BLACKETT LABORATORY

Mathematical Formulae

$\sin(A \pm B) = \sin A \cos B \pm \cos A \ \sin B$	$\sin(2A) = 2 \sin A \cos A$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\cos 2A = \cos^2 A - \sin^2 A$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$= 2\cos^2 A - 1 = 1 - 2\sin^2 A$
$\sin(\pi/2 - A) = \cos A$	$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin A + \sin B = +2 \sin\left(\dfrac{A+B}{2}\right) \cos\left(\dfrac{A-B}{2}\right)$	$\sin C \cos D = +\frac{1}{2}[\sin(C+D) + \sin(C-D)]$
$\sin A - \sin B = +2 \cos\left(\dfrac{A+B}{2}\right) \sin\left(\dfrac{A-B}{2}\right)$	$\cos C \cos D = +\frac{1}{2}[\cos(C+D) + \cos(C-D)]$
$\cos A + \cos B = +2 \cos\left(\dfrac{A+B}{2}\right) \cos\left(\dfrac{A-B}{2}\right)$	$\sin C \sin D = -\frac{1}{2}[\cos(C+D) - \cos(C-D)]$
$\cos A - \cos B = -2 \sin\left(\dfrac{A+B}{2}\right) \sin\left(\dfrac{A-B}{2}\right)$	

ARITHMETIC SERIES

$$S_n = a + (a+d) + (a+2d) + (a+3d) + \cdots [a + (n-1)d] = \tfrac{n}{2}[2a + (n-1)d]$$

GEOMETRIC SERIES

$$S_n = a + ar + ar^2 + ar^3 + \cdots ar^{(n-1)} = a\tfrac{(1-r^n)}{(1-r)}$$

For $|r| < 1$ $\quad \lim_{n\to\infty} S_n = \dfrac{a}{(1-r)}$

BINOMIAL SERIES

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \cdots$$

TAYLOR SERIES

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} \cdots\cdots \text{ where } f'(a) = \left(\frac{df}{dx}\right)_{x=a}.$$

MACLAURIN SERIES ($a = 0$)

$$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} \cdots\cdots$$

Examples (all with $a = 0$).

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad \tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots \quad |x| < 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \cdots$$

For a sphere of radius r, area$=4\pi r^2$, volume$=(4/3)\pi r^3$.

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$\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$	$\cosh x = \dfrac{e^{x} + e^{-x}}{2}$
$\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$	$\sinh x = \dfrac{e^{x} - e^{-x}}{2}$
$e^{ix} = \cos x + i \sin x$	$e^{x} = \cosh x + \sinh x$
$e^{-ix} = \cos x - i \sin x$	$e^{-x} = \cosh x - \sinh x$
$1 = \cos^2 x + \sin^2 x$	$1 = \cosh^2 x - \sinh^2 x$

$$\lim_{x \to 0} \frac{\sin ax}{ax} = 1$$

For integer n:
$n! = n(n-1)(n-2) \cdots \cdots 2 \cdot 1$
$0! = 1$

$\dfrac{d}{dx}(ax)$	$= a$	$\int x^n \, dx$	$= \dfrac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
$\dfrac{d}{dx}(ax^n)$	$= nax^{n-1}$	$\int \cos x \, dx$	$= \sin x + C$
$\dfrac{d}{dx}\left(\dfrac{a}{x}\right)$	$= -\dfrac{a}{x^2}$	$\int \sin x \, dx$	$= -\cos x + C$
$\dfrac{d}{dx}(\sin ax)$	$= +a \cos ax$	$\int \sec^2 x \, dx$	$= \tan x + C$
$\dfrac{d}{dx}(\cos ax)$	$= -a \sin ax$	$\int \mathrm{cosec}^2 x \, dx$	$= -\cot x + C$
$\dfrac{d}{dx}(\tan ax)$	$= a \sec^2 ax$	$\int \tan x \, dx$	$= -\ln \lvert \cos x \rvert + C$
$\dfrac{d}{dx}(\sec ax)$	$= a \sec ax \tan ax$	$\int e^{ax} \, dx$	$= \dfrac{e^{ax}}{a} + C$
$\dfrac{d}{dx}(e^{ax})$	$= ae^{ax}$	$\int a^x \, dx$	$= \dfrac{a^x}{\ln a} + C$
$\dfrac{d}{dx}(\sinh x)$	$= \cosh x$	$\int \dfrac{1}{x} \, dx$	$= \ln \lvert x \rvert + C$
$\dfrac{d}{dx}(\cosh x)$	$= \sinh x$	$\int \dfrac{f'(x)}{f(x)} \, dx$	$= \ln \lvert f(x) \rvert + C$
$\int \dfrac{dx}{x^2 + a^2}$	$= \dfrac{1}{a} \tan^{-1}\left(\dfrac{x}{a}\right) + C$	$\int \dfrac{dx}{\sqrt{a^2 - x^2}}$	$= \sin^{-1}\left(\dfrac{x}{a}\right) + C$
$\int \dfrac{dx}{\sqrt{x^2 \pm a^2}}$	$= \ln\left(x + \sqrt{x^2 \pm a^2}\right) + C$	$\int \dfrac{x \, dx}{\sqrt{x^2 \pm a^2}}$	$= \sqrt{x^2 \pm a^2} + C$

Speed of light in vacuo	c	3.00	$\times 10^{8}$	$\mathrm{ms^{-1}}$	
Permittivity constant	ϵ_0	8.85	$\times 10^{-12}$	$\mathrm{Fm^{-1}}$	
Permeability constant	μ_0	4π	$\times 10^{-7}$	$\mathrm{Hm^{-1}}$	
Electronic charge (-)	e	1.60	$\times 10^{-19}$	C	
Mass of an electron	m_e	9.11	$\times 10^{-31}$	kg	$= 0.511\ \mathrm{MeV}/c^2$
Mass of a proton	m_p	1.67	$\times 10^{-27}$	kg	$= 938\ \mathrm{MeV}/c^2$
Planck's constant	h	6.63	$\times 10^{-34}$	Js	
	$\hbar = h/2\pi$	1.05	$\times 10^{-34}$	Js	
Gravitational constant	G	6.67	$\times 10^{-11}$	$\mathrm{Nm^2kg^{-2}}$	
Earth's gravity	g	9.81		$\mathrm{ms^{-2}}$	
Gas constant	R	8.31		$\mathrm{JK^{-1}mol^{-1}}$	
Avogadro's number	N_A	6.02	$\times 10^{23}$	$\mathrm{mol^{-1}}$	
Boltzmann's constant	$k_B = R/N_A$	1.38	$\times 10^{-23}$	$\mathrm{JK^{-1}}$	$= 8.62 \times 10^{-5}\ \mathrm{eV\ K^{-1}}$
Stefan's constant	σ	5.67	$\times 10^{-8}$	$\mathrm{W\,m^{-2}K^{-4}}$	
Atomic mass unit	u	1.66	$\times 10^{-27}$	kg	
Bohr magneton	$\mu_B = e\hbar/2m_e$	9.27	$\times 10^{-24}$	$\mathrm{JT^{-1}}$	
Rydberg constant	R(Rydberg)	1.10	$\times 10^{7}$	$\mathrm{m^{-1}}$	$= 13.6\ \mathrm{eV}/hc$
Fine-structure constant	$\alpha = e^2/(4\pi\varepsilon_0\hbar c)$	7.30	$\times 10^{-3}$		$= 1/137$