Pressurisation in microstructured optical fibre drawing

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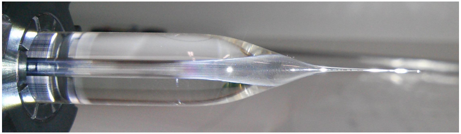

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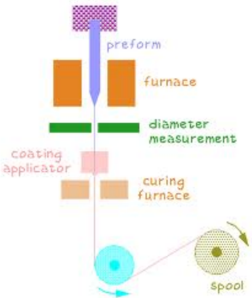
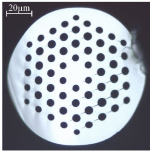
Microstructured optical fibres (MOFs)



Fibre drawing

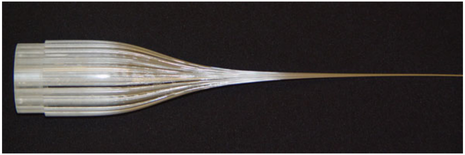
Have a large number of wavelength-scale channels which make for novel optical properties.



Cross section

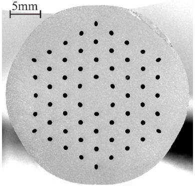
Fabricated by drawing a heated preform onto a rotating spool.

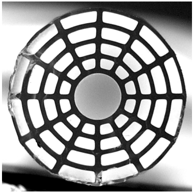
$$\text{Draw ratio} = \frac{\text{draw speed}}{\text{feed speed}} \gg 1.$$



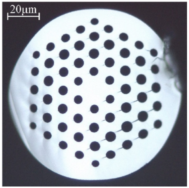
Photos: IPAS, Adelaide

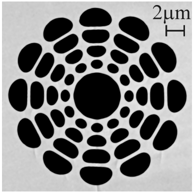
Fibre drawing modifies the geometry





preform





fibre

During fibre drawing the cross section deforms under surface tension and channel pressurisation.

Goal: understand this deformation.

Forward problem:

For given preform geometry and draw parameters . . . what fibre geometry?

Inverse problem:

For a desired fibre geometry . . . what preform geometry and draw parameters?

Mathematical model

- Consider: surface tension, pressurisation of channels, varying viscosity.
- Neglect: inertia, gravity.

Model with internal pressure

3D steady Stokes' flow; slender geometry ($\epsilon = \sqrt{S_0}/L$).

Axial model

$$\chi^2(\tau)\,u(\tau) = 1$$

$$\frac{\mathrm{d}\chi}{\mathrm{d}\tau} - \frac{\chi}{12}\tilde{\Gamma}(\tau) = -\mathcal{T}$$

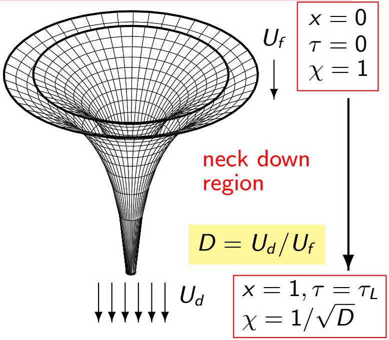
Cross-plane model

$$\nabla \cdot U = 0, \quad -\nabla p + \nabla^2 U = 0 \quad \text{in fluid}$$

$$\sigma \cdot \hat{n} = -\left(\kappa + \mathcal{P}^{(k)}\chi\right)\hat{n} \quad \text{on surface}$$

$$0 = G_\tau + vG_y + wG_z \quad \text{on surface}$$

- τ is a Lagrangian reduced time co-ordinate, $\tau = \gamma^* \int_0^t \frac{dt}{\mu^* \chi}$;
- $\chi^2(\tau)$ is the cross-sectional area;
- $\tilde{\Gamma}(\tau)$ is the scaled total circumference.



If there's axially varying pressure in the channels

$$(3\mu^* S u_x)_x + \frac{1}{2}\gamma^* \Gamma_x + \gamma^* \sum_{k=1}^{N} \mathcal{P}_x^{(k)} A^{(k)} = 0$$

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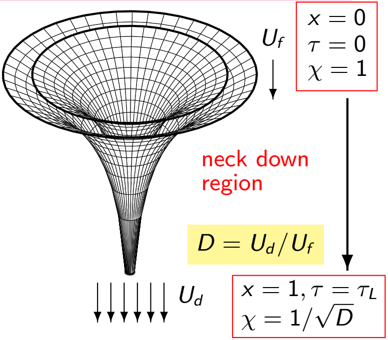
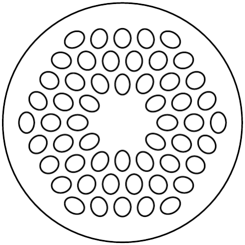


Solving the 2D cross-plane model

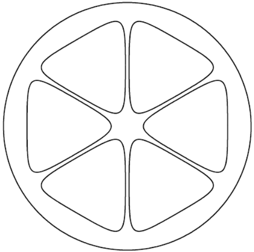
Straightforward for the fundamental case of a tubular fibre.

For more general geometries have a suite of numerical solution techniques based on complex variable methods:

- Elliptical pore model: channel shape are ellipses (Buchak et al, JFM '15)

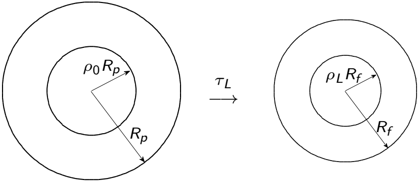


- Spectral method: more general channel shapes (Buchak & Crowdy, JCP '16)

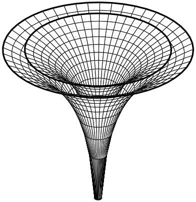


Tubular fibres

The two boundaries are constrained to be concentric circles - it's convenient to describe them by the ratio of their radii, $\rho(\tau)$ (with $0 < \rho < 1$).



initial preform with ρ_0 final fibre with ρ_L



The governing equations, which are solved from $\chi = 1$ to $1/\sqrt{D}$, are:

$$\frac{\mathrm{d}\chi}{\mathrm{d}\tau} = \frac{\sqrt{\pi}}{6}\chi\,(1+\rho)^{\frac{1}{2}}\,(1-\rho)^{-\frac{1}{2}} - \mathcal{T},$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}\tau} = -\frac{\sqrt{\pi}}{2}\,(1+\rho)^{\frac{3}{2}}\,(1-\rho)^{\frac{1}{2}} + \frac{1}{2}\mathcal{P}\rho\chi.$$

Additionally, it is sometimes useful to solve for the axial position x:

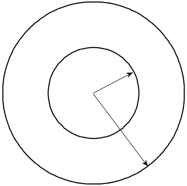
$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = \frac{\mu^*}{\gamma^*\chi}$$

Tubular fibres: summary of experiments

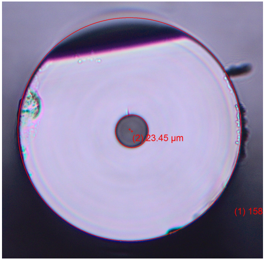
Six extruded tubular preforms were drawn to fibre.

- Account for taper from extrusion process.
- Initial geometry either $\rho_0 = 0.16$ or $\rho_0 = 0.515$.
- Systematically vary one operational parameter at a time: furnace temperature (and therefore tension), draw speed or pressurisation.
- Take microscope measurements of fibre outer and inner diameter from the drum.

Preform: 10mm OD with ρ_0

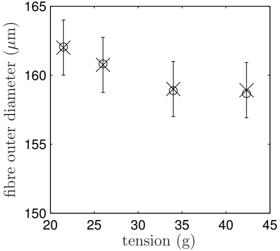


Fibre: 160μm OD with ρ_L

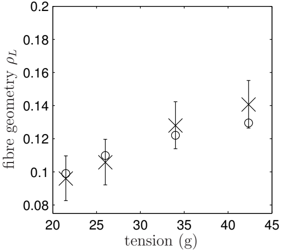


Results

Experiment 2

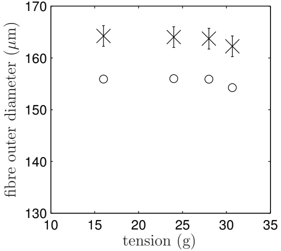


(d)

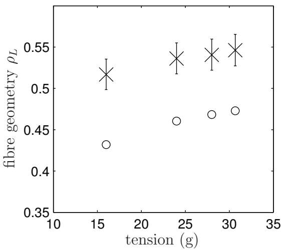


$\rho_0 = 0.16$, $U_\mathrm{f} = 1.4$mm/min,
$U_\mathrm{d} = 6.1$m/min, $T_\mathrm{f} = 940$–905°C

Experiment 4



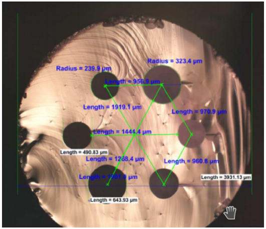
(d)



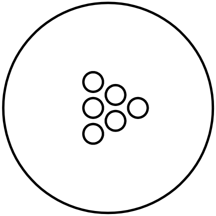
$\rho_0 = 0.515$, $U_\mathrm{f} = 1.4$mm/min,
$U_\mathrm{d} = 5.9$m/min, $T_\mathrm{f} = 980$–950°C

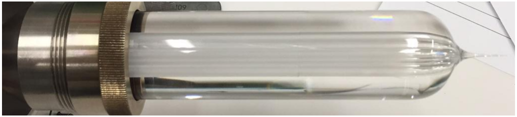
Multi-hole preforms: two experiments

1. From the literature (Luzi et al. JLT 2012):
   - Six holed preform, drawn with a range of (quite large) pressurisations.
   - Relative small outer diameter (3.93 mm), small draw ratio (D=935)

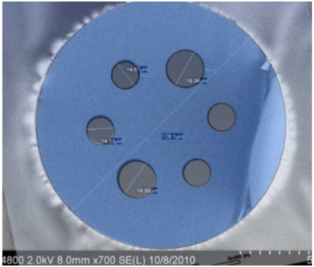


2. A new purpose-designed experiment:
   - Six holed preform, drawn at a range of fibre tensions
   - Large outer diamter (30 mm), large draw ratio (D=45500)

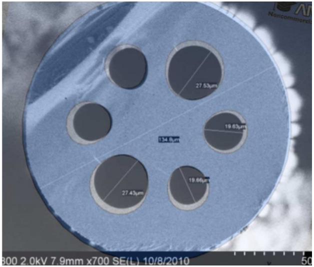




Multi-hole preform: Luzi et al.

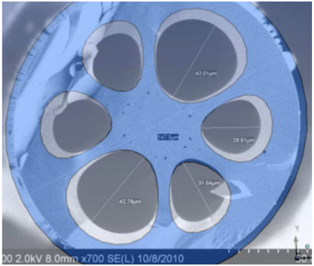


pressure=0 mbar



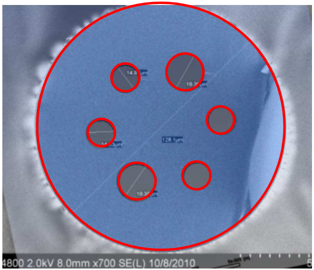
pressure=150 mbar

pressure=250 mbar

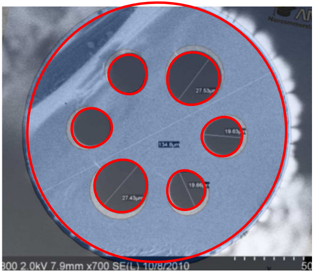


pressure=300 mbar

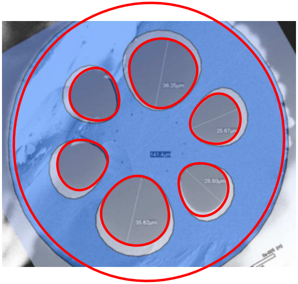
Multi-hole preform: Luzi et al. (more)



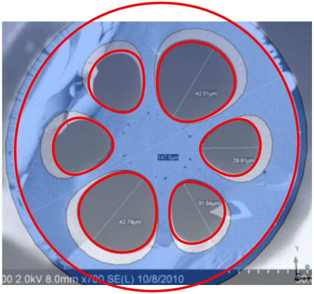
pressure=0 mbar



pressure=150 mbar



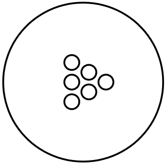
pressure=250 mbar



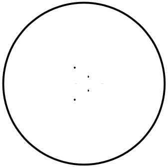
pressure=300 mbar

Multi-hole preforms (VA preform):

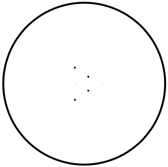
prefrom:



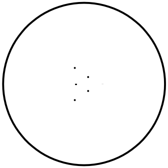
tension=15 g



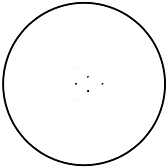
tension=19 g



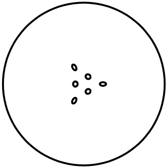
tension=27 g



tension=37 g



tension=55 g

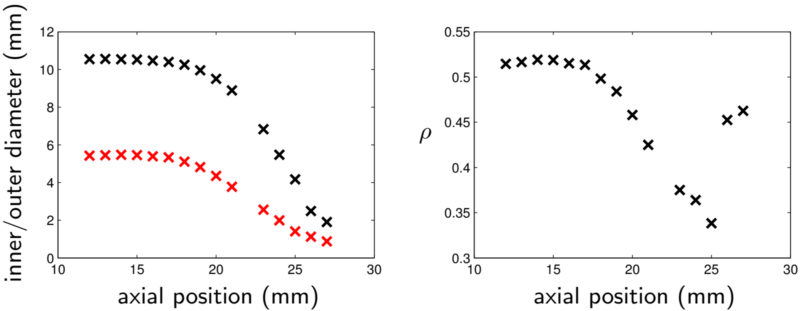


Summary of experiments

The modelling is a good match for some of the experiments, but not others - what's going on here?

- Where they don't match it looks like the geometries are 'inflated', could this be a pressure effect?
- Discrepancies more pronounced for larger channels.

The cooled preform from experiment 4 ($\rho_0 = 0.515$) was sliced up into 1mm sections.

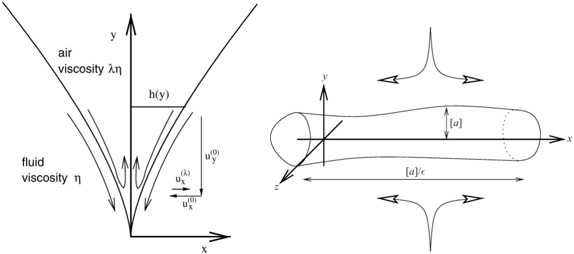


Air entrainment in channels

There is evidence that a some kind of pressure is induced in the channels:

- Pressure 'felt' more strongly in larger channels.
- It seems to increase with tension (maybe draw speed too).
- It varies axially.

A possible explanation is that the air in the channels is entrained as the fibre is drawn (suggested by Eggers PRL '01 and Howell & Siegel JFM '04).



Suggests we need to model the flow *in* the channels.

Air entrainment in channels (cont.)

Recall that the axial momentum equation from earlier was:

$$(3\mu^* S u_x)_x + \frac{1}{2}\gamma^* \Gamma_x + \gamma^* \sum_{k=1}^{N} \mathcal{P}_x^{(k)} A^{(k)} = 0$$

Air entrainment in channels (cont.)

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Treat air as a 'weakly viscous fluid', that is $\mu_{\mathrm{air}} = \epsilon^2 \lambda \mu_0$.

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Rederiving the above equation, the last term drops off to give the usual

$$\frac{\mathrm{d}\chi}{\mathrm{d}\tau} - \frac{\chi}{12}\tilde{\Gamma}(\tau) = -\mathcal{T}$$

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$$\frac{\mathrm{d}\chi}{\mathrm{d}\tau} - \frac{\chi}{12}\tilde{\Gamma}(\tau) = -\mathcal{T}$$

and the pressure gradient in the channels is:

$$\frac{\mathrm{d}p_{a0}^{(j)}}{\mathrm{d}\tau} = \frac{\mu^*(x)}{\gamma^*} \frac{\lambda}{\chi^5} \frac{1}{Q\tilde{A}^{(j)}}$$

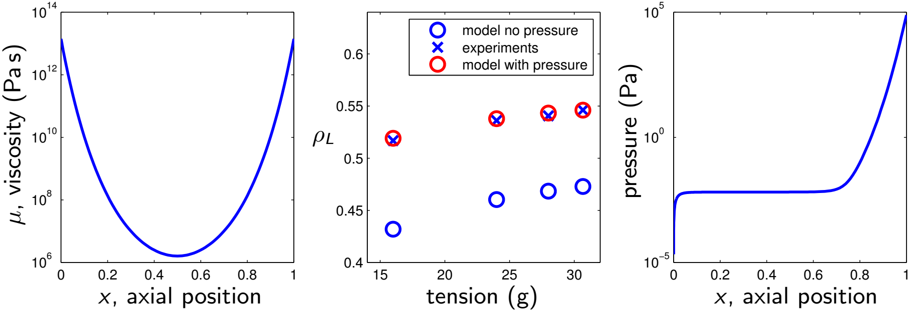
- $\tilde{A}^{(j)}$ is the channel area;
- Q depends on the channel geometry, straightforward expressions for circular and elliptical cross-sections.

Another look at the large channel tube

For a tube the pressure gradient is:

$$\frac{\mathrm{d}p_{a0}}{\mathrm{d}\tau} = \frac{\mu^*(x)}{\gamma^*}\,\frac{\lambda}{\chi^5}\,\frac{8\pi(1-\rho^2)}{\rho^2}$$

In the absence of knowledge about how the viscosity $\mu^*(x)$ varies axially, choose a profile based on a Gaussian temperature profile.

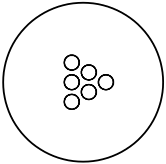


Another look at the 6-hole preform

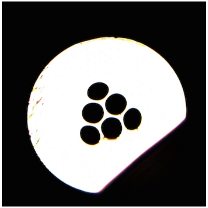
Can we get the kind of inflation we saw in the experiments?

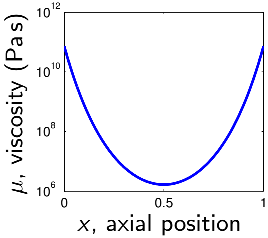
Choose a (slightly) less steep viscosity curve:

prefrom:

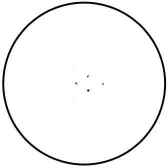


tension=37 g

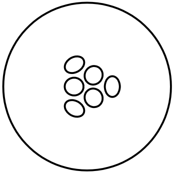


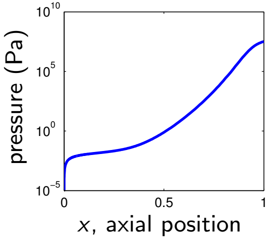


no pressure:



pressure:



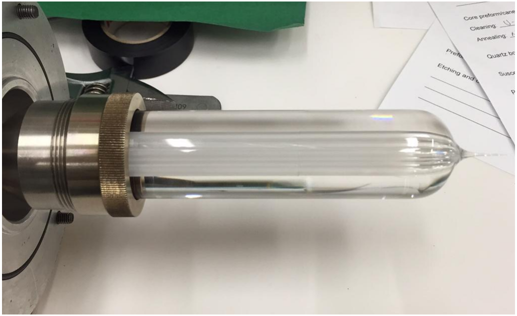


Reasonable match for the geometry - pressure quite large for this case.

Future work

- Extend to non-elliptical geometries.
- Modelling of the glass temperature, to get more realistic viscosity profiles.

Thanks!



Some papers:
Chen et al., *Asymptotic Modelling of a Six-Hole MOF*, J. Lightwave Tech. 34(24) 2016.
Buchak & Crowdy, *Surface-tension-driven Stokes flow: A numerical method based on conformal geometry*, JCP 317, 2016.