Topological properties of surfaces flows

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13th March 2017
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2. Rep of uniform flows with pt vortices
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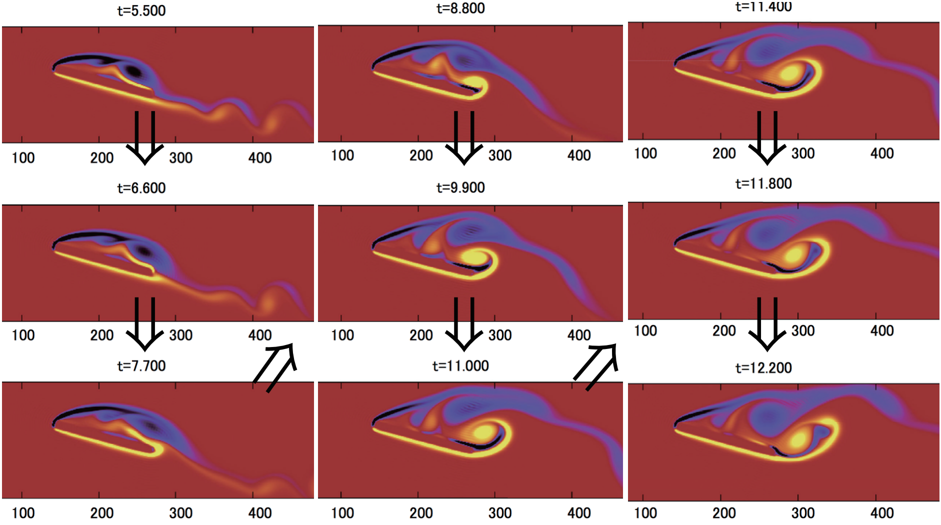
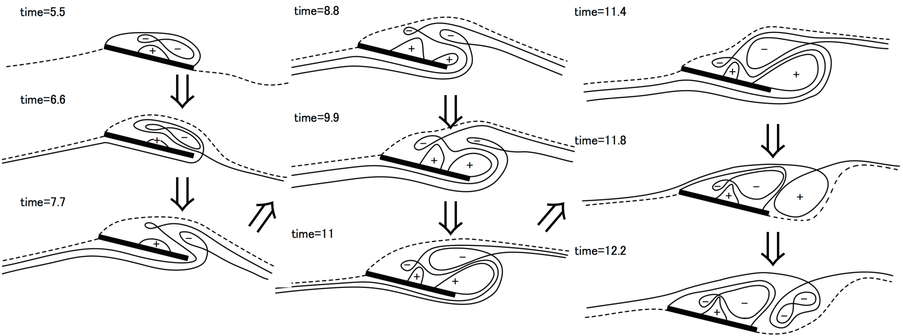
A problem and a topological approch

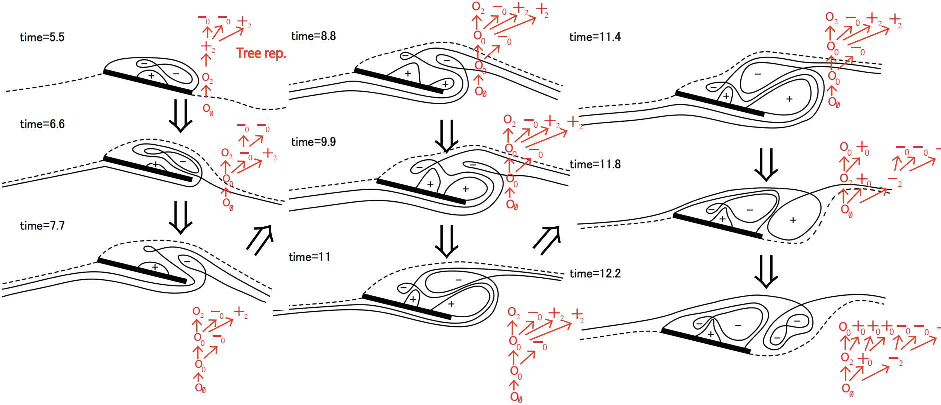
Find extremal lift-to-drag ratios



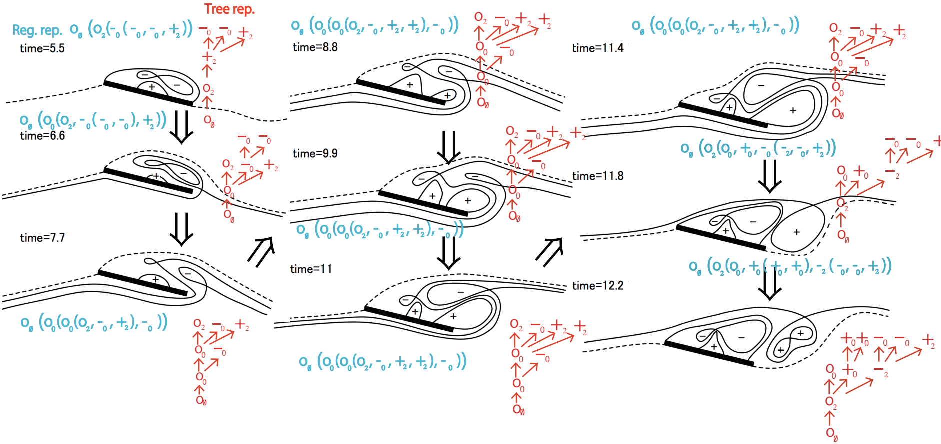
Find extremal lift-to-drag ratios



Find extremal lift-to-drag ratios



Find extremal lift-to-drag ratios



Find extremal lift-to-drag ratios

time = 5.5

$o_{\emptyset}\,(o_2(-_0\,(-_0\,, -_0\,, +_2\,))$

time = 6.5 $\Downarrow$

$o_{\emptyset}\,(o_0(o_2, -_0(-_0\,, -_0), +_2))$

time = 7.5 $\Downarrow$

$o_{\emptyset}\,(o_0(o_0(o_2, -_0\,, +_2), -_0\,))$

$\Nearrow$

time = 8.8

$o_{\emptyset}\,(o_0(o_0(o_2, -_0\,, +_2, +_2), -_0\,))$

$\|$

time = 9.9

$o_{\emptyset}\,(o_0(o_0(o_2, -_0\,, +_2, +_2), -_0\,))$

$\|$

time = 11

$o_{\emptyset}\,(o_0(o_0(o_2, -_0\,, +_2, +_2), -_0\,))$

$//$

time = 11.4

$o_{\emptyset}(o_0(o_0(o_2, -_0\,, +_2, +_2), -_0\,))$

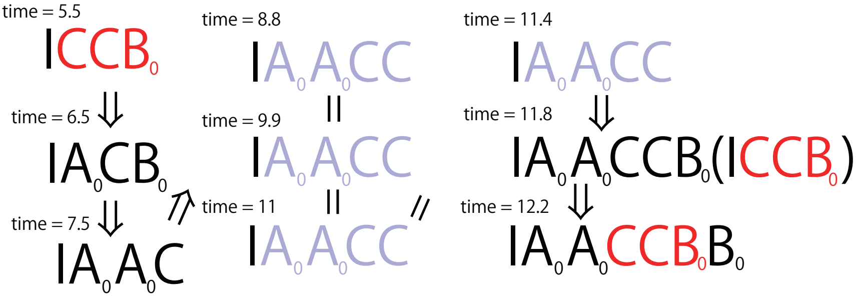
time = 11.8 $\Downarrow$

$o_{\emptyset}(o_2(o_0, +_0, -_0\,(-_2, -_0, +_2))$

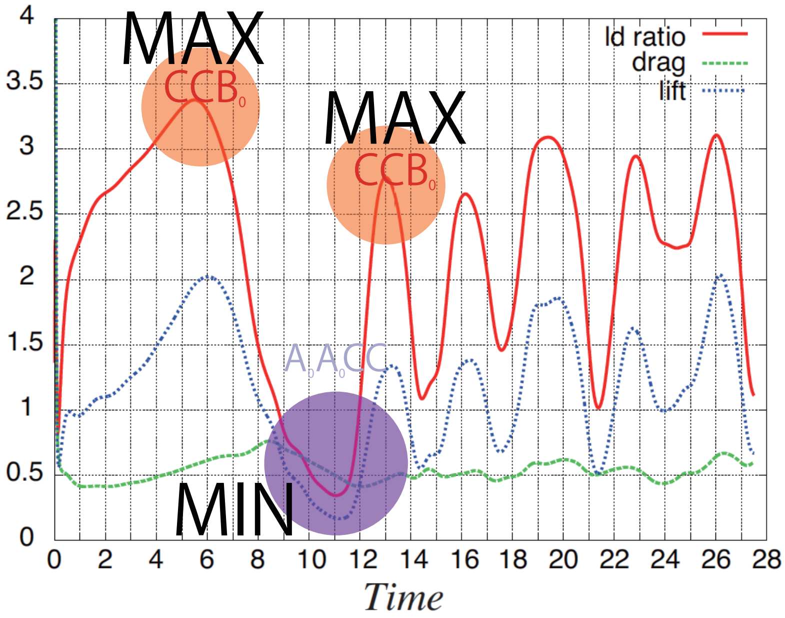
time = 12.2 $\Downarrow$

$o_{\emptyset}(o_2(o_0, +_0(+_0, +_0), -_2\,(-_0, -_0, +_2)))$

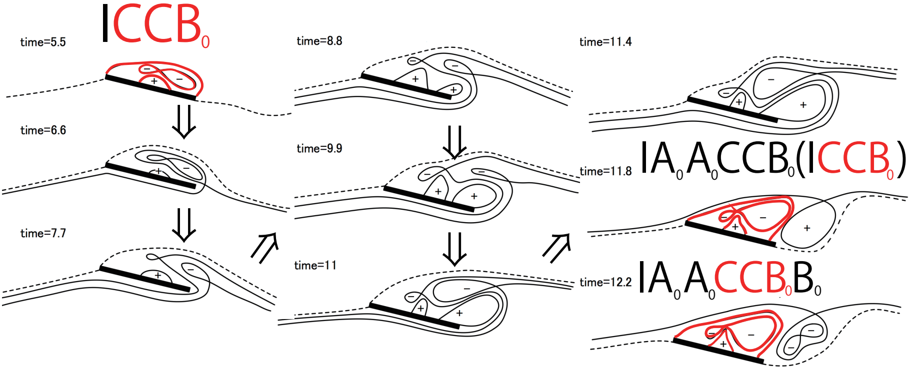
Find extremal lift-to-drag ratios



Evolutions of the lift, the drag and the lift-to-drag ratio



Find high lift-to-drag ratios



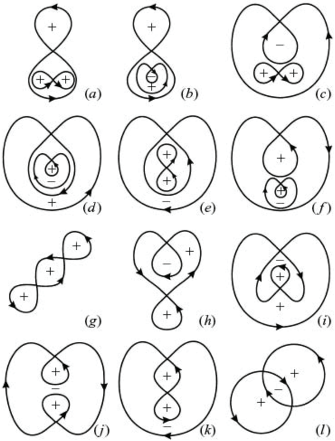
Theoritical background

Topological classification of streamline patterns for Hamiltonian surface flows

◦ Aref and Brøns (1998)
Case: planes without boundaries

◦ Kidambi and Newton (2000)
Case: spheres without boundaries

◦ Moffatt H.K. (2001)
Case: tori without boundaries



Aref & Brøns (JFM:1998)

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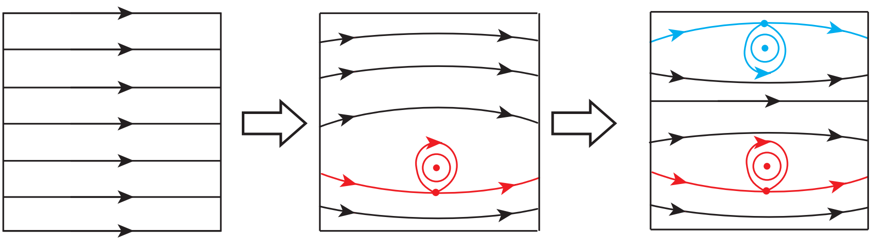
Uniform flows with vortices $\Longrightarrow$ Reduced tree structures

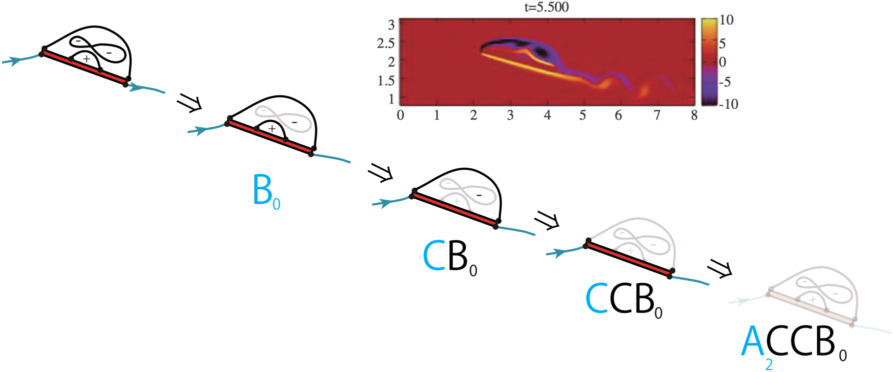
$$W(z) = Uz \implies \varepsilon$$

$$W(z) = Uz + i\kappa \log z \implies A_0$$

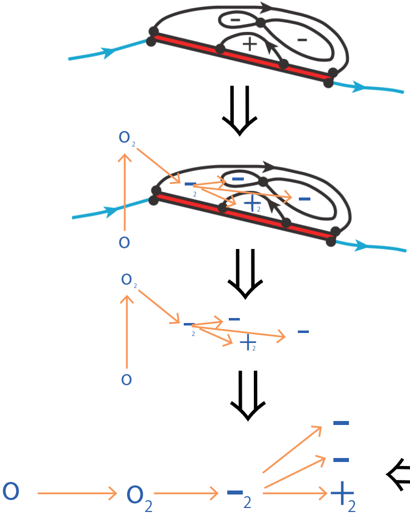
$$W(z) = Uz + i\kappa \log z - i\kappa \log(z-a) \implies A_0 A_0$$

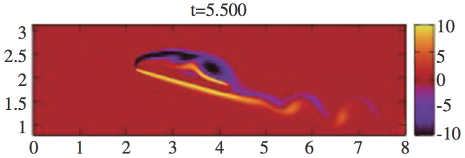


Uniform flows with vortices $\Longrightarrow$ Reduced tree structures



Uniform flows with vortices $\Longrightarrow$ Tree structures





Word representation
(Reduced tree representation)

$$AC_2CB_0$$

$$\Uparrow$$

$$\Leftrightarrow \quad o(o_2, -_2(-, -, +_2))$$

Tree representation

Tree representation (Joint work with T. Sakajo)

We construct algorithmically an injection:

$$\begin{array}{c}\exists \text{ inj} : \{\text{generic unif flow w/ vortices}\}/\sim \\ \downarrow \\ \{\text{tree}\} \\ \wr\| \\ \{\text{regular expression}\}\end{array}$$

The injection is called a tree representation of uniform flows with vortices

Uniform flows $\Longrightarrow$ Tree structures

Local stream topological structure $\Longrightarrow$ Letter

Global stream topological structure $\Longrightarrow$ Label + Edge

Topology of a flow $\Longrightarrow$ Labelled graph

Topology of a time-dep flow $\Longrightarrow$ Seq of labelled graphs

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Surface flows $\Longrightarrow$ Tree structures

A flow is of finite type $\overset{\text{def}}{\Longleftrightarrow}$ 1. Each stagnation point is non-degenerate

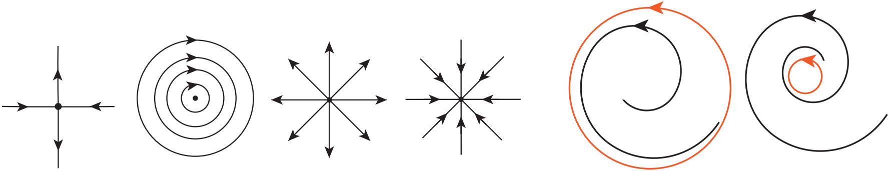
2. There are at most finitely many limit cycles

and 3. There are no quasi-minimal sets

Theorem 1

A flow of finite type on a compact surface can be reconstructed by finite data.

quasi-minimal set = orbit closure of a non-closed recurrent point

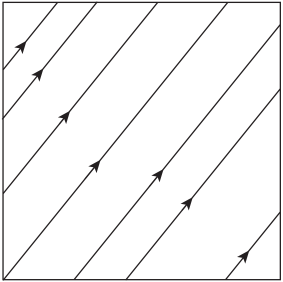


saddle center source sink Limit cycles

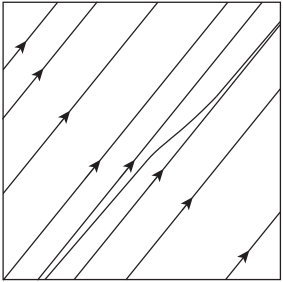
Necessity of the condition 3 (Non-existence of quasi-minimal sets)

Remark 1

The set of topological equivalent classes of minimal flows (resp. Denjoy flows) on a torus is uncountable.



Minimal flow



Denjoy flow

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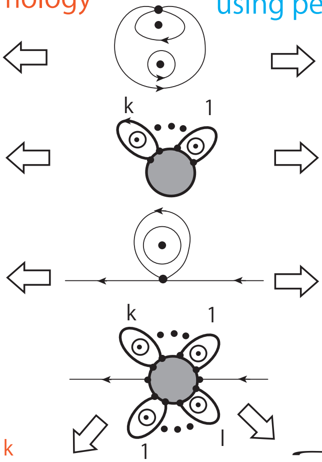
Good data str and Bad data str (Joint work with T.Yokoyama)

Hard to compute by computers using persistent homology

Easy to compute by computers using persistent homology



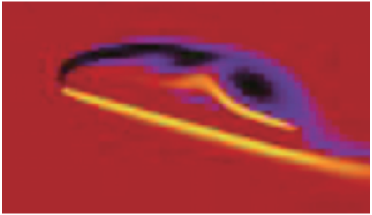
$+(+,-)$ ⇐ ⇒ β_{i+}

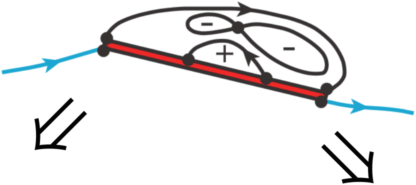
$(\overbrace{+_2, \dots, +_2}^{k})$ ⇐ ⇒ $\beta_{2+}(\overbrace{c_+, \dots, c_+}^{k})$

$(o, +)$ ⇐ ⇒ a_+

$(o_2(\overbrace{-_2, \dots, -_2}^{l}), \overbrace{+_2, \dots, +_2}^{k})$ ⇙ ⇘ $a_2(\overbrace{c_+, \dots, c_+}^{k}, \overbrace{c_-, \dots, c_-}^{l})$

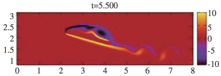
Good data structures and Bad data structures



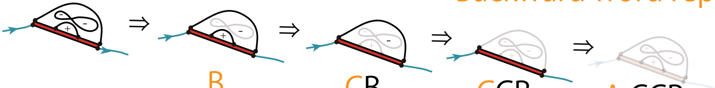


$o(o_2, -_2(-, -, +_2))$ $\alpha_2(c_-(\beta_{o-}, c_+))$

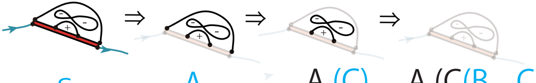
Good data structures and Bad data structures



Backward word rep



B_0 $\quad$ CB_0 $\quad$ CCB_0 $\quad$ A_2CCB_0



ε $\quad$ A_2 $\quad$ $A_2(C)$ $\quad$ $A_2(C(B_0, C))$

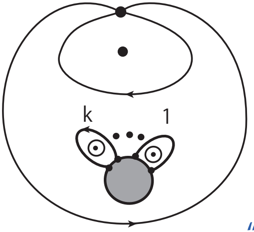
Forward word rep

ε $\quad$ A_2 $\quad$ A_2C $\quad$ A_2CB_0C

Reduced forward word rep

Good data structures and Bad data structures

Data str of topology of a Hamiltonsurface flow
= Regular tree grammar + Cyclic order



cyclic order

$$\beta_{i+}(\beta_{2+}(\overbrace{c_{+}, \dots, c_{+}}^{k}))$$

“Regular tree grammar”

Summary

We construct the following injective representation concretely

$$\mathrm{inj} : \{\text{ Uniform flow with vortices }\} \to \{\text{ finite labelled graphs }\}$$

We show the existence of following injective representation

$$\mathrm{inj} : \{\text{ surface flow of finite type}\} \to \{\text{ finite data }\}$$

Surface flows and Data structures

Topology of a flow $\Longrightarrow$ Regular tree grammar $+$ Cyclic order
Resolution $\Longrightarrow$ Depth of nodes
Good data structure $\Longrightarrow$ Persistent $\Longrightarrow$ Easy to implement
Bad data structure $\Longrightarrow$ Sensitive to error $\Longrightarrow$ Hard to implement

Appendix

Idea

Spatial setting

Rough representation : Space $\mapsto$ Homotopy group

Finer representation : Space $\mapsto$ Cell decomposition

We construct analogical tools as follows:

Flow setting

Rough representation : Hamiltonian flow $\mapsto$ Word

Finer representation : Flow $\mapsto$ Graph

Uniform flows $\implies$ Tree structures

vortex + orientation + (un)bdd-ness $\implies$ Letter

(ss-)saddle connection diagram $\implies$ Label + Edge

Snapshot of a unif flow w/ vortices $\implies$ Labelled graph

Uniform flow w/ vortices $\implies$ Seq of labelled graphs

Surface flows and Data structures

Topology of a flow	$\Longrightarrow$	Regular tree grammar + Cyclic order		
Resolution	$\Longrightarrow$	Depth of nodes		
Good data structure	$\Longrightarrow$	Persistent	$\Longrightarrow$	Easy to implement
Bad data structure	$\Longrightarrow$	Sensitive to error	$\Longrightarrow$	Hard to implement