated oscillation events more rapidly than the Monte Carlo simulator.
- Inverse network can potentially extract oscillation parameters from experimental data in DUNE and other experiments. Also, statistical confidence intervals can be calculated using the differentiable generator.

Future works include adding systematic error parameters incurred inherently by DUNE to the input parameters $\boldsymbol{p}$.

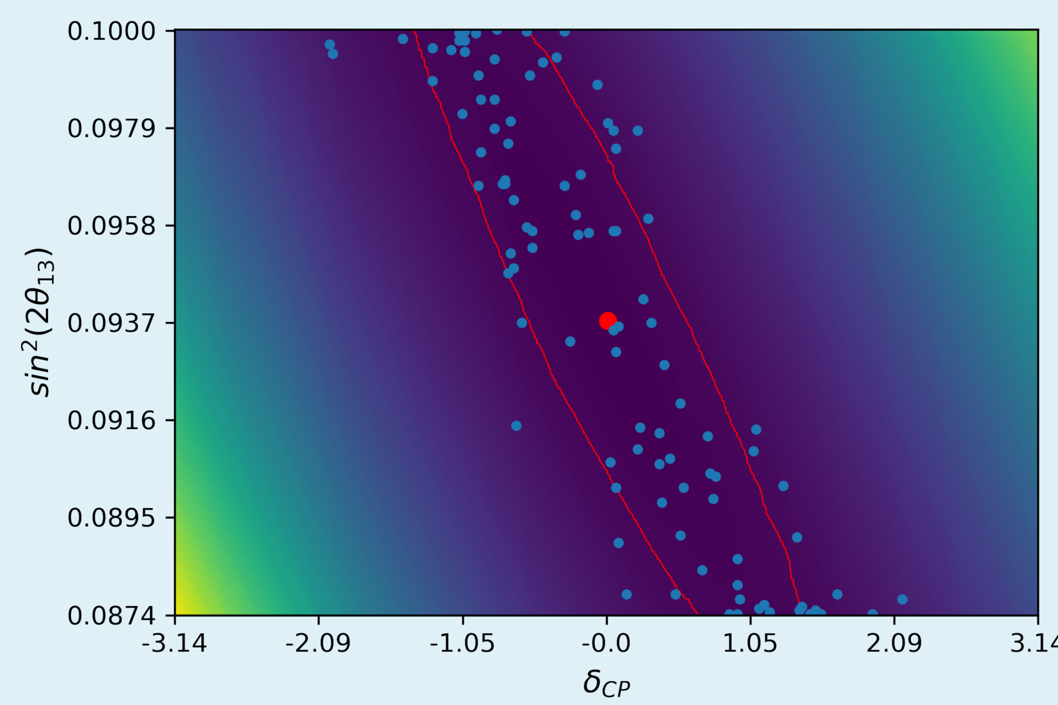

Figure 4. The likelihood surface with the maximum indicated by the red dot and 1σ confidence interval for an example toy experiment (red contour line). The blue dots are random throws within the Poisson error of the toy experiment, calculated to show that they fall within the contour at the given confidence interval.

References

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