Electric Dipole Moments in a generic CP violating theory, And its application to the MSSM

Matthias Le Dall

September 25, 2009

Supervised by Dr. Arttu Rajantie

Imperial College London
Blackett Laboratory
Theoretical Physics

Submitted in partial fulfilment of the requirements for the degree of Master of Science of Imperial College London

I dedicate this dissertation

To my parents, to my brothers and to my sister,

for the great support they have given me.

Abstract

In this dissertation, we will try to give the motivations for the search for the electric dipole moment of the neutron and particularly for the use of the minimal supersymmetric standard model to compute it. In this aim, a review of the calculation of the electric dipole moment of the neutron in the standard model, and specifically using quantum chromodynamics, has first been made. Then we will explain two methods to compute an expression for diagrams contributing to the electric dipole moments of fermions, using a generic theory containing CP-violating interactions. Finally, these results will be applied to the case of the minimal supersymmetric standard model.

Acknowledgements

I thank Dr. Arttu Rajantie for the very helpful discussions we had. I also want to thank Ali Mozaffari for having read my dissertation and for his constructive criticism.

Contents

Introduction

Experimentally we discovered symmetries in physics when we identified similarities in particles that seemed entirely different. For instance we saw that all particles with a spin will behave similarly, regardless to the value of their spin. Such observations allow to classify particles in classes. These experimental observations are very important as it simplifies the construction of theories. Instead of considering all particles with spin in the theory, we will rather consider a general particle with a general spin and mass, from what we will be able to explain the behavior of all particles having these same characterictics. A symmetry cannot be proved, it is postulated as it relies only on observations. We want to include these symmetries when building the theories. The first reason is simply to agree with the observation of symmetries. The second reason, is that, if the symmetry postulated is indeed a true symmetry, the theory will go beyond what we know from experiments and will have a great predictive power. This is why the search for symmetries in physics is so intense.

As a correlation to the symmetries, there are the broken symmetries. We first build beautiful theories with nice symmetries in it, to eventually break them, in order to understand better the theory itself. Is not that paradoxal? In fact, no, because physics is only a matter of scale. At the begining of the last century, physicists understood that matter does not look the same at different length scales. At the macroscopic scale, matter is a continuum, while at the atomic scale, we can roughly think of matter as an electron rotating around a nucleus. From then, we thought of matter as a Matryoshka doll of particles, each particle is made of smaller particles. The smaller the particle we want to observe is, the higher the energy needed to observe it is. Physics is therefore a matter of energy scale, as well as matter, it does not look the same at different energy scales. This fact is now explained by broken symmetries. A symmetry is broken at an energy scale lower than the scale at which the symmetry is still a true symmetry. A famous example of such broken symmetry, is the broken electroweak symmetry, which gives different masses to different observed particles.

Discrete symmetry is a crucial kind of symmetry in physics. Discrete symmetries give rise to six quark flavours, eight quark colours, one elementary electric charge, etc. The oldest discrete symmetry to be tested is parity. Newton's theory is parity invariant, and this is well observed in every day experiences. Though, parity symmetry is broken by electroweak interactions [1]. The first test of parity was done by Purcell and Ramsey in 1949 in the domain of strong interactions [2]. They supposed the Hamiltonian for a nonrelativistic charged fermion of spin $\vec{S}$ placed in an electric field $\vec{E}$ and magnetic field $\vec{B}$, which can be expressed as:

$$H = -(d\vec{E} + \mu\vec{B}) \cdot \frac{\vec{S}}{S} \tag{1}$$

where μ is its magnetic dipole moment and d its electric dipole moment. They argued that, as a theory for strong interactions was still unknown, it was not possible to derive parity violation, thus one must assume parity symmetry. As $\vec{E}\cdot\vec{S}$ is both P- and T-odd, a non zero electric dipole moment exists if and only if these two transformations are broken. Experimental datas show that d is of the order of $10^{-18}e \cdot cm$, which was at that time far below the observable scale. That agreed with their conclusion that d must be zero. Experimentally, CP violation or T violation if one assumes CPT invariance of physics, has been observed via the mixing of Kaons [3]. It provides the first experimental hint that an EDM can exist. This observation has been explained by the Kobayashi-Maskawa mechanism [4], thanks to which the theory went beyond in predicting the existence of three generations of matter. This existence of the third generation was experimentally confirmed when the top quark was first observed at the Fermilab in 1995 [5].

This was only when quantum chromodynamics came to life that time reversal non invariance was first theoretically possible. Indeed, QCD contains the so-called θ-term which is CP-odd. That was the first time that d could theoretically be non zero. The θ parameter is known to be tuned to 10^{-9}, which is the strong CP problem. It turns out that, this term has many impressive fundamental implications. Among others, it is responsible for the non triviality of the vacuum structures of gauge theories [6]. Also, its mixing to the quark masses in QCD gives a prediction of the EDM. The small value of θ implies a value for d of the order $10^{-27}e\cdot$cm. This value is almost ten orders of magnitude below the experimental datas. The standard model turns out to be insufficient to predict the right d.

The smallest theory beyond the standard model is the minimal supersymmetric standard model. It postulates the existence of a superpartner particle to each particle of the standard model, according to the, yet supposed, supersymmetry. The masses of the "ordinary" particles and their superpartners are the same under supersymmetry. These new particles are still experimentally unobserved, which is now understood to be a manifestation of the fact that supersymmetry must be broken at low energies, putting the masses of the superpartners well above the masses of the SM particles, although the lowest supersymmetric particle, LSP, is hoped to be seen at the LHC. The MSSM has more than 100 parameters, among which there are more than 10 CP-odd ones. There can potentially be more contributions to the EDM than there are in the SM, making the MSSM greatly hoped to predict the right amount of EDM. The attempt to make the MSSM agree with the observations put a lot of constraints on these new parameters. In fact, the CP-violation experiments, through the measurements of the electric dipole moment of the neutron, are the most sensitive experiments we dispose nowadays. This is why the research for the EDMs is so intensive.

The fact that supersymmetric particles have masses well above the masses of the SM particles, i.e. that supersymmetry is broken, implies that the MSSM is a theory living at a high energy scale and that the SM is a low energy limit of it. In other words, the SM is an effective theory obtained by integrating out heavy fields in the MSSM action. In particular, the effective Lagrangian includes the following CP-odd dimension 5 term:

$$\mathcal{L} = \frac{d}{4}\overline{\psi}[\gamma^\mu, \gamma^\nu]\gamma_5\psi F_{\mu\nu} \tag{2}$$

wich gives the fermion ψ an electric dipole moment d via its interaction with the electromagnetic field $F_{\mu\nu}$. This operator gets contributions from the MSSM through interactions of the kind of the ones in figure 1.

The first section of my dissertation will expose the basic notions of discrete symmetries, from their definitions to their actions on quantum fields. In the second section, we

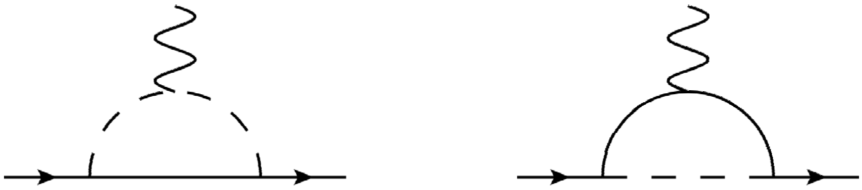


Figure 1: Diagrams giving contributions to the electric dipole moment d of the external fermion line in the MSSM. The internal scalar and fermion line are supersymmetric particles.

will look at the birth of the θ-term, in order to turn to the review of its implications on the electric dipole moment of the neutron in QCD. In the last section, we will give a way of deriving the electric dipole moment of a fermion in the case of a general model, and then review how these results are applied to the case of the electric dipole moment of the neutron in the minimal supersymmetric standard model.

1 Discrete transformations

1.1 P transformation

The P transformation is referred as the parity transformation. It consists in reversing spacial axis, i.e. it turns $\vec{r}$ into $-\vec{r}$. We write the action of P on $\vec{r}$ as: $P(\vec{r}) = -\vec{r}$. Also $\vec{v} = \frac{d\vec{r}}{dt}$, so we have [7]:

$$P(\vec{v}) = -\vec{v}, \tag{3}$$

from what it follows that $P(\vec{p}) = -\vec{p}$ with $\vec{p} = m\vec{v}$. And therefore, the action of the parity transformation on the Newton's law is given by:

$$P\left(\frac{d\vec{p}}{dt}\right) = -\frac{d\vec{p}}{dt} \qquad \Leftrightarrow \qquad P(\vec{F}) = -\vec{F} \tag{4}$$

Where $\vec{F} = d\vec{p}/dt$. In order to find the action of P on the electric and magnetic field, we can let P act on the Lorentz's force. Indeed

$$\vec{F}_{lorentz} = q(\vec{v} \wedge \vec{B} + \vec{E}) \tag{5}$$

So using equations (3), (4) and (5) we find :

$$P(\vec{E}) = -\vec{E} \quad , \qquad P(\vec{B}) = \vec{B} \tag{6}$$

In fact, (6) expresses the fact that $\vec{E}$ is a vector and that $\vec{B}$ is a pseudovector. The angular momentum is given by

$$\vec{J} = \vec{r} \wedge \vec{p}, \tag{7}$$

from what we can deduce that :

$$P(\vec{J}) = \vec{J} \tag{8}$$

Moreover, as the spin $\vec{S}$ is mathematically identical to the angular momentum, we can assume that :

$$P(\vec{S}) = \vec{S} \tag{9}$$

Finally, as S is a scalar, we automatically have $P(S) = S$. So, putting all these elements together, we will find that :

$$P(\vec{E} \cdot \vec{S}) = -\vec{E} \cdot \vec{S} \quad , \qquad P(\vec{B} \cdot \vec{S}) = \vec{B} \cdot \vec{S} \tag{10}$$

$$P(H) = (d\vec{E} - \mu\vec{B}) \cdot \frac{\vec{S}}{S} \tag{11}$$

In fact, similarly to equations (6), equations (10) express the fact that $\vec{E} \cdot \vec{S}$ is a pseudoscalar, and that $\vec{B} \cdot \vec{S}$ is a scalar.

1.2 T transformation

T transformation is time reversal, i.e. it turns t into $-t$, if t is the time variable. So the action of T on t is: $T(t) = -t$. In a very straight foward way, we can use the same techniques as in section 1.1 to find how T acts on quantities. We have:

$$T(\vec{E}) = \vec{E} \quad , \qquad T(\vec{B}) = -\vec{B} \tag{12}$$

$$T(\vec{E} \cdot \vec{S}) = -\vec{E} \cdot \vec{S} \quad , \qquad T(\vec{B} \cdot \vec{S}) = \vec{B} \cdot \vec{S} \tag{13}$$

$$T(H) = (d\vec{E} - \mu\vec{B}) \cdot \frac{\vec{S}}{S} \tag{14}$$

We see through (11) and (14) that H is PT symmetric:

$$PT(H) = H = TP(H) \tag{15}$$

Where $PT(H)$ means first applying the T transformation to H and then applying the P transformation.

1.3 C transformation

It is important to note that C transformation implies the existence of antiparticles, which do not exist within classical mechanics. The notion of antiparticle is intrisic to quantum field theories. For the present case of a classical particle, it would therefore make no sense to search a C transformation as we did for P and T transformations.

1.4 $\mathcal{P}$, $\mathcal{T}$ and $\mathcal{C}$ operators

We would like to define operators representing the action of P, T and C transformations on quantum fields. In order for that, we first say that in classical mechanics parity transformation does not affect the time coordinate, so it should commute with a translation in time. Then, by virtue of the principle of correspondance between classical and quantum mechanics, we would require the time translation operator to commute with the parity operatoy. Since, in quantum mechanics the time translation operator is represented by $U = \exp(-iH\Delta t)$, we would require $[\mathcal{P}, H] = 0$ which implies that P is a symmetry. However, we know from experiments that weak interactions violate parity. So this is not a good way to define this operator. The same is true for the $\mathcal{C}$ operator since, following the same arguments, we find that $[\mathcal{C}, H] = 0$, which is in contradiction with C not being a good symmetry of weak interactions.

Though, we need to define these operators. In order to define them, we note that electromagnetic interaction are invariant under both P and C transformations. We will therefore define the $\mathcal{P}$ and $\mathcal{C}$ operators so that they are good symmetries of the electromagnetic interaction. When we will then consider other interactions, e.g. weak and strong ones, we will still use as $\mathcal{P}$ and $\mathcal{C}$ operators the ones defined previously. In the case where the considered interaction is not invariant under the action of either $\mathcal{P}$ or $\mathcal{C}$, will say that this interaction breaks either P or C symmetry.

The definition of the $\mathcal{T}$ operator is not so simple. Indeed, unlike $\mathcal{P}$ or $\mathcal{C}$, $\mathcal{T}$ is an antiunitary operator. We can see this fact through the action of the T transformation on the fundamental relation of quantum mechanics:

$$[r_i, p_j] = i\delta_{ij} \tag{16}$$

where r_i is the i-th cartesian component of the vector position, and p_j is the j-th cartesian component of the momentum. As r_i is T-even while p_j is T-odd, the only way for (16) to be always true is to say that i must also be T-odd. That means:

$$\mathcal{T} i \mathcal{T}^\dagger = -i \qquad \Leftrightarrow \qquad \mathcal{T}\mathcal{T}^\dagger = -\mathbf{1} \tag{17}$$

which precisely shows the antiunitarity of $\mathcal{T}$. The same can be seen through the Schrodinger equation for a non relativistic particle:

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\vec{\nabla}^2\psi \tag{18}$$

Since the classical equations of motion are time-reversal invariant, and since $\partial/\partial t$ is T-odd while $\vec{\nabla}$ is T-even, we must impose:

$$\mathcal{T} i \mathcal{T}^\dagger = -i \tag{19}$$

which again implies the antiunitarity of $\mathcal{T}$.

Adopting the metric $\eta^{\mu\nu} = \mathrm{diag}\,(1,-1,-1,-1)$, we can define $\mathcal{P}$, $\mathcal{C}$ and $\mathcal{T}$ operators satisfying the following properties:

$$\mathcal{P}^\dagger\mathcal{P} = \mathbf{1} \quad , \qquad \mathcal{C}^\dagger\mathcal{C} = \mathbf{1} \quad , \qquad \mathcal{T}\mathcal{T}^\dagger = -\mathbf{1} \tag{20}$$

$$P : \partial_\mu \rightarrow \partial^\mu \quad , \qquad T : \partial_\mu \rightarrow -\partial^\mu \tag{21}$$

Their action on a complex scalar, electromagnetic and spinor fields are as follows [8]:

$$\mathcal{P}^\dagger\phi(\vec{r},t)\mathcal{P} = \eta_P\phi(-\vec{r},t) \tag{22a}$$

$$\mathcal{T}^\dagger\phi(\vec{r},t)\mathcal{T} = \eta_T\phi(\vec{r},-t) \tag{22b}$$

$$\mathcal{C}^\dagger\phi(\vec{r},t)\mathcal{C} = \eta_C\phi(\vec{r},t)^\dagger \tag{22c}$$

$$\mathcal{P}^\dagger A_\mu(\vec{r},t)\mathcal{P} = A^\mu(-\vec{r},t) \tag{22d}$$

$$\mathcal{T}^\dagger A_\mu(\vec{r},t)\mathcal{T} = A^\mu(\vec{r},-t) \tag{22e}$$

$$\mathcal{C}^\dagger A_\mu(\vec{r},t)\mathcal{C} = -A_\mu(\vec{r},t) \tag{22f}$$

$$\mathcal{P}^\dagger\psi(\vec{r},t)\mathcal{P} = \xi_P S_P\psi(-\vec{r},t) \tag{22g}$$

$$\mathcal{T}^\dagger\psi(\vec{r},t)\mathcal{T} = \xi_T S_T\psi(\vec{r},-t) \tag{22h}$$

$$\mathcal{C}^\dagger\psi(\vec{r},t)\mathcal{C} = \psi_C(\vec{r},t) \tag{22i}$$

Where η_P, η_T, η_C, ξ_P, ξ_T and ξ_C are pure phases, i.e. they can be written as $\eta_P = \exp(i\alpha_P)$ and $\xi_P = \exp(i\beta_P)$, and similarly for the others. The charged conjugated spinor field $\psi_C(\vec{r},t)$ is defined as $\psi_C(\vec{r},t) = \xi_C S_C \overline{\psi}^T(\vec{r},t)$. Also the definitions of the matrices S_p, S_T

and S_C are:

$$S_T \gamma^\mu S_T^\dagger = \gamma_\mu^* \tag{23a}$$

$$S_P^{-1} \gamma^\mu S_P = \gamma_\mu \tag{23b}$$

$$S_C \gamma^\mu S_C^{-1} = -(\gamma^\mu)^T \tag{23c}$$

These definition have not been written in any particular representation for the gamma matrices. For further details on how the $\mathcal{P}$, $\mathcal{T}$ and $\mathcal{C}$ are defined, see [8].

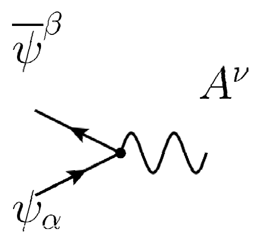
2 Electric dipole moments in the Standard Model

We saw that the Hamiltonian for a classical particle, in an electric and magnetic field is of the form:

$$H = -(d\vec{E} + \mu\vec{B}) \cdot \frac{\vec{S}}{S} \tag{24}$$

We also saw, equations (11) and (14), that the d term, namely $\vec{E} \cdot \vec{S}/S$, is P-odd and T-odd. As we know, classical Hamiltonians are invariant under space and time inversions. Therefore, a non-zero parameter d may exist if both P and T are broken. If we assume CPT invariance of physics, the P non invariance of H would imply a CT non invariance, and the T non invariance would imply a CP non invariance. This point is going to be crucial, since it tells us that electric dipole moments get contributions from CP-odd terms in the Hamiltonian, or Lagrangian, of the particle. So, in our attempt to generalise the electric dipole moments of classical particles to the relativistic case, we will look for CP-odd operators. As a first generalisation, we can write down the following Lagrangian [9]:

$$H_{CP-odd} = -d\vec{E} \cdot \frac{\vec{S}}{S} \qquad \rightarrow \qquad \mathcal{L} = -\frac{id}{2}\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu} \tag{25}$$



$$= \quad -idk_{\mu}\left(\sigma^{\mu\nu}\gamma_5\right)_{\alpha}{}^{\beta}$$

Figure 2: fermion-fermion-photon interaction led by (27). This interaction gives the fermion an electric dipole moment whose value is d

Which can be seen to be a good generalisation through its transformation under P, T and C transformations. This can also be seen by looking at the non-relativistic limit [7]. In order to include possible EDMs into the standard model, we need to include this interaction term in the Lagrangian of standard model, which is as said an effective term. By doing so, the Lagrangian will be:

$$\mathcal{L}_{Effective\ Theory} = \mathcal{L}_{SM} + \mathcal{L}_{EDM} \tag{26}$$

Where we have defined:

$$\mathcal{L}_{EDM} = -\frac{id}{2}\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu} \tag{27}$$

Here $F_{\mu\nu}$ is the electromagnetic field strength, $\sigma^{\mu\nu}$ has been defined in section 1.4 and the matrix γ_5 is defined as $\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3$. This new term in the Lagrangian brings a new fermion-fermion-photon interaction, whose Feynman rule is on figure 2.

It is interesting to notice that such an interaction already exists inside the standard model, from the term $e\overline{\psi}\gamma^{\mu}\psi A_{\mu}$ in QED. We may be tempted to say that $\mathcal{L}_{EDM}$ does not bring any new interaction. But in fact, if we perform a dimensional analysis in d dimensions, with $[\psi] = (d-1)/2$ and $[F_{\mu\nu}] = d/2$, we obtain $[d] = 1 - d/2$, which is relevant for $d \geq 3$, which makes the theory considered above effective. This is a hint for the high energy processes contributing to the electric dipole moments of the fermions. We can consider high energy processes coming from the standard model, but also coming from theories beyond the standard model, which will taken to be the minimal supersymmetric standard model. Figure 3 illustrates how the standard model and the

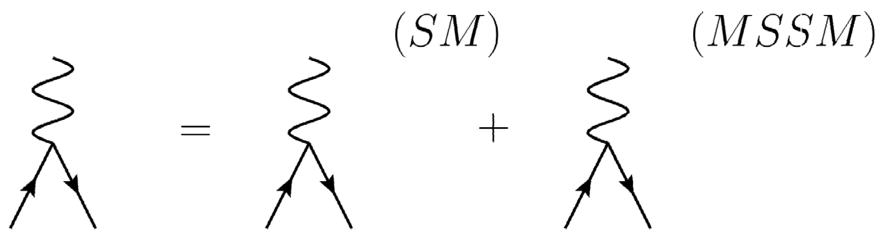


Figure 3: Contributions of the SM and MSSM to the EDM

minimal supersymmetric standard model give contributions the the EDM.

2.1 θ-term

The θ-term is a purely topological term, in the sense that it arises only when we consider the boundary topology of the gauge group of the theory. It is not an obvious term, but it turns out that it has deep implications on the vacuum of the theory itself, and that it contributes to the electric dipole moments. Here we are therefore interested in the possible vacua of the gauge group of the theory, whatever it is. So we will make the following assumption:

$$\mathcal{Z} = \int \mathcal{D}A_\mu e^{iS[A_\mu]} \approx \sum_{vac} e^{iS[A_{\mu\, vac}]} \tag{28}$$

Where $\sum_{vac}$ represents the fact that the sum is to be done over all the different vacua of the theory, and A_μ is a gauge field. This assumption hides another one, that is that for (28) to be true, the action has to be finite. Therefore, studying the vacua of the theory is the same as studying the finiteness of the action. In order to understand the effect of the finiteness of the action, let us look at the gauge theory [10]:

$$S[A] = \frac{1}{4} \int d^d x \epsilon^E_{ijkl} F^a_{ij} F^a_{kl} \tag{29}$$

where the sum over repeated indices is implicit, and F^a_{ij} is the field strength, ϵ^E is the totally antisymmetric tensor in euclidean space, i, j, k and l are the space indices and a is the gauge group index. We can regard this as the action of a gauge theory either in a d-dimensional euclidean space, with space indices running from 1 to d, or in a $d+1$-

dimensional spacetime, with spacetime indices running from 0 to d in the temporal gauge $A_0^a = 0$. From now on we will consider the action in a d-dimensional euclidean space. For this action to be finite we must require at least:

$$F_{ij}^a \underset{|\vec{x}|\to\infty}{\sim} |\vec{x}|^{-\frac{d+1}{2}} \tag{30}$$

where $\vec{x}$ is the vector position in the d-dimensional space. That implies the condition

$$A_i^a \underset{|\vec{x}|\to\infty}{\sim} |\vec{x}|^{-\frac{d-1}{2}} \tag{31}$$

By using the gauge invariance of the theory, written in (32):

$$it^a A_i^a(\vec{x}) \quad \to \quad it^a A_i^a(\vec{x}) + g^{-1}(\vec{x})\partial_i g(\vec{x}) \tag{32}$$

the condition (31) is equivalent to say that the gauge field approaches a pure gauge at infinity. In other words:

$$it^a A_i^a(\vec{x}) \quad \xrightarrow[|\vec{x}|\to\infty]{} \quad g^{-1}(\vec{x})\partial_i g(\vec{x}) \tag{33}$$

Where $g(\vec{x})$ is a direction dependent element of the gauge group G, and the gauge coupling constant has been included into the generators t^a. As we can see in (33), at inifnity the gauge field is unchanged if we multiply $g(\vec{x})$ by any fixed element g_0 of G. We can use this freedom and replace $g(\vec{x})$ by $\widetilde{g}(\vec{x}) = g_0 * g(\vec{x})$ in such way that $\widetilde{g}(\vec{x}_u) = \mathbf{1}$, for any one direction $\vec{x}_u$. In addition, by virtue of group theory, the relation (33) remains the same if we muliply $\vec{x}$ by a constant α. We can use this other freedom to transform $\vec{x}$ as $\vec{x}_0 = \alpha\vec{x}$, in such way that $|\vec{x}_0| = 1$. The relation (33) can equivalently be written as:

$$it^a A_i^a(\vec{x}) \quad \xrightarrow[|\vec{x}|\to\infty]{} \quad \widetilde{g}^{-1}(\vec{x}_0)\partial_i\widetilde{g}(\vec{x}_0) \tag{34}$$

which clearly shows that, at infinity, the gauge field is a mapping between the unit sphere $S_{d-1} = \{\vec{x} \mid |\vec{x}| = 1\}$ and the gauge group manifold $G = \{g(\vec{x})\}$. Group theory tells us that there are different classes of such mappings, and that equivalent mappings belong to the same class.

As a simple illustration, we can take the mapping $S_1 \to U(1)$, which can be characterised by the integer ν called the winding number. For instance we can consider

$$g_\nu(\theta) = e^{i\nu\theta} \tag{35}$$

where the parameter θ is the polar coordinate on S_1. In order to understand the meaning of the winding number, one can simply make $\theta \to \theta + 2\pi$, which obviously transforms any point on S_1 into itself. However, at the same time, $g_\nu(\theta)$ might have to go around S_1 more than once to get back where it originally was. The following relation illustrates this fact:

$$g_\nu(\theta + 2\pi) = g_\nu(\theta)e^{i2\pi\nu} \tag{36}$$

In the case where $\nu = \pm 1$, the mapping is an isomorphism. In order for two mappings to be equivalent, they must have the same winding number. The winding number therefore characterises the class of the mapping. Actually, the winding number is a general feature of this kind of mappings which can be more complicated.

Another useful quantity is the Cartan-Maurer integral invariant. It is very useful to characterise the topology of different manifolds because it is topologically invariant. Using this quantity, one can show [10]:

$$\int d^4x \epsilon^E_{ijkl} F^a_{ij} F^a_{kl} = 64\pi^2\nu \tag{37}$$

And:

$$S[A_\mu] = 8\pi^2|\nu| \tag{38}$$

We would now like to get a Minkowskian action. In order for that, we recall that in the Euclidean space $x_i = (x_1, x_2, x_3, x_4)$ while in Minkowski spacetime with $x_4 = ix_0$, $x_\mu = (x_0, x_1, x_2, x_3)$. Also, $\epsilon^{E1234} = 1$ becomes in Minkowski spacetime $\epsilon^{1230} = -1$. And finally, by the definition of F^a_{ij} we notice that by going to Minkowski spacetime, we must have $F^a_{34} = -iF^a_{30}$. Therefore $\epsilon^E_{ijkl}F^a_{ij}F^a_{kl} = i\epsilon^{\mu\nu\sigma\rho}F^a_{\mu\nu}F^a_{\sigma\rho}$, so that (37) becomes:

$$\nu = -\frac{1}{64\pi^2}\int d^4x\epsilon^{\mu\nu\sigma\rho}F^a_{\mu\nu}F^a_{\sigma\rho} \tag{39}$$

We previously saw that different winding numbers can be used to characterise different field configurations. It might therefore be important to include this parameter into the path integral formulation of operators. The expectation value of the operator $\mathcal{O}$ is

$$\langle\mathcal{O}\rangle_\Omega = \frac{\sum_\nu f(\nu)\int_\nu \mathcal{D}\Phi\mathcal{O}[\Phi]exp(iS_\Omega[\Phi])}{\sum_\nu f(\nu)\int_\nu \mathcal{D}\Phi exp(iS_\Omega[\Phi])} \tag{40}$$

Where we consider field configurations only over a large spacetime volume Ω, S_Ω is the integral of the Lagragian over this volume and Φ is a generic field of the theory. The subscript ν on the integral means that we are to integrate only over the fields with winding number ν. In order to determine the form of $f(\nu)$, one uses the following argument. Suppose, we split the volume Ω into two volumes Ω_1 and Ω_2 such that $\mathcal{O}$ is only in Ω_1. Field configurations in Ω_1 have a winding number ν_1, and similarly in Ω_2. According to the definition of the winding number in complex analysis, one has $\nu_1 + \nu_2 = \nu$. We can rewrite (40) as:

$$\langle\mathcal{O}\rangle_\Omega = \frac{\sum_{\nu_1,\nu_2} f(\nu_1+\nu_2)\int_{\nu_1} \mathcal{D}\Phi\mathcal{O}[\Phi]exp(iS_{\Omega_1}[\Phi])\int_{\nu_2}\mathcal{D}\Phi exp(iS_{\Omega_2}[\Phi])}{\sum_{\nu_1,\nu_2} f(\nu_1+\nu_2)\int_{\nu_1} \mathcal{D}\Phi exp(iS_{\Omega_1}[\Phi])\int_{\nu_2}\mathcal{D}\Phi exp(iS_{\Omega_2}[\Phi])} \tag{41}$$

But in fact, physically this is as if we omitted Ω_2, since the operator $\mathcal{O}$ is not in it. Therefore $\langle\mathcal{O}\rangle$ should also be written as:

$$\langle\mathcal{O}\rangle_\Omega = \frac{\sum_{\nu_1} f(\nu_1)\int_{\nu_1} \mathcal{D}\Phi\mathcal{O}[\Phi]exp(iS_{\Omega_1}[\Phi])}{\sum_{\nu_1} f(\nu_1)\int_{\nu_1} \mathcal{D}\Phi exp(iS_{\Omega_1}[\Phi])} \tag{42}$$

In order for (41) and (42) to be always true, we must have the following relations for $f(\nu)$:

$$f(\nu_1+\nu_2) = f(\nu_1)f(\nu_2) \quad \Leftrightarrow \quad f(\nu) = e^{i\theta\nu} \tag{43}$$

Combining (39), (40) and (43), one obtains that we must include in the lagrangian the term:

$$\mathcal{L}_\theta = -\frac{\theta}{64\pi^2}\epsilon^{\mu\nu\sigma\rho}F^a_{\mu\nu}F^a_{\sigma\rho} = -\frac{\theta}{32\pi^2}F^a_{\mu\nu}\widetilde{F}^{a\mu\nu} \tag{44}$$

$\widetilde{F}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\sigma\rho}F^a_{\sigma\rho}$ is the dual field of $F^a_{\mu\nu}$. This term is the so-called θ-term. In fact, another form for this term is possible, depending on what the definition of the gauge field is. Here, we have chosen to include the coupling constant into the generators, but if we choose not to do so we get:

$$\mathcal{L}_\theta = -\frac{g^2\theta}{32\pi^2}F^a_{\mu\nu}\widetilde{F}^{a\mu\nu} \tag{45}$$

This term has been of particular interests for many reasons. Firstly, as we have just seen, the θ-term is purely topological and contains a lot of informations about the vacua of quantum fields theories where it appears. In QCD for instance, it is a the center of the studies of the instantons and the tunneling phenomena. This is a very interesting aspect of physics, but I will not give more details on the subject, further reading can be found at [11]. This term has another particularity, which is that it can be written as a total derivative. If we write $G_{\mu\nu} = G^a_{\mu\nu}t^a$, where t^a are the generators of the gauge group satisfying the normalisation condition $tr(t^a t^b) = \delta^{ab}/2$, we have:

$$\mathcal{L}_\theta = -\frac{g^2\theta}{32\pi^2}\epsilon_{\mu\nu\sigma\rho}tr[G_{\mu\nu}G_{\sigma\rho}] \tag{46}$$

with $G_{\mu\nu} = \partial_\mu A_\nu - \partial\mu A_\nu + [A_\mu, A_\nu]$. Then we can write:

$$\mathcal{L}_\theta = \partial_\mu\left(-\frac{g^2\theta}{8\pi^2}J^\mu\right) \tag{47}$$

Where we have defined the current:

$$J^\mu = \epsilon^{\mu\nu\sigma\rho}tr[A_\nu(\partial_\sigma A_\rho + \frac{1}{3}[A_\sigma, A_\rho])] \tag{48}$$

This current is called the Chern-Simons current. The non triviality of the vacuum is such that this term does not vanish as we integrate it over the spacetime. It therefore has non trivial contributions to the theory. Finally, we notice that it is a CP violating term. Indeed, according to (22) we can show that:

$$\mathcal{P}^{\dagger}\mathcal{C}^{\dagger}\mathcal{L}_{\theta}\mathcal{C}\mathcal{P} = -\mathcal{L}_{\theta} \tag{49}$$

As the EDMs get contributions from CP violating terms, the equation (49) tells that the θ-term might have something to do with the electric dipole moment. This term has to be taken into account in our study of the electric dipole moments.

2.2 Chiral transformation

At first, we want to understand the effect of the θ-term on the Lagrangian for fermions. Let us look at the chiral transformation:

$$\psi(x) \quad \rightarrow \quad e^{i\alpha(x)\gamma^5}\psi(x) \tag{50}$$

We can show [10] that a transformation of the form:

$$\psi(x) \quad \rightarrow \quad U(x)\psi(x) \tag{51a}$$

$$U(x) = e^{i\gamma^5\alpha(x)t} \tag{51b}$$

With t a general Hermitian matrix and $\alpha(x)$ is a general real parameter, implies a change in the fermion measure as follows:

$$\mathcal{D}\psi\mathcal{D}\overline{\psi} \quad \rightarrow \quad \exp\left(i\int d^4x\,\alpha(x)A(x)\right)\mathcal{D}\psi\mathcal{D}\overline{\psi} \tag{52a}$$

$$A(x) = -\frac{1}{16\pi^2}\epsilon^{\mu\nu\sigma\rho}F^a_{\mu\nu}(x)F^b_{\sigma\rho}(x)tr[t^a t^b t] \tag{52b}$$

Where the trace is done over all fermion species and t_a and t_b are the generators of the gauge group. In the case of (50), we have $t = \mathbf{1}$. Therefore the transformation (50) implies:

$$\mathcal{D}\psi\mathcal{D}\overline{\psi} \quad \rightarrow \quad \exp\left(-\frac{i}{32\pi^2}\epsilon^{\mu\nu\sigma\rho}\int d^4x F^a_{\mu\nu}F^b_{\sigma\rho}\delta^{ab}\alpha(x)\right)\mathcal{D}\psi\mathcal{D}\overline{\psi} \tag{53}$$

By choosing the generators to satisfy the normalisation condition $tr(t^at^b) = \delta^{ab}/2$, which is relevant for the analysis of quarks as the gauge group is $SU(3)$. The transformation (50) therefore brings in the Lagrangian a new term:

$$\mathcal{L} \quad \rightarrow \quad \mathcal{L} - \frac{1}{32\pi^2}\epsilon^{\mu\nu\sigma\rho}F^a_{\mu\nu}(x)F^a_{\sigma\rho}(x)\alpha(x) \tag{54}$$

It is now obvious that if we turn to the case of QCD, writing explicitly the strong coupling constant and making the phase $\alpha(x) = k \times \theta/2$ gobal, we obtain:

$$\mathcal{L} \quad \rightarrow \quad \mathcal{L}' \quad = \quad \mathcal{L} - k\frac{g^2\theta}{64\pi^2}\epsilon^{\mu\nu\sigma\rho}F^a_{\mu\nu}(x)F^a_{\sigma\rho}(x) \tag{55a}$$

$$= \quad \mathcal{L} - k\mathcal{L}_\theta \tag{55b}$$

k is nothing but a constant of proportionality. Here we have illustrated one of the effects of the chiral transformations (50), that is that the θ-term is equivalent to a given chiral transformation. Now, we will illustrate another implication of chiral transformations. Consider the Lagrangian for massless fermions:

$$\mathcal{L} \quad = \quad i\overline{\psi}\gamma^\mu D_\mu\psi \tag{56a}$$

$$= \quad i\overline{\psi}\gamma^\mu\partial_\mu\psi - g\overline{\psi}\gamma^\mu\psi A_\mu \tag{56b}$$

Whith possibly indices on fermions labelling different fermion species. This Lagrangian has the following symmetry leading to the corresponding conserved Noether's current:

$$\psi(x) \quad \rightarrow \quad e^{i\eta_a t_a}\psi(x) \tag{57a}$$

$$j_a^\mu(x) = \overline{\psi}(x)\gamma^\mu t_a \psi(x) \tag{57b}$$

Where η_a are real constant parameters, the currents $j_a^\mu(x)$ are called the vector currents. In fact, the Lagrangian (56) also has the following symmetry:

$$\psi(x) \quad \rightarrow \quad e^{i\gamma^5 \xi_a t_a}\psi(x) \tag{58}$$

And more specifically:

$$\psi(x) \quad \rightarrow \quad e^{i\alpha\gamma^5}\psi(x) \tag{59}$$

Here α is a real constant parameter. This is a symmetry according to the following properties of γ^5:

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{60a}$$

$$(\gamma^5)^\dagger = \gamma^5 \tag{60b}$$

$$\left\{\gamma^5, \gamma^\mu\right\} = 0 \tag{60c}$$

This symmetry leads to the Noether's current:

$$\delta\mathcal{L} = \frac{\delta\mathcal{L}}{\delta\psi}\delta\psi + \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi)}\delta(\partial_\mu\psi) \tag{61a}$$

$$= -\alpha\partial_\mu\left(j^{5\mu}\right) \tag{61b}$$

$$= 0 \tag{61c}$$

$$j^{5\mu} = \overline{\psi}\gamma^\mu\gamma^5\psi \tag{61d}$$

The conserved current $j^{5\mu}$ is called the axial current. The symmetry (58) is called a chiral symmetry because it is a true symmetry only if the fermions are massless. Indeed, we may want to include massive fermions to the Lagrangian (56) and see the implications of (58):

$$\mathcal{L} = i\overline{\psi}\gamma^\mu D_\mu\psi - m\overline{\psi}\psi \tag{62}$$

The equation for the dynamical fields ψ and $\overline{\psi}$ are respectively:

$$\begin{aligned} \partial_\mu\overline{\psi}\gamma^\mu &= ig\overline{\psi}\gamma^\mu A_\mu + im\overline{\psi} && \text{(63a)} \\ \gamma^\mu\partial_\mu\psi &= -ig\gamma^\mu\psi A_\mu - im\psi && \text{(63b)} \end{aligned}$$

Thanks to these equations, we can compute the derivative of the axial current:

$$\begin{aligned} \delta\mathcal{L} &= \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi)}\delta\psi\right) && \text{(64a)} \\ &= \alpha\partial_\mu\left(i\overline{\psi}\gamma^\mu\gamma^5\psi\right) && \text{(64b)} \\ &= -2im\alpha\overline{\psi}\gamma^5\psi && \text{(64c)} \end{aligned}$$

We clearly see that the axial currents $j^{5\mu}$ are conserved when the fermions are massless, i.e. when the fermions are chiral fermions. Hence the transformation (58) is a chiral symmetry, it is an approximate symmetry. Also, the last relation shows that the transformation (59) brings in the Lagrangian (62) an additional term. Thus, under (59) we have:

$$\begin{aligned} \mathcal{L} \quad \rightarrow \quad \mathcal{L}' &= \mathcal{L} + \delta\mathcal{L} && \text{(65a)} \\ &= \mathcal{L} + i\overline{\psi}\gamma^5\mathcal{M}\psi && \text{(65b)} \\ \mathcal{M} &= k \times m\alpha\mathbf{1} && \text{(65c)} \end{aligned}$$

Where k is a real constant of proportionality. In the most general case of (58), we have $\mathcal{M} = km\xi_a t_a$. We have here illustrated another implication of the chiral symmetry. Now we can join the two implications explained above, and say that, the θ-term is equivalent to a chiral transformation which is itself equivalent to adding a term proportional to $\overline{\psi}\gamma^5\psi$ in the Lagrangian. We can therefore remove the θ-term (44) from the Lagrangian and introduce an equivalent dependance on the θ parameter in the term (65).

2.3 EDM of the neutron in QCD

In this section we will review the calculation of the electric dipole moment of the neutron in QCD, mainly following [12]. The Lagrangian of QCD with the θ-term is:

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \overline{\psi}_i(i\gamma^\mu D_\mu - m_i)\psi_i - \frac{g^2\theta}{64\pi^2}\epsilon^{\mu\nu\sigma\rho}G^a_{\mu\nu}G^a_{\sigma\rho} \tag{66}$$

Which can be equivalently written as:

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \overline{\psi}_i(i\gamma^\mu D_\mu - m_i)\psi_i + i\overline{\psi}_i\gamma^5\mathcal{M}\psi_i \tag{67}$$

$G^a_{\mu\nu}$ is the gluon field strength with $a = 1, ..., 8$ and $i = 1, ..., 6$ labels the quark flavors up, down, charm, strange, top, bottom. Also, as this redefinition is due to the chiral symmetry, we must suppose that m is small. That is why we only consider the three lightest quark flavors up, down and strange. We can show [12] that, to first order in θ, the matrix $\mathcal{M}$ is:

$$\mathcal{M} = -i\theta\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\mathbf{1} \tag{68}$$

Thus, the mass Lagrangian is of the form:

$$\begin{aligned}
\mathcal{L}_m &= \sum_{i=u,d,s}(m_i\overline{\psi}_i\psi_i) - i\theta\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\sum_{i=u,d,s}(\overline{\psi}_i\gamma^5\psi_i) && \text{(69a)}\\
&= \sum_{i=u,d,s}(m_i\overline{\psi}_i\psi_i) - \Delta\mathcal{L} && \text{(69b)}
\end{aligned}$$

As $m_s \gg m_d, m_u$, one can make the approximation:

$$\Delta\mathcal{L} = i\theta \frac{m_u m_d}{m_u + m_d} \sum_{i=u,d,s} (\overline{\psi}_i \gamma^5 \psi_i) \tag{70}$$

Using (22) we see that this is a CP violating operator, it therefore has implications in the electric dipole moment. But that is not surprising since this operator is equivalent to the θ-term which was already known to have implications in the EDM. We know that the interaction leading to the EDM is an interaction between a hadron, the neutron, and the electromagnetic field, as in figure 2. Also, we must include CP violating operators. In the model described above, the only term in the Lagrangian containing CP-violating operators is $\Delta\mathcal{L}$, which can be treated as an interaction term due to the fact that the value of θ is small. The following relations are respectively the amplitude of the EDM interaction from the point of view of the hadronic electromagnetic interaction and the amplitude from the point of view of the interaction the first must contribute to.

$$\mathcal{A}_\mu^{CP} = \langle n(p_{out})| T J_\mu(0) \exp\left(i \int d^4x\, \Delta\mathcal{L}\right) |n(p_{in})\rangle \tag{71a}$$

$$\mathcal{A}_\mu^{EDM} = -d_n \overline{a}_{out} \sigma_{\mu\nu} \gamma^5 k^\nu a_{in} \tag{71b}$$

$n(p_{in}) = n_{in}$, $n(p_{out}) = n_{out}$, $\overline{a}_{in}$ and a_{out} are respectively the in- and out-states, and the in- and out-amplitudes of the neutron, and T is the time ordering operator, and $k^\nu = (p_{in} - p_{out})^\nu$. Both above relations must agree, so:

$$\langle n_{out}| T J_\mu(0) \exp\left(i \int d^4x\, \Delta\mathcal{L}\right) |n_{in}\rangle = -d_n \overline{a}_{out} \sigma_{\mu\nu} \gamma^5 k^\nu a_{in} \tag{72}$$

So, to first order in θ, one has:

$$\theta \frac{m_u m_d}{m_u + m_d} T \langle n_{out}| J_\mu(0) \int d^4x \sum_{i=u,d,s} \left(\overline{\psi}_i(x) \gamma^5 \psi_i(x)\right) |n_{in}\rangle = d_n \overline{a}_{out} \sigma_{\mu\nu} \gamma^5 k^\nu a_{in} \tag{73}$$

The left hand side term can be calculated by inserting a complete set of states $\{|X\rangle\}$, between the two operators J_μ and $\sum_{i=u,d,s} \overline{\psi}_i \gamma^5 \psi_i$, we will have:

$$\mathcal{A}_\mu^{CP} = -\theta \frac{m_u m_d}{m_u + m_d} \int d^4x \sum_X \sum_{i=u,d,s} \\ \langle n_{out}| \Big(J_\mu(0)|X\rangle\langle X|\, \overline{\psi}_i(x)\gamma^5\psi_i(x) \; + \overline{\psi}_i(x)\gamma^5\psi_i(x)|X\rangle\langle X|\, J_\mu(0) \Big) |n_{in}\rangle \tag{74}$$

The set of complete states $\{|X\rangle\}$ contains multiparticle states of stable hadrons entering the one-loop diagrams, $\{|X\rangle\} = |N\rangle, |N\pi\rangle, |N\pi\pi\rangle, ...$ Where N is a nucleon and π is a pion. One can argue that, in the soft-pion limit where $m_\pi \rightarrow 0$, the most singular contribution, of order $O(m_\pi^2 ln(m_\pi^2))$, to the electric dipole moment comes from the term with $|X\rangle = |N\pi\rangle$. So the above relation reduces to:

$$\mathcal{A}_\mu^{CP} = -\theta \frac{m_u m_d}{m_u + m_d} \int d^4x \sum_{i=u,d,s} \\ \Big(\langle n_{out}|J_\mu(0)|N\pi_{soft}\rangle\langle N\pi_{soft}|\, \overline{\psi}_i(x)\gamma^5\psi_i(x)\, |n_{in}\rangle \\ + \langle n_{out}|\overline{\psi}_i(x)\gamma^5\psi_i(x)|N\pi_{soft}\rangle\langle N\pi_{soft}|\, J_\mu(0)\, |n_{in}\rangle \Big) \tag{75}$$

The term $\langle n|J_\mu(0)|N\pi_{soft}\rangle$, where $|n\rangle$ means either $|n_{in}\rangle$ or $|n_{out}\rangle$, can be calculated using current algebra [13]. The computation shows that the terms where the photon couples to the nucleon only give a contribution of order $O(m_\pi^2)$. Also, at low energy the photon does not couple to π^0, because at low energy this pion is seen neutral by the photon, while at high energies the pion will be seen as a set of quarks, which have an electromagnetic charge. The nucleon being composed of neutrons and protons and given the fact that the photon does not couple to the neutron at low energies, for the same reasons as for the neutral pion, the calculation of this term therefore reduces to the calculation of $\langle n|J_\mu|p\pi^\pm\rangle$. Finally, by charge conservation, the state $|p\pi^+\rangle$ must be ruled out. Therefore, only the term $\langle n|J_\mu|p\pi^-\rangle$ gives the contribution to the electric dipole moment to order $O(m_\pi^2 ln(m_\pi^2))$. That is the reason why the interactions giving the EDMs are as in figure 4.

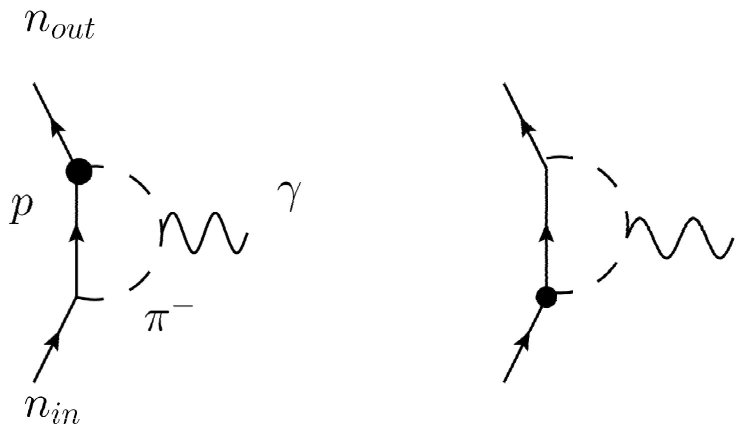


Figure 4: Diagrams giving contribution to order $O(m_\pi^2 ln(m_\pi^2))$ to the EDM of the neutron in QCD. The black dot vertex represents the CP violating interaction.

On the other hand, the term $\langle N\pi_{soft}|\overline{\psi}_i(x)\gamma^5\psi(x)|n\rangle$ is computed via the following Lagrangian:

$$\mathcal{L}_{\pi NN} = \phi_\pi^a \overline{\psi}_N \tau^a (ig_{\pi NN}\gamma^5 + \overline{g}_{\pi NN})\psi_N \tag{76}$$

The three pions are described by the scalar fields ϕ_π^a, the nucleon is described by the fermion field ψ_N and τ^a are the Pauli matrices. It truns out [12] that the CP violating coupling $\overline{g}_{\pi NN}$, between a pion and two nucleons, is given by:

$$\overline{g}_{\pi NN} = i\theta \frac{m_u m_d}{m_u + m_d} \sum_{i=u,d,s} \langle \pi^a N_{out}|\overline{\psi}_i \gamma^5 \psi_i|N_{in}\rangle \tag{77}$$

By invoking the same arguments as in the case of $\langle n|J_\mu(0)|N\pi_{soft}\rangle$, we realise that the contribution from $\overline{g}_{\pi NN}$ to order $O(m_\pi^2 ln(m_\pi^2))$ comes from:

$$\overline{g}_{\pi NN} = i\theta \frac{m_u m_d}{m_u + m_d} \sum_{i=u,d,s} \langle \pi^- p|\overline{\psi}_i \gamma^5 \psi_i|n\rangle \tag{78}$$

In addition, the CP non-violating interaction between two nucleons and a pion, is given by $g_{\pi NN}$ in (76). And again, can be precisely reduced to the proton-nucleon-pion interaction.

Finally, the interaction between two identical scalars and a photon is given by the Feynamn rule:

$$\langle \pi^-(p+k)|J_\mu|\pi^-(p)\rangle = -(2p+k)_\mu \tag{79}$$

So, the computation of d_n reduces to the simple calculation of the one-loop diagrams of figure 4, with the CP-violating and CP-non violating vertices respectively given by $\overline{g}_{\pi NN}$ and $g_{\pi NN}$ and the photon interaction with the pion given by (79). The result is:

$$\frac{d_n}{e} = \frac{\overline{g}_{\pi NN} g_{\pi NN}}{4\pi^2 M_N} \ln\left(\frac{M_N}{m_\pi}\right) \tag{80}$$

A numerical application, with the experimental value of $g_{\pi NN}$, $|\overline{g}_{\pi NN}| = 0.038|\theta|$ from (77) and of $\ln(M_N/m_\pi) = 1.9$, leads to:

$$\frac{d_n}{e} = 5.2 \cdot 10^{-18}\theta \cdot m \tag{81}$$

The value of θ is known to be very small, $\theta \approx 10^{-9}$, so the value of the EDM of the neutron is predicted to be $d_n \approx 10^{-27} e \cdot m$, which is well below the value the experimentalists measure.

3 Electric dipole moments in the MSSM

3.1 General case

3.1.1 Set up

The diagrams [14] entering the MSSM part in figure 3 are displayed on figure 5. One reason why the mass insertions are needed, is because the interaction of figure 2 changes the chirality of fermions. Indeed, if we write the Dirac fermion into right- and left-handed components:

$$\psi = \psi_R + \psi_L \tag{82}$$

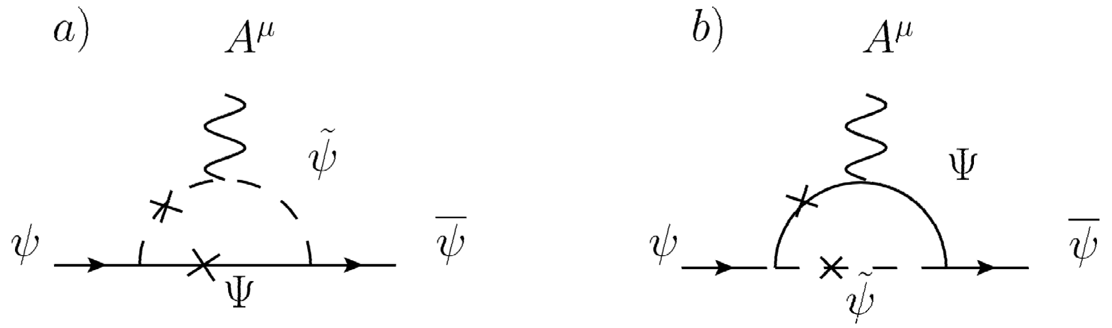


Figure 5: One loop diagrams leading to the electric dipole moment of a fermion in the MSSM. The cross symbol represents a mass insertion. The field ψ is the fermion field, the field $\tilde{\psi}$ is the sfermion field. In the diagram a) the field Ψ can either be a gaugino $\tilde{g}$, a neutralino χ^0 or a chargino χ^+, while in the diagram b) the field Ψ can only be a chargino.

with $\psi_R = -\gamma_5\psi_R$ and $\psi_L = \gamma_5\psi_L$, then:

$$\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu} = \overline{\psi_R}\sigma^{\mu\nu}\psi_L F_{\mu\nu} - \overline{\psi_L}\sigma^{\mu\nu}\psi_R F_{\mu\nu} \tag{83}$$

Which is an interaction changing an incoming right-handed fermion into a outgoing left-handed fermion, and changing an incoming left-handed fermion into an outgoing right-handed fermion. The simplest terms in the Lagrangian which mixes both chiralities are the mass terms coming from the superpotential and the soft SUSY breaking terms, hence the mass insertions. Another reason why the mass insertions are needed is due to the fact that the supersymmetric particles are experimentally not observed. To explain this "unobservation" of supersymmetric particles, we suppose that supersymmetric particle masses are much higher than the masses of ordinary particles. This fact is mathematically implemented by taking the supersymmetric particle momenta to be large. Thus, the ordinary particle masses can be treated as interaction terms.

We will here concentrate on the calculation of the EDM via the diagram a) in figure 5. We still have to keep in mind that this calculation is not complete as many other diagrams contribute, e.g. the diagram b) on the same figure. As explained above, the diagrams in figure 5 have high energy internal particles, so they contribute to the effective operator (27). In order to obtain an expression for d, we will compute the Feynman rule led by the diagram a) of figure 5, then by comparison with the Feynman rule of figure 2, we will be able to identify what goes into d.

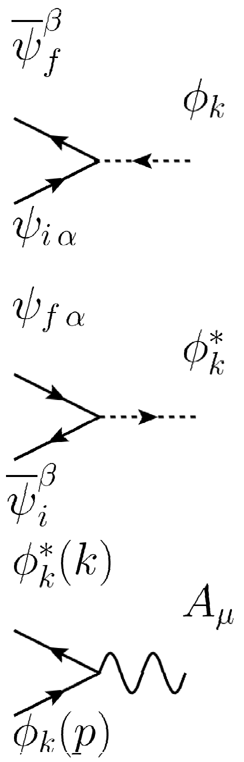
We want the computation to be as general as possible, so that the EDM of an actual particle will be a particular case of this computation. In order for that, we take the general case of a light external fermion labelled ψ_f, a heavy scalar ϕ_k and a heavy fermion ψ_i. In the case of the MSSM, the heavy fermion can be a gaugino, neutralino or a chargino. The light fermion is the particle of which we want to compute the EDM and the heavy scalar will be its associated sfermion. Thus the subscript f, k and i on the fields label different particles. The part of the MSSM Lagrangian expressing the interactions between these three fields has the general form [14]:

$$-\mathcal{L}_{int} = \sum_{i,k} \overline{\psi_f} \left(K_{ik} \frac{1-\gamma_5}{2} + L_{ik} \frac{1+\gamma_5}{2} \right) \psi_i \phi_k + H.c \tag{84}$$

Where $H.c$ stands for Hermitian conjugated. The heavy fermion and scalar have charges respectively Q_i and Q_k, and masses m_i and m_k. The Feynman rules associated with the Lagrangian (84) are shown in a) and b) of figure 6, the Feynman rule c) on the same figure is the one of the interaction between a scalar and the photon. A complete set of Feynman rules for the MSSM can be found at [15].

Before starting the computation, we must make a little precision. We did not draw the arrows on the internal scalar and fermion lines on figure 5 for simplicity, but as these fields are complex, we should have drawn them. We can ask the question of how many configurations of the vertices in figure 6 lead to the same form of diagram as the left diagram in figure 5. The first obvious configuration is figure 7. Actually, we can think of another one, which is the same with the two vertices on the fermion line exchanged. However, as we must keep the heavy fermion ψ_i as an internal line, we realise that this other configuration is exactly the same as the previous one. There is therefore only one possible configuration, shown in figure 8. Now we can compute compute $(\mathcal{D}'^{\mu})_{\alpha}{}^{\beta}$, in the notation explained in 8. We have:

$$(\mathcal{D}'^{\mu})_{\alpha}{}^{\beta} = -eQ_k m_i m_k^2 \int \frac{d^4q}{(2\pi)^4} \frac{(2(p-q)-k)^{\mu}\Gamma_{\alpha}{}^{\beta}}{(q^2-m_i^2)^2((p-q)^2-m_k^2)^2((p-q-k)^2-m_k^2)} \tag{85}$$



a) $= -i\left(K_{ik}\frac{1-\gamma_5}{2} + L_{ik}\frac{1+\gamma_5}{2}\right)_{\alpha}{}^{\beta}$

b) $= -i\left(K^*_{ik}\frac{1+\gamma_5}{2} + L^*_{ik}\frac{1-\gamma_5}{2}\right)_{\alpha}{}^{\beta}$

c) $= -ieQ(p+k)^{\mu}$

Figure 6: Feynman rules entering the diagrams of figure 5. The Feynman rules a) and b) are the ones associated with the interaction Lagrangian (84), while the Feynman rule c) is the one between a scalar and a photon. In principle, the diagram c) can change the type of particle, change of flavour for example. So there can be an extra Kronecker's δ^{ij} to take this change into account. But we will stay in the simpler case where we omit these possible changes.

Also, we should really have summed over all indices i and k because they label different particles and therefore the diagrams involving these particles should be added up. But this sum will be omitted now and we keep it in mind until the last step, where we will put it back. We have defined:

$$\Gamma = \left[K_{ik}\frac{1-\gamma^5}{2} + L_{ik}\frac{1+\gamma^5}{2}\right](\gamma^{\sigma}q_{\sigma} + m_i)(\gamma^{\rho}q_{\rho} + m_i)\left[K^*_{ik}\frac{1+\gamma^5}{2} + L^*_{ik}\frac{1-\gamma^5}{2}\right] \tag{86a}$$

$$= \Gamma_1^{\sigma\rho}q_{\sigma}q_{\rho} + 2m_i\Gamma_2^{\sigma}q_{\sigma} + m_i^2\Gamma_3 \tag{86b}$$

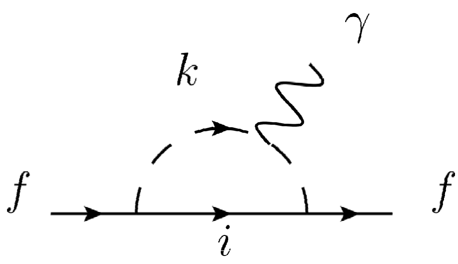


Figure 7: Configuration of vertices giving a diagram of the same form as the left one in 5.

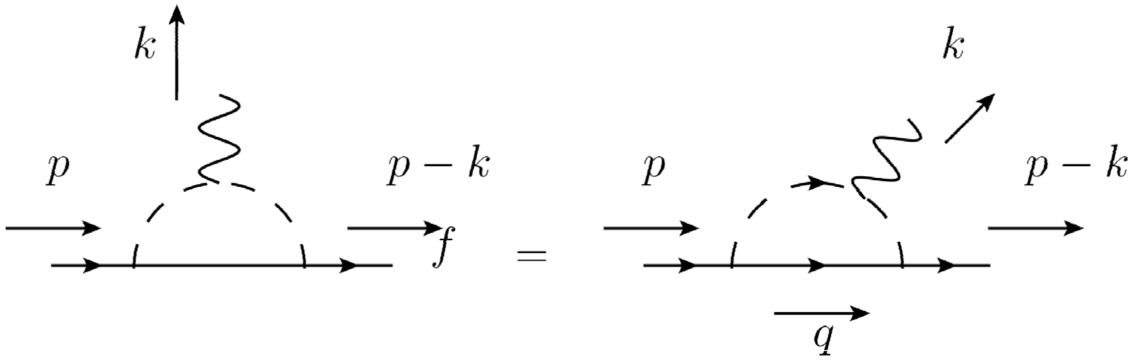


Figure 8: When we put arrows on fermion and scalar lines, it turns out that there is only one configuration of the vertices that give the same form of diagram. The diagram on the left hand side is called $\mathcal{D}^{\mu}$, and the one on the right hand side is $\mathcal{D}'^{\mu}$. In this diagram and in figure 7, the mass insertions are understood.

With, $\{\gamma_5, \gamma^{\mu}\} = 0$ and $\gamma_5^2 = \mathbf{1}$ we can write:

$$\Gamma_1^{\sigma\rho} = \left[K_{ik}\frac{1-\gamma_5}{2} + L_{ik}\frac{1+\gamma_5}{2}\right]\gamma^{\sigma}\gamma^{\rho}\left[K_{ik}^{*}\frac{1+\gamma_5}{2} + L_{ik}^{*}\frac{1-\gamma_5}{2}\right] \tag{87a}$$

$$= \gamma^{\sigma}\gamma^{\rho}Z_5 \tag{87b}$$

$$\Gamma_2^{\sigma} = \left[K_{ik}\frac{1-\gamma_5}{2} + L_{ik}\frac{1+\gamma_5}{2}\right]\gamma^{\sigma}\left[K_{ik}^{*}\frac{1+\gamma_5}{2} + L_{ik}^{*}\frac{1-\gamma_5}{2}\right] \tag{87c}$$

$$= \frac{1}{2}\gamma^{\sigma}\Lambda_5 \tag{87d}$$

$$\Gamma_3 = \left[K_{ik}\frac{1-\gamma_5}{2} + L_{ik}\frac{1+\gamma_5}{2}\right]\left[K_{ik}^{*}\frac{1+\gamma_5}{2} + L_{ik}^{*}\frac{1-\gamma_5}{2}\right] \tag{87e}$$

$$= Z_5 \tag{87f}$$

Where we have defined $\Lambda_5 = |K_{ik}|^2 + |L_{ik}|^2 + \gamma_5(|K_{ik}|^2 - |L_{ik}|^2)$ and

$Z_5 = \mathfrak{Re}(L_{ik}K_{ik}^{*}) + i\gamma_5\mathfrak{Im}(L_{ik}K_{ik}^{*})$ and we have suppressed the spinors indices α and β, but are trivial to put back in. We can simplify further by using the relation $\{\gamma^\sigma, \gamma^\rho\} = 2\eta^{\sigma\rho}$, so that $\gamma^\sigma\gamma^\rho q_\sigma q_\rho = q^2$. Therefore:

$$\Gamma = (q^2 + m_i^2) Z_5 + m_i q_\sigma \gamma^\sigma \Lambda_5 \tag{88}$$

As we want to apply this model to the case where the internal particles are MSSM particles, we can assume that the internal lines carry momenta much higher than the ones carried by external momenta. But at the same time, we want to see how the photon momentum enters in the calculation. We can therefore make the assumption $q^\sigma \gg p^\sigma$, so that:

$$(p-q)^2 = (p-q)^\sigma (p-q)_\sigma \approx q^2 \tag{89}$$

$$(p-q-k)^2 = (p-q-k)^\mu (p-q-k)_\mu \approx (q+k)^2 \tag{90}$$

So, the expression for $\mathcal{D}'^\mu$ becomes:

$$\mathcal{D}'^\mu = eQ_k m_i m_k^2 \left[Z_5 \int \frac{d^4q}{(2\pi)^4} \frac{(q^2+m_i^2)(2q+k)^\mu}{\Sigma(q,k;m_i,m_k)} + m_i \gamma^\sigma \Lambda_5 \int \frac{d^4q}{(2\pi)^4} \frac{q_\sigma (2q+k)^\mu}{\Sigma(q,k;m_i,m_k)} \right] \tag{91}$$

For convenience of writing, $\Sigma(q,k;m_i,m_k) = (q^2 - m_i^2)^2 (q^2 - m_k^2)^2 ((q+k)^2 - m_k^2)$ has been defined. If we look at the last term, we find that it is not of interest to us. Indeed:

$$m_i \gamma^\sigma \Lambda_5 \int \frac{d^4q}{(2\pi)^4} \frac{q_\sigma (2q+k)^\mu}{\Sigma(q,k;m_i,m_k)} = 2 m_i \gamma^\sigma \Lambda_5 \int \frac{d^4q}{(2\pi)^4} \frac{q_\sigma q^\mu}{\Sigma(q,k;m_i,m_k)} \tag{92}$$

Because of Lorentz invariance we impose the condition:

$$\int \frac{d^4q}{(2\pi)^4} \frac{q_\sigma}{\Sigma(q,k;m_i,m_k)} = 0 \tag{93}$$

And the remaining term can be expressed in two different ways:

$$
\begin{aligned}
2m_i\gamma^\sigma\Lambda_5\int\frac{d^4q}{(2\pi)^4}\frac{q_\sigma q^\mu}{\Sigma(q,k;m_i,m_k)} &= m_i\gamma^\sigma\Lambda_5\frac{\eta^\mu_\sigma}{2}\int\frac{d^4q}{(2\pi)^4}\frac{q^2}{\Sigma(q,k;m_i,m_k)} && \text{(94a)}\\
&= 2m_i\gamma^\sigma\Lambda_5\frac{k_\sigma k^\mu}{k^2}\int\frac{d^4q}{(2\pi)^4}\frac{q^2}{\Sigma(q,k;m_i,m_k)} && \text{(94b)}
\end{aligned}
$$

But in both cases, this term does not have the tensor structure we want to have. So the expression for $\mathcal{D}'^\mu$ will become:

$$
\begin{aligned}
\mathcal{D}'^\mu &= eQ_k m_i m_k^2 Z_5\int\frac{d^4q}{(2\pi)^4}\frac{(q^2+m_i^2)(2q+k)^\mu}{\Sigma(q,k;m_i,m_k)}+N.I.T && \text{(95a)}\\
&= eQm_i m_k^2 Z_5 k^\mu\int\frac{d^4q}{(2\pi)^4}\frac{(q^2+m_i^2)}{\Sigma(q,k;m_i,m_k)}+N.I.T && \text{(95b)}
\end{aligned}
$$

Where $N.I.T$ stands for "Non Important Terms". The second line has been derived with the use of (93) as we can always define $\Sigma'(q,k;m_i,m_k)=\Sigma(q,k;m_i,m_k)/(q^2+m_i^2)$. Here again we can say that we are looking for a term proportional to γ_5, so we can simply keep the term $i\gamma_5\mathfrak{Im}(L_{ik}K^*_{ik})$ in Z_5. We therefore end up with:

$$
\mathcal{D}'^\mu = ieQ_k m_i m_k^2\mathfrak{Im}(L_{ik}K^*_{ik})\gamma_5 k^\mu\int\frac{d^4q}{(2\pi)^4}\frac{(q^2+m_i^2)}{\Sigma(q,k;m_i,m_k)}+N.I.T \tag{96}
$$

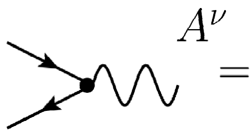
Now, using figure 8 we end up with:

$$
\mathcal{D}^\mu = ieQ_k m_i m_k^2\mathfrak{Im}(L_{ik}K^*_{ik})\gamma_5 k^\mu\int\frac{d^4q}{(2\pi)^4}\frac{(q^2+m_i^2)}{\Sigma(q,k;m_i,m_k)}+N.I.T \tag{97}
$$

From now on, we will drop the $N.I.T$ term. Having a closer look at (27) in momentum space, i.e. $\partial_\mu A_\nu \to -ik_\mu A_\nu$, and using the relation $[\gamma^\mu,\gamma^\nu]k_\mu = 2(k^\nu - \gamma^\nu\gamma^\mu k_\mu)$ which is easy to show using the Dirac algebra, we realise that:

$$
\mathcal{L}_{EDM} = -id\overline{\psi}k^\nu\gamma_5\psi A_\nu + id\overline{\psi}\gamma^\nu\gamma^\mu k_\mu\gamma_5\psi A_\nu \tag{98}
$$

And so we obtain the Feynman rule of figure 9.



$$= \quad dk^\nu\gamma_5 - d\gamma^\nu\gamma^\mu\gamma_5 k_\mu$$

Figure 9: Other way of writing the Feynman rule associated with this diagram

The analogy between (97) and figure 9 tells that:

$$d = ieQ_k m_i m_k^2 \mathfrak{Im}(L_{ik}K_{ik}^*) \int \frac{d^4q}{(2\pi)^4} \frac{q^2 + m_i^2}{(q^2 - m_i^2)^2(q^2 - m_k^2)^2((q+k)^2 - m_k^2)} \tag{99}$$

Now that we have seen how does k^μ enters the calculation, we can assume $q^\mu \gg k^\mu$, so that:

$$d = ieQ_k m_i m_k^2 \mathfrak{Im}(L_{ik}K_{ik}^*) \int \frac{d^4q}{(2\pi)^4} \frac{q^2 + m_i^2}{(q^2 - m_i^2)^2(q^2 - m_k^2)^3} \tag{100}$$

In order to compute the integral in (100), we will use the dimensional regularisation scheme, i.e. we will promote the 4-dimensional integration to a d-dimensional integration. We will introduce the mass dimension 1 parameter μ and the parameter $\epsilon = 4 - d$:

$$\int \frac{d^4q}{(2\pi)^4} \quad \to \quad \mu^\epsilon \int \frac{d^dq}{(2\pi)^d} \tag{101}$$

Also, in order not to be confused with the value of d in 4 dimensions and in d dimensions, we introduce the notation $d^{(d)}$ for the electric dipole moment d in d dimensions. So, according to this notation, (100) is in fact the expression for $d^{(4)}$ and according to the dimensional regularisation scheme we have:

$$d^{(4)} \quad \to \quad d^{(d)} = ieQ_k \mu^\epsilon m_i m_k^2 \mathfrak{Im}(L_{ik}K_{ik}^*) \int \frac{d^dq}{(2\pi)^d} \frac{q^2 + m_i^2}{(q^2 - m_i^2)^2(q^2 - m_k^2)^3} \tag{102}$$

The two following subsections will expose two different methods to compute $d^{(4)}$ and $d^{(d)}$.

3.1.2 First method: Feynman parameters

The first method to compute the integral in (102) makes use of the Feynman parameters. The general formula for the Feynman parameters is [16]:

$$\frac{1}{A_1^{\alpha_1} A_2^{\alpha_2} \dots A_n^{\alpha_n}} = \int_0^1 dx_1 dx_2 \dots dx_n \delta(\sum_i x_i - 1) \frac{\prod_i x_i^{\alpha_i - 1}}{[\sum_i x_i A_i]^{\sum_i \alpha_i}} \frac{\Gamma(\sum_i \alpha_i)}{\prod_i \Gamma(\alpha_i)} \tag{103}$$

Applying this formula to (102) and using the property of Gamma functions for integers $\Gamma(n+1) = n!$, we obtain:

$$d^{(d)} = 12ieQ\mu^\epsilon m_i m_k^2 \mathfrak{Im}(L_{ik} K_{ik}^*) \int_0^1 dy\,(1-y)y^2 \int \frac{d^d q}{(2\pi)^d} \frac{(q^2 + m_i^2)}{(q^2 - \Delta)^5} \tag{104}$$

Where we have defined $\Delta = m_i^2 - y(m_i^2 - m_k^2)$. The usual techniques of integration in a d-dimensional Minkowski spacetime allow us to compute the integral in (104). We have [16]:

$$\int \frac{d^d q}{(2\pi)^d} \frac{q^2}{(q^2 - \Delta)^5} = \frac{id\Gamma(4 - \frac{d}{2})}{2(4\pi)^{\frac{d}{2}} \Gamma(5)} \Delta^{\frac{d}{2} - 4} \tag{105a}$$

$$\int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2 - \Delta)^5} = \frac{-i\Gamma(5 - \frac{d}{2})}{(4\pi)^{\frac{d}{2}} \Gamma(5)} \Delta^{\frac{d}{2} - 5} \tag{105b}$$

Putting these expressions back into (104), we have:

$$d^{(d)} = -\frac{\mu^\epsilon e Q_k m_i m_k^2 d}{4(4\pi)^{\frac{d}{2}}} \Gamma(4 - \frac{d}{2}) \mathfrak{Im}(L_{ik} K_{ik}^*) \int_0^1 dy\,(1-y)y^2 \left[\Delta^{\frac{d}{2} - 4} - \frac{2m_i^2 \Gamma(5 - \frac{d}{2})}{d\Gamma(4 - \frac{d}{2})} \Delta^{\frac{d}{2} - 5} \right] \tag{106}$$

In order to compute it, we define:

$$I_d = \int_0^1 dy\,(1-y)y^2\left(\Delta^{\frac{d}{2}-4} - \frac{2m_i^2}{d}\frac{\Gamma(5-\frac{d}{2})}{\Gamma(4-\frac{d}{2})}\Delta^{\frac{d}{2}-5}\right) \tag{107a}$$

$$= I'_d - \frac{2m_i^2}{d}\frac{\Gamma(5-\frac{d}{2})}{\Gamma(4-\frac{d}{2})}I''_d \tag{107b}$$

With:

$$I'_d = m_k^{d-8}(1-\omega)^{\frac{d}{2}-4}\int_0^1 dy(1-y)y^2\left(y+\frac{\omega}{1-\omega}\right)^{\frac{d}{2}-4} \tag{108a}$$

$$I''_d = m_k^{d-10}(1-\omega)^{\frac{d}{2}-5}\int_0^1 dy(1-y)y^2\left(y+\frac{\omega}{1-\omega}\right)^{\frac{d}{2}-5} \tag{108b}$$

We have introduced the very convenient parameter $\omega = m_i^2/m_k^2$. We first compute this without worrying about the value of d. In this way we find:

$$I'_d = \frac{4m_k^{d-8}(1-\omega)^{-2}}{(d-6)(d-4)}\left(1-\frac{4}{d-2}\frac{2+\omega^{\frac{d}{2}-1}}{1-\omega}+\frac{24}{(d-2)d}\frac{1-\omega^{\frac{d}{2}}}{(1-\omega)^2}\right) \tag{109a}$$

$$I''_d = \frac{4m_k^{d-10}(1-\omega)^{-2}}{(d-8)(d-6)}\left(1-\frac{4}{d-4}\frac{2+\omega^{\frac{d}{2}-2}}{1-\omega}+\frac{24}{(d-4)(d-2)}\frac{1-\omega^{\frac{d}{2}-1}}{(1-\omega)^2}\right) \tag{109b}$$

We therefore obtain:

$$\begin{aligned}I_d =& \frac{4m_k^{d-8}}{(d-6)(d-8)(1-\omega)^2}\\ &\times\left[1-\frac{2\Gamma(5-\frac{d}{2})}{d\Gamma(4-\frac{d}{2})}\omega - \frac{4}{(d-2)(1-\omega)}\left(2+\omega^{\frac{d}{2}-1}-\frac{2(d-2)\Gamma(5-\frac{d}{2})}{d(d-4)\Gamma(4-\frac{d}{2})}(2+\omega^{\frac{d}{2}-2})\omega\right)\right.\\ &\left.+\frac{24}{d(d-2)(1-\omega)^2}\left(1-\omega^{\frac{d}{2}}-\frac{2\Gamma(5-\frac{d}{2})}{(d-4)\Gamma(4-\frac{d}{2})}(1-\omega^{\frac{d}{2}-1})\omega\right)\right]\end{aligned} \tag{110}$$

We can use the property of Gamma functions $\Gamma(z+1) = z\Gamma(z)$, which leads to:

$$
\begin{aligned}
I_d =& \frac{4m_k^{d-8}}{(d-6)(d-8)(1-\omega)^2} \\
\times& \left[1 + \frac{(d-8)\omega}{d} - \frac{4}{(d-2)(1-\omega)}\left(2 + \omega^{\frac{d}{2}-1} + \frac{(d-2)(d-8)}{d(d-4)}(2+\omega^{\frac{d}{2}-2})\omega\right)\right. \\
&\left. + \frac{24}{d(d-2)(1-\omega)^2}\left(1 - \omega^{\frac{d}{2}} + \frac{d-8}{d-4}(1-\omega^{\frac{d}{2}-1})\omega\right)\right]
\end{aligned} \tag{111}
$$

And therefore, inserting this back into (106) we get:

$$
\begin{aligned}
d^{(d)} =& -\frac{eQ_k m_i}{(4\pi)^{\frac{d}{2}} m_k^2}\left(\frac{\mu}{m_k}\right)^{4-d} \frac{d}{(d-6)(d-8)}\Gamma(4-\frac{d}{2})\mathfrak{Im}(L_{ik}K_{ik}^*) \\
&\times \frac{1}{(1-\omega)^2}\left[1 + \frac{(d-8)\omega}{d}\right. \\
&- \frac{4}{(d-2)(1-\omega)}\left(2 + \omega^{\frac{d}{2}-1} + \frac{(d-2)(d-8)}{d(d-4)}(2+\omega^{\frac{d}{2}-2})\omega\right) \\
&\left. + \frac{24}{d(d-2)(1-\omega)^2}\left(1 - \omega^{\frac{d}{2}} + \frac{d-8}{d-4}(1-\omega^{\frac{d}{2}-1})\omega\right)\right]
\end{aligned} \tag{112}
$$

At this point, we have to remember that we computed this for only one specie of heavy fermion and heavy scalar, and according to the remark made immediately after (85), we simply sum (112) over all possible values for i and k. So:

$$
\begin{aligned}
d^{(d)} =& \sum_{i,k}\frac{eQ_k m_i}{(4\pi)^{\frac{d}{2}} m_k^2}\left(\frac{\mu}{m_k}\right)^{4-d} \frac{d}{(d-6)(d-8)}\Gamma(4-\frac{d}{2})\mathfrak{Im}(K_{ik}L_{ik}^*) \\
&\times \frac{1}{(1-\omega)^2}\left[1 + \frac{(d-8)\omega}{d}\right. \\
&- \frac{4}{(d-2)(1-\omega)}\left(2 + \omega^{\frac{d}{2}-1} + \frac{(d-2)(d-8)}{d(d-4)}(2+\omega^{\frac{d}{2}-2})\omega\right) \\
&\left. + \frac{24}{d(d-2)(1-\omega)^2}\left(1 - \omega^{\frac{d}{2}} + \frac{d-8}{d-4}(1-\omega^{\frac{d}{2}-1})\omega\right)\right]
\end{aligned} \tag{113}
$$

We have used the fact that $\mathfrak{Im}(K_{ik}L_{ik}^*) = -\mathfrak{Im}(L_{ik}K_{ik}^*)$. The expression (113) seems to be finite only in $d \neq 2, 4, 6, 8$. But in fact, this is an example that the dimensional regularisation scheme may be misleading. Indeed, we must take this expression for $d = 4 - \epsilon$ and look at the divergent pieces as $\epsilon \to 0$. And despite its apparences of being divergent in $d = 4$, it is likely that some divergent pieces cancel each other, so that only well behave term remain.

We now compute $d^{(4)}$, we go back to the formula (111) and substitute $d = 4 - \epsilon$, we get:

$$
\begin{aligned}
I_{4-\epsilon} = \frac{4m_k^{-4}(1-\epsilon\ln m_k)}{(2+\epsilon)(4+\epsilon)(1-\omega)^2}\Bigg[1 - \frac{4+\epsilon}{4-\epsilon}\omega + 2\frac{4+2\omega-\omega\epsilon\ln\omega}{(2+\epsilon)(1-\omega)} \\
+ 12\frac{2-2\omega^2+\epsilon\omega^2\ln\omega+(1+\epsilon)\omega^2\ln\omega}{(4-\epsilon)(2-\epsilon)(1-\omega)^2} \\
+ \frac{(4+\epsilon)\omega^2}{(4-\epsilon)(1-\omega)}\ln\omega + \frac{4(4+\epsilon)\omega}{(4-\epsilon)(1-\omega)}\frac{2+\epsilon^2}{(2-\epsilon)\epsilon}\Bigg]
\end{aligned}
\tag{114}
$$

We have used the fact that $a^\epsilon = \exp(\epsilon\ln a) = 1 + \epsilon\ln a + 0(\epsilon^2)$. A divergent piece in $1/\epsilon$ appears as expected. So the dipole moment in $4-\epsilon$ dimensions is:

$$
\begin{aligned}
d^{(4-\epsilon)} = & -\frac{eQ_k m_i m_k^{-2}}{(4\pi)^2(1-\omega)^2}\mathfrak{Im}(L_{ik}K_{ik}^*)\Gamma(2+\frac{\epsilon}{2})\frac{(1+\epsilon\ln\mu)(1-\epsilon\log m_k)(4-\epsilon)}{(1-\frac{\epsilon}{2}\ln 4\pi)(2+\epsilon)(4+\epsilon)} \\
& \times\Bigg[1 - \frac{4+\epsilon}{4-\epsilon}\omega + \frac{2}{(2+\epsilon)(1-\omega)}(4+2\omega-\omega\epsilon\ln\omega) \\
& + \frac{12}{(4-\epsilon)(2-\epsilon)(1-\omega)^2}\big(2-2\omega^2+\epsilon\omega^2\ln\omega+(1+\epsilon)\omega^2\ln\omega\big) \\
& + \frac{(4+\epsilon)\omega^2}{(4-\epsilon)(1-\omega)}\ln\omega + \frac{4(4+\epsilon)\omega}{(4-\epsilon)(1-\omega)}\frac{2+\epsilon^2}{(2-\epsilon)\epsilon}\Bigg]
\end{aligned}
\tag{115}
$$

By expanding this to the first order in ϵ, and taking the limit $\epsilon \to 0$, we get:

$$\lim_{\epsilon\to 0} d^{(4-\epsilon)} = d^{(4)} = \sum_{i,k} \frac{eQ_k m_i}{2(4\pi)^2 m_k^2} \mathfrak{Im}(K_{ik}L_{ik}^*) \frac{1}{(1-\omega)^2}$$
$$\times \left[1 - \omega + 3\frac{(1+\omega)}{1-\omega} + \frac{1}{1-\omega}\left(1 + \frac{3\omega^2}{2(1-\omega)}\right)\ln\omega \right. \tag{116}$$
$$\left. + \frac{1}{1-\omega}\left(\ln\frac{4\pi\mu^2}{m_k^2} + \frac{2\omega}{\epsilon}\right)\right]$$

Where we have restored the sum over all possible i and k according to the remark done after (85). The last term in $1/\epsilon$ was expected from (113). Despite its apparences, this term is not worrying. We did not consider all the diagrams having contributions to $d^{(4)}$. If we did so, it is very likely that other diagrams would bring similar divergent pieces that would cancel that one. Thus, the final result would naturally be finite. But this term was also expected for another reason. We did consider the bare parameters of the Lagrangian, which are not physical. The physical parameters are the renormalised ones. So, if we want to remove this divergences from (116), we could use a renormalisation scheme. According to the the form of the divergent piece, we may use the $\overline{MS}$ scheme. Thus, we may add couterterms in the Lagrangian in such way that they cancel this term. At the same time, the $\overline{MS}$ scheme would also help in removing the abritray parameter μ.

For comparison, we can compare this result, (116), to the one found by Tarek Ibrahim and Pran Nath in [14] shown in (117). We see that, although the actual expressions are not the same, they have very similar structures. This again can be justified. In [14], Tarek Ibrahim and Pran Nath displayed the the full expression for d, (118), taking into account the two kind of diagrams of figure 5. Here the same remark can again be made, that the other diagram may have terms such that, when added to this diagram, (117) are the terms that remain.

$$d_{TP} = \sum_{i,k} \frac{eQ_k m_i}{2(4\pi)^2 m_k^2} \mathfrak{Im}(K_{ik}L_{ik}^*) \frac{1}{(\omega-1)^2}\left(1 + \omega + \frac{2\omega}{1-\omega}\ln\omega\right) \tag{117}$$

The full expression for d that Tarek Ibrahim and Pran Nath found is:

$$d = \sum_{i,k} \frac{em_i}{(4\pi)^2 m_k^2} \mathfrak{Im}(K_{ik} L_{ik}^*) \frac{1}{2(1-\omega)^2} \left[Q_i \left(3 - \omega + \frac{2}{1-\omega} \ln \omega \right) + Q_k \left(1 + \omega + \frac{2\omega}{1-\omega} \ln \omega \right) \right] \tag{118}$$

3.1.3 Second method: partial fraction expansion

The second method to compute (102) uses the partial fraction expansion. The partial fraction expansion consists in reducing uniquely a rational function into a sum of fractions as follows. Consider a function $f(x)$ of the form:

$$f(x) = \frac{h(x)}{(x-a_1)^{\alpha_1}(x-a_2)^{\alpha_2} \cdots (x-a_n)^{\alpha_n}} \tag{119}$$

Where $h(x)$ is any function of x. $f(x)$ has a pole of order α_1 at $x = a_1$, a pole of order α_2 at $x = a_2$, etc. We can actually write $f(x)$ as:

$$f(x) = A_0 + \frac{A_{11}}{x-a_1} + \frac{A_{12}}{(x-a_1)^2} + \cdots + \frac{A_{1\alpha_1}}{(x-a_1)^{\alpha_1}} + \frac{A_{21}}{x-a_2} + \cdots + \frac{A_{n\alpha_n}}{(x-a_n)^{\alpha_n}} \tag{120}$$

The integral we must compute is the one in (102). In order for that, we will first look for an expansion in fractions of a function of the form:

$$f(x) = \frac{x+c}{(x-a)^2(x-b)^3} = A_0 + \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \frac{B_1}{x-b} + \frac{B_2}{(x-b)^2} + \frac{B_3}{(x-b)^3} \tag{121}$$

In the case we want to apply this to, we have $x = q^2$, $a = m_i^2$, $b = m_k^2$ and $c = a = m_i^2$. We can use the following techniques to find the constants A's and B's:

$$(x-a)^2 f(x)\big|_{x=a} = A_2 = \frac{a+c}{(a-b)^3} \tag{122a}$$

$$(x-a)^3 f(x)\big|_{x=b} = B_3 = \frac{b+c}{(a-b)^2} \tag{122b}$$

And:

$$\lim_{x\to\infty} f(x) = 0 = A_0 \quad \Rightarrow \quad A_0 = 0 \tag{123a}$$

$$\lim_{x\to\infty} (x-a)f(x) = 0 = A_1 + B_1 \quad \Rightarrow \quad A_1 = -B_1 \tag{123b}$$

So:

$$\frac{x+c}{(x-a)^2(x-b)^3} = A_1\left(\frac{1}{x-a} - \frac{1}{x-b}\right) + \frac{B_2}{(x-b)^2} + \frac{a+c}{(a-b)^3(x-a)^2} + \frac{b+c}{(a-b)^2(x-b)^3} \tag{124}$$

We can now simply put everything on the same denominator, and identify the coefficients of the terms in x^3, x^2, x and the remaining constants on the right and left hand side. Thus, identifying the coefficients of the terms in x^3 and x^2 leads to the following system of equations:

$$0 = A_1(a-b) + B_2 + \frac{a+c}{(a-b)^3} \tag{125a}$$

$$0 = A_1(a-b)(a+2b) + B_2(2a+b) - \frac{(b+c)(a-b) - 3b(a+c)}{(a-b)^3} \tag{125b}$$

Note that any other identification would have led to the same result as the expansion is unique. Also, we could have written 4 different equations, but as we are looking for only two unknown constants there would have been 2 redundant equations. The solutions of this system are:

$$A_1 = -\frac{2a+b+3c}{(a-b)^4} \tag{126a}$$

$$B_2 = \frac{a+b+2c}{(a-b)^3} \tag{126b}$$

The function $f(x)$ can therefore be written equivalently as:

$$f(x) = \frac{x+c}{(x-a)^2(x-b)^3} = \frac{2a+b+3c}{(a-b)^4}\left(\frac{1}{x-b} - \frac{1}{x-a}\right) + \frac{1}{(a-b)^3}\left(\frac{a+c}{(x-a)^2} + \frac{a+b+2c}{(x-b)^2}\right) + \frac{b+c}{(a-b)^2}\frac{1}{(x-b)^3} \tag{127}$$

If we define:

$$\mathcal{I}_d = \int \frac{d^d q}{(2\pi)^d} \frac{q^2 + m_i^2}{(q^2 - m_i^2)^2 (q^2 - m_k^2)^3} \tag{128}$$

The application of the formula (127) to (128) gives:

$$\mathcal{I}_d = \frac{1}{m_k^2(\omega-1)^2} \int \frac{d^d q}{(2\pi)^d} \left[\frac{5\omega+1}{m_k^4(\omega-1)^2} \left(\frac{1}{q^2 - m_k^2} - \frac{1}{q^2 - m_i^2} \right) + \frac{1}{m_k^2(\omega-1)} \left(\frac{3\omega+1}{(q^2-m_k^2)^2} + \frac{2\omega}{(q^2-m_i^2)^2} \right) + \frac{\omega+1}{(q^2-m_k^2)^3} \right] \tag{129}$$

Here again, we can use the formula [16]:

$$\int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{\frac{d}{2}}} \frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)} \Delta^{\frac{d}{2}-n} \tag{130}$$

And we obtain:

$$\mathcal{I}_d = \frac{i}{(4\pi)^{\frac{d}{2}} m_k^2 (\omega-1)^2} \left[\frac{(5\omega+1)\Gamma(1-\frac{d}{2})}{m_k^4(\omega-1)^2} \left(m_i^{d-2} - m_k^{d-2} \right) + \frac{\Gamma(2-\frac{d}{2})}{m_k^2(\omega-1)} \left(2\omega m_i^{d-4} + (3\omega+1) m_k^{d-4} \right) - \frac{\Gamma(3-\frac{d}{2})}{2} (\omega+1) m_k^{d-6} \right] \tag{131}$$

By factorising by powers of m_k, we can make ω appear in such way that:

$$\mathcal{I}_d = \frac{i m_k^{d-8}}{(4\pi)^{\frac{d}{2}}(\omega-1)^2}\Bigg[\Gamma(1-\frac{d}{2})\frac{(5\omega+1)(\omega^{\frac{d}{2}-1}-1)}{(\omega-1)^2}+ \\ \Gamma(2-\frac{d}{2})\frac{1+\omega(3+2\omega^{\frac{d}{2}-2})}{\omega-1}-\frac{1}{2}\Gamma(3-\frac{d}{2})(\omega+1)\Bigg] \tag{132}$$

Going back to (102) and summing over i and k gives another expression for d in d dimensions:

$$d^{(d)} = \sum_{i,k}\frac{eQ_k m_i}{(4\pi)^{\frac{d}{2}}m_k^2}\left(\frac{\mu}{m_k}\right)^{4-d}\mathfrak{Im}(K_{ik}L_{ik}^*)\frac{1}{(\omega-1)^2} \\ \Bigg[\Gamma(1-\frac{d}{2})\frac{(5\omega+1)(\omega^{\frac{d}{2}-1}-1)}{(\omega-1)^2}+ \\ \Gamma(2-\frac{d}{2})\frac{1+\omega(3+2\omega^{\frac{d}{2}-2})}{\omega-1}-\frac{1}{2}\Gamma(3-\frac{d}{2})(\omega+1)\Bigg] \tag{133}$$

We now turn to the special case of 4 dimensions, $d=4$. As we did in 3.1.2, we set in (133) $d=4-\epsilon$ and make ϵ go to 0. Setting $d=4-\epsilon$, we get:

$$d^{(4-\epsilon)} = \frac{eQ_k m_i}{(4\pi)^2 m_k^2}\mathfrak{Im}(K_{ik}L_{ik}^*)\frac{1}{(1-\omega)^2}\left(1+\frac{\epsilon}{2}\ln\frac{4\pi\mu^2}{m_k^2}\right) \\ \times\Bigg[\Gamma\left(\frac{\epsilon}{2}-1\right)\left(\frac{5\omega+1}{\omega-1}-\frac{5\omega+1}{2(\omega-1)^2}\epsilon\ln\omega\right) \\ +\Gamma\left(\frac{\epsilon}{2}\right)\left(\frac{1+\omega}{\omega-1}-\frac{\omega}{\omega-1}\epsilon\ln\omega\right)-\frac{1}{2}\Gamma\left(1+\frac{\epsilon}{2}\right)(\omega+1)\Bigg] \tag{134}$$

We can use the formula for the Gamma functions:

$$\Gamma(-l+\frac{\epsilon}{2}) = \frac{(-1)^l}{l!}\left(\frac{2}{\epsilon}+\sum_{p=1}^{l}\frac{1}{p}-\gamma+O(\epsilon)\right) \tag{135}$$

Where $\gamma \approx 0.577216$ is the Euler-Mascheroni constant and $l \geq 0$. If $l=0$ then the sum over p in (135) reduces to 1. So by expanding the Gamma functions, and keeping only the terms to the first order and taking the limit $\epsilon \to 0$, we find:

$$\lim_{\epsilon\to 0} d^{(4-\epsilon)} = d^{(4)} = \sum_{i,k} \frac{eQ_k m_i}{2(4\pi)^2 m_k^2} \mathfrak{Im}(K_{ik}L_{ik}^*) \frac{1}{(\omega-1)^2}$$
$$\left[-1 - \omega + \frac{22\omega+6}{1-\omega} - \frac{2}{\omega-1}\left(1+2\omega\frac{\omega-4}{\omega-1}\right)\ln\omega \right. \tag{136}$$
$$\left. + 4\frac{3\omega+1}{1-\omega}\left(\frac{2}{\epsilon}-\gamma\right) - \frac{\omega(5+\omega)}{(\omega-1)^2}\ln\frac{4\pi\mu^2}{m_k^2} \right]$$

Here again, the formula is quite similar to (116) and (117). Particularly, we clearly notice the same pole at $\epsilon = 0$, which as we discussed in section 3.1.2 can be naturally removed.

3.1.4 Comments on the results

In the above section, we have computed the electric dipole moment in d and 4 dimensions, using a general model of a fermion, interacting with two heavy fields, one fermion and one scalar. These results can, in principle, be applied to any model as no restrictions, other than CP-violating, have been put on the interactions. In fact, we could also perform the same study in the case even more general, where fields are considered neither light nor heavy. This assumption has been made during the calculation, but we could have found a result without it. We have used two methods to compute the electric dipole moment, one using the techniques of the Feynman parameters and the other using partial function expansions. The method with Feynman parameters led to:

$$d^{(4)} = \sum_{i,k} \frac{eQ_k m_i}{2(4\pi)^2 m_k^2} \mathfrak{Im}(K_{ik}L_{ik}^*) \frac{1}{(1-\omega)^2}$$
$$\times \left[1 - \omega + 3\frac{(1+\omega)}{1-\omega} + \frac{1}{1-\omega}\left(1+\frac{3\omega^2}{2(1-\omega)}\right)\ln\omega \right. \tag{137}$$
$$\left. + \frac{1}{1-\omega}\left(\ln\frac{4\pi\mu^2}{m_k^2} + \frac{2\omega}{\epsilon}\right) \right]$$

And the partial function expansion techniques led to:

$$d^{(4)} = \sum_{i,k} \frac{eQ_k m_i}{2(4\pi)^2 m_k^2} \mathfrak{Im}(K_{ik} L_{ik}^*) \frac{1}{(\omega - 1)^2}$$
$$\left[-1 - \omega + \frac{22\omega + 6}{1 - \omega} - \frac{2}{\omega - 1} \left(1 + 2\omega \frac{\omega - 4}{\omega - 1} \right) \ln \omega \right. \tag{138}$$
$$\left. + 4 \frac{3\omega + 1}{1 - \omega} \left(\frac{2}{\epsilon} - \gamma \right) - \frac{\omega(5 + \omega)}{(\omega - 1)^2} \ln \frac{4\pi\mu^2}{m_k^2} \right]$$

The first thing that we notice is that they seem different while they are the same quantity $d^{(4)}$. In spite of the fact that this seems worrying, what is more important is the behavior of these according to the dimensionful parameters. Indeed, the parameters ω is given by m_i^2/m_k^2, and so is a dimensionless parameters, thus non measurable. What is in fact more worth is the prefactors.

The first important parameters are the masses m_i and m_k, which are characteristic energy scales. In terms of the masses, the prefactor goes as $1/M_{characteristic}$. This is a very important observation, because it does tell a lot about the theory itself. $d^{(d)}$ is experimentally measurable and measured, thus the order of magnitude of it gives a range of order of magnitudes for the characteristic energy scale of the theory.

The second important parameter in the prefactor, is the imaginary part of the coupling matrices L and K. We see that if the product of the matrices is real valued, then the electric dipole moment vanishes. So, similarly to the masses, this parameter is measurable through $d^{(4)}$. But, unlike the case of the masses, this imaginary part will give more precise informations on the theory, e.g. which couplings are complex, which ones are not, etc.

The minimal supersymmetric standard model is a direct and very important example of such models. Indeed, the MSSM contains interactions as the one that has just been described, the MSSM therefore predicts EDMs of particles. The MSSM is our best hope for an extension of the standard model, but it is not yet complete in the sense that there are still many free parameters, more than one hundred. One of the most sensitive electric dipole moment is the one of the neutron. The prediction of the MSSM to it involves many parameters, many phases, many couplings which are not precisely known. The experiments, hopefully, will help us in determining the value, or at least the range, of

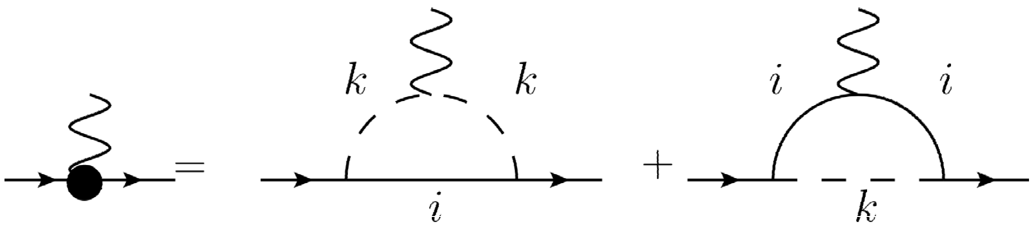


Figure 10: Full diagrams giving the EDM of the external fermion. The indices i and k represent the type of particle, in the same notation as section 3.1. Here again, the convention for the particles "i and "k" are the same as stated in figure 5.

each of these so that the theory will be complete, as is the SM.

3.2 EDM of the neutron

Here we will review the results of the literature, we will mainly use [14] and [18]. The SU(6) quark model is a model of nonrelativistic quarks in 4 dimensions. It states that in this limit, the electric dipole moment of the neutron d can simply be related to the ones of the up and down quarks, respectively noted d_u and d_d with charges q_u and q_d, by:

$$d = q_d d_u + q_u(2d_d) = -\frac{1}{3}(d_u - 4d_d) \tag{139}$$

The calculation of d becomes the calculation of d_u and d_d. We can use this model as we are not interested in the dynamics of the particles. Considering the general Lagrangian of (84) and the diagrams of figure 10, Tarek Ibrahim and Pran Nath [14] have the following results:

$$d = \sum_{i,k} \frac{em_i}{(4\pi)^2 m_k^2} \mathfrak{Im}(K_{ik}L_{ik}^*) \frac{1}{2(1-\omega)^2} \left[Q_i \left(3 - \omega + \frac{2}{1-\omega} \ln \omega \right) + Q_k \left(1 + \omega + \frac{2\omega}{1-\omega} \ln \omega \right) \right] \tag{140}$$

The part proportional to Q_i comes from the diagrams where the photon line is connected to the fermion line labeled i, and the part proportional to Q_k comes from the other diagram in figure 10. The computation of d will consist in computing the contributions to d_u and d_d from the chargino, neutralino and gluino by applying the formula (10). Since the gluino and neutralino are neutral particles, when the quark interacts with these fields we can use a generic field q. However, we will have to explicitly deal with two different fields u and d when they interact with the chargino.

The Lagrangian encoding the quark-squark-gluino interaction is:

$$\mathcal{L}_{q\tilde{q}\tilde{g}} = \sum_{q=u,d} \sqrt{2} g_s \sum_{j,k=1}^{3} \sum_{a=1}^{8} \left(\overline{q^j} T^a_{jk} \frac{1-\gamma_5}{2} \tilde{g}_a \tilde{q}^k_R - \overline{q^j} T^a_{jk} \frac{1+\gamma_5}{2} \tilde{g}_a \tilde{q}^k_L \right) + H.c \tag{141}$$

Where g_s is the strong interaction coupling. The index a labels the eight gluino colours, the indices j, k label the three quark and squark colours and q is either the up or down quark. Usually, one prefers dealing with mass eigenstates of fields instead of left-right handed "basis". This operation is done in the annexe. The generators T^a are the Gell-Mann matrices λ^a, so taking this into acccount and inserting the mass eigenstate fields in the Lagrangian, via the formula (172), we have:

$$\mathcal{L}_{q\tilde{q}\tilde{g}} = \sum_{q=u,d} \sum_{j,k=1}^{3} \sum_{a=1}^{8} \sum_{l=1}^{2} \left(\overline{q^j} (\sqrt{2} g_s \lambda^a_{jk} D_{q2l}) \frac{1-\gamma_5}{2} \tilde{g}_a \tilde{q}^k_l - \overline{q^j} (\sqrt{2} g_s \lambda^a_{jk} D_{q1l}) \frac{1+\gamma_5}{2} \tilde{g}_a \tilde{q}^k_l \right) + H.c \tag{142}$$

Finally, as the Gell-Mann matrices are essentially $\lambda = \pm 1, \pm i$, we see that the Lagrangian (142) is of the same form as (84). The result is then:

$$d_q^{gluino} = -\frac{e g_s^2 Q_{\tilde{q}}}{6\pi^2} \sum_{l=1}^{2} \frac{m_{\tilde{g}}}{m^2_{\tilde{q}_l}} \mathfrak{Im}(D_{q2l} D^*_{q1l}) \left(1 + \omega + \frac{2\omega}{1-\omega} \ln \omega \right) \tag{143}$$

With, $\omega = m^2_{\tilde{g}} / m^2_{\tilde{q}_l}$ and $e > 0$ is the electric charge. Using the equations from (169) to (176), one gets:

$$\begin{aligned}
\mathfrak{Im}(D_{u2l}D^*_{u1l}) &= \frac{(-1)^{l+1}}{2}\sin\phi_u \sin\theta_u &\text{(144a)}\\
&= \frac{m_u}{M^2_{\tilde{u}1}-M^2_{\tilde{u}2}}(m_0|A_u|\sin\alpha_u + |\mu|\sin\theta_\mu \cot\beta) &\text{(144b)}\\
\mathfrak{Im}(D_{d2l}D^*_{d1l}) &= \frac{(-1)^{l+1}}{2}\sin\phi_d \sin\theta_d &\text{(144c)}\\
&= \frac{m_d}{M^2_{\tilde{d}1}-M^2_{\tilde{d}2}}(m_0|A_d|\sin\alpha_d + |\mu|\sin\theta_\mu \tan\beta) &\text{(144d)}
\end{aligned}$$

The Lagrangian encoding the chargino-quark-squark interaction is:

$$\mathcal{L}_{q\tilde{q}\lambda} = -g\tilde{u}_L^\dagger\overline{\lambda}\frac{1+\gamma_5}{2}d - g\tilde{d}_R^\dagger\overline{\lambda^C}\frac{1+\gamma_5}{2}u - g\tilde{u}_L\overline{d}\frac{1-\gamma_5}{2}\lambda - g\tilde{d}_L\overline{u}\frac{1-\gamma_5}{2}\lambda^C \tag{145}$$

By changing these states to the mass eigenstates, i.e. diagonalising the mass matrix, one finds the contributions from the chargino to the up- and down-type quark EDMs as:

$$\begin{aligned}
d_u^{chargino} &= -\sum_{i,k=1}\frac{e^3 m_{\tilde{\chi}_i^+}}{16\pi^2\sin^2\theta_W m^2_{\tilde{d}k}}\mathfrak{Im}\Big(\kappa_u V^*_{i2}D_{d1k}(U^*_{i1}D^*_{d1k} - \kappa_d U^*_{i2}D_{d2k^*})\Big)\\
&\times\left[Q_{\tilde{d}}\left(1+\omega+\frac{2\omega}{1-\omega}\ln\omega\right) + (Q_u - Q_{\tilde{d}})\left(3-\omega+\frac{2}{1-\omega}\ln\omega\right)\right]
\end{aligned}\tag{146}$$

Where $\omega = m^2_{\tilde{\chi}_i^+}/m^2_{\tilde{d}k}$. And, similarly:

$$\begin{aligned}
d_d^{chargino} &= -\sum_{i,k=1}\frac{e^3 m_{\tilde{\chi}_i^+}}{16\pi^2\sin^2\theta_W m^2_{\tilde{u}k}}\mathfrak{Im}(\kappa_d V^*_{i2}D_{u1k}(U^*_{i1}D^*_{u1k} - \kappa_u U^*_{i2}D_{u2k^*}))\\
&\times\left[Q_{\tilde{u}}\left(1+\omega+\frac{2\omega}{1-\omega}\ln\omega\right) + (Q_d - Q_{\tilde{u}})\left(3-\omega+\frac{2}{1-\omega}\ln\omega\right)\right]
\end{aligned}\tag{147}$$

Where, according to (166) and (167), we have defined:

$$\kappa_u = \frac{m_u}{\sqrt{2}M_W \sin\beta} = \frac{f_u}{g} \tag{148a}$$

$$\kappa_d = \frac{m_d}{\sqrt{2}M_W \cos\beta} = \frac{f_d}{g} \tag{148b}$$

The Lagrangian of the neutralino-quark-squark interaction is [18]:

$$\mathcal{L}_{q\tilde{q}\lambda_A} = -\frac{1}{\sqrt{2}}\left[(\tilde{u}_L^\dagger\ \tilde{d}_L^\dagger)\begin{pmatrix} g\overline{\lambda}_3 + \frac{g'}{3}\overline{\lambda}_0 & g(\overline{\lambda}_1 - i\overline{\lambda}_2) \\ g(\overline{\lambda}_1 + i\overline{\lambda}_2) & -g\overline{\lambda}_3 + \frac{g'}{3}\overline{\lambda}_0 \end{pmatrix}\frac{1+\gamma_5}{2}\begin{pmatrix} u \\ d \end{pmatrix} + g'\delta_q Q_u \tilde{q}_R^\dagger \overline{\lambda}_0 \frac{1-\gamma_5}{2} q_R\right] + H.c \tag{149}$$

The parameter δ_q such that $\delta_u = -2$ and $\delta_d = 1$ has been introduced and $Q_u = 2/3$ is the charge of the up-quark. One has:

$$d_q^{neutralino} = \sum_{k=1}^{2}\sum_{i=1}^{4} \frac{e^3 Q_{\tilde{q}} m_{\tilde{\chi}_i^0}}{16\pi^2 \sin^2\theta_W m_{\tilde{q}k}^2} \mathfrak{Im}(\eta_{qik}) \left(1 + \omega + \frac{2\omega}{1-\omega}\ln\omega\right) \tag{150}$$

Where $T_{3u,d} = 1/2, -1/2$. Also:

$$\begin{aligned} \eta_{qik} =& (\sqrt{2}\tan\theta_W Q_q X_{1i} D_{q2k} - \kappa_q X_{bi} D_{q1k}) \\ & \left[-\sqrt{2}\Big(\tan\theta_W (Q_q - T_{3q}) X_{1i} + T_{3q} X_{2i}\Big) D_{q1k}^* + \kappa_q X_{bi} D_{q2k}^*\right] \end{aligned} \tag{151}$$

The matrix X is the matrix diagonalising the neutralino mass matrix, as in the annexe. In the case of the up-quark $b = 4$ and $b = 3$ in the case of the down-quark.

The full quark EDM is then given by:

$$d_q = d_q^{gluino} + d_q^{chargino} + d_q^{neutralino} \tag{152}$$

And finally, in order to obtain the full neutron EDM from the contributions of the gluino, the chargino and the neutralino, one has to insert (152) into (139).

Conclusion

In this dissertation, we have first reviewed the calculation of the electric dipole moment of the neutron in QCD. The θ-parameter enters the calculation and gives a contribution to the EDM. However, due to the fact that this parameter is known to be very small, the electric dipole moment of the neutron computed in QCD is too small compared to what the experiments measure. The minimal supersymmetric standard model, smallest theory beyond the SM, is greatly hoped to resolve this and to predict the right amount of EDM, because it contains many new CP-violating phases that are likely to contribute to this observable.

We then derived expressions for the electric dipole moment of a fermion in a generic theory containing CP-violating interactions. The calculation showed that the EDM goes as $1/M_{characteristic}$, which is the inverse of the characteristic energy scale in the theory. In addition, it turned out that the phases of the CP-violating terms of the Lagrangian also enter the calculation. The experiments of the electric dipole moments can therefore be used as a probe of the theory. Not only, the experimental value would allow to estimate $M_{characteristic}$, but it would put constraints on the possible values of parameters that enter the calculation in the theory.

As a concrete application, we reviewed the calculation of the EDM of the neutron using the MSSM. The parameters that come in are the masses of the gluinos, the squarks, the gauginos and the higgsinos, and the phases of the Higgs mass term and of the Yukawa couplings. So, as expected, we can estimate important quantities of the theory. In fact, as the electric dipole moment seems to be small, of the order of $10^{-18} e \cdot cm$, M_{SUSY} is expected to be large. Actually, the lightest supersymmetric particle is expected to be seen at the LHC, which reach energies in the range of 10 TeV. So, if we can set the MSSM parameters so that they agree with experiments, that would be a strong hint and probe for physics beyond the standard model.

Appendix A: Brief review of the MSSM

The aim of the annexes A and B is not to give a detailed review of the MSSM, but simply to set the notations. For a comprehensive review of SUSY and the MSSM phenomenology refer to [18], [17] and for a full set of Feynman rules in the MSSM go to [15].

The gauge group of the MSSM is the same as the one of the SM, i.e. $SU(3) \times SU(2) \times U(1)_Y$. The standard model matter fields are promoted to supermutliplets. The SM fermion fields become chiral scalar superfields. In the case of the quarks fields, we have:

$$\begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} \quad \to \quad \hat{Q}_i \equiv \begin{pmatrix} \hat{u}_i \\ \hat{d}_i \end{pmatrix} \tag{153a}$$

$$u_{iR} \quad \to \quad \hat{U}_i \tag{153b}$$

$$d_{iR} \quad \to \quad \hat{D}_i \tag{153c}$$

Where $i = 1, 2, 3$ labels the three generations, from now on we will refer to the quark fields u_i and d_i simply under the name q_i. Each field $\hat{q}_i$, $\hat{Q}_i$ is in fact a supermultiplet:

$$\hat{q}_{iL} = \tilde{q}_{iL} + i\sqrt{2}\theta q_{iL} + i\bar{\theta}\theta\mathcal{F}_{q_L} \tag{154a}$$

$$\hat{q}_{iR} = \tilde{q}^{\dagger}_{iR} + i\sqrt{2}\theta q_{iR} + i\bar{\theta}\theta\mathcal{F}_{q_L} \tag{154b}$$

The θ's are the Grassmann variables, coordinates in superspace. The fields $\tilde{q}_i$ are scalar fields, named sfermions for scalar-fermions. These are the scalar superpartners of the fermion fields q_i and have the same gauge group charges as their partner fields of the SM. The subscript L, R on scalar fields refer to the chirality of their superpartner fields. The term $\mathcal{F}$ is the so-called auxiliary $\mathcal{F}$ field. This auxiliary field does not have any dynamics, i.e. no kinetic terms, it is needed in order to have as many degrees of freedom on-shell as off-shell.

The gauge boson fields are promoted to gauge superfields as follows:

$$B_\mu \quad \longrightarrow \quad \hat{B} = (\lambda_0, B_\mu, \mathcal{D}_j) \tag{155a}$$

$$W_{j_\mu} \quad \longrightarrow \quad \hat{W}_j = (\lambda_j, W_{j\mu}, \mathcal{D}_{Wj}) \tag{155b}$$

$$g_{k\mu} \quad \longrightarrow \quad \hat{g}_k = (\tilde{g}_k, G_{k\mu}, \mathcal{D}_{gk}) \tag{155c}$$

With $j = 1, 2, 3$ labels the three $SU(2)$ gauge fields and $k = 1, \ldots, 8$ labels the eight $SU(3)$ gauge fields. The field λ_0 is the bino, superpartner of the $U(1)_Y$ gauge field B_μ. The λ_j are the winos, superpartners of the $SU(2)$ gauge fields $W_{j\mu}$. The fields $\tilde{g}_k$ are the gluino fields, superpartner of of the gluon fields $G_{k\mu}$. The fields $\mathcal{D}$ are the so-called $\mathcal{D}$ auxiliary fields, auxiliary fields of the gauge fields, as $\mathcal{F}$, they do not have any dynamics. The Lagrangian of a SUSY theory is entirely specified by the superpotential $\hat{W}$. Indeed, one can show that the full Lagrangian for chiral superfields can be written as:

$$\begin{aligned}\mathcal{L}_{SUSY} = &\sum_i (\partial_\mu \phi_i)^\dagger (\partial^\mu \phi_i) + \frac{i}{2} \sum_i \overline{\psi}_i \gamma^\mu \partial_\mu \psi_i - \sum_i \left| \frac{\partial \hat{W}}{\partial \hat{\phi}_i} \right|^2_{\hat{\phi}=\phi} \\ &- \frac{1}{2} \sum_{i,j} \left(\left. \frac{\partial^2 \hat{W}}{\partial \hat{\phi}_i \partial \hat{\phi}_j} \right|_{\hat{\phi}=\phi} \overline{\psi}_i \frac{1-\gamma_5}{2} \psi_j + H.c \right)\end{aligned} \tag{156}$$

Where the indices i, j span all the particles of the theory, ϕ_i are the scalar fields of the theory and ψ_i are the fermion fields. The equality $\hat{\phi} = \phi$ means equating the superfield to its scalar component in the end. In terms of the superfields, the superpotential of the MSSM is:

$$\hat{W} = \mu \sum_{a=1}^{2} \hat{H}_u^a \hat{H}_{da} + \sum_{i,j=1}^{3} \sum_{a,b=1}^{2} \left((f_u)_{ij} \epsilon_{ab} \hat{Q}_i^a \hat{H}_u^b \hat{U}_j + (f_d)_{ij} \hat{Q}_i^a \hat{H}_{da} \hat{D}_j + (f_e)_{ij} \hat{L}_i^a \hat{H}_{da} \hat{E}_j \right) \tag{157}$$

The superfields $\hat{L}_i$ and $\hat{E}_i$ are the superfields of the electron left-handed doublet and right-handed singlet under $SU(2)$ respectively, they are built in a similar way to those of the quarks. The indices a, b are the $SU(2)$ indices and i, j are the generation indices, and ϵ_{ab} is the totally antisymmetric tensor defined by $\epsilon_{12} = 1$. The two superfields $\hat{H}_u$ and $\hat{H}_d$ are the two Higgs superfields of the MSSM. As shown in (158), the usual Higgs field of the SM is promoted to the superfield $\hat{H}_u$, while the other Higgs superfield is introduced in order to give masses to the down-type quarks, which cannot be done with only the up-type Higgs superfield. Actually, it turns out that the latter field is very convenient to cancel anomalies which are otherwise present. Finally, the matrices f_u, f_d and f_e are Yukawa-like matrices. The term proportional to μ is the so-called μ term of the MSSM.

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \rightarrow \quad \hat{H}_u = \begin{pmatrix} \hat{h}_u^+ \\ \hat{h}_u^0 \end{pmatrix} \tag{158a}$$

$$\hat{H}_d = \begin{pmatrix} \hat{h}_d^- \\ \hat{h}_d^0 \end{pmatrix} \tag{158b}$$

We could also write down additional terms in the superpotential that violate lepton number conservation. Although these are not allowed in the standard model because of gauge invariance, these are perfectly correct in the MSSM. As we are not here interested in lepton number violating processes, we will assume the R-parity symmetry, so that such terms are forbidden. As we know that supersymmetry is broken, we also introduce soft SUSY breaking terms. These are called soft breaking terms because they do not introduce ultraviolet divergences. These terms consist in the scalar masses, the gaugino masses, and linear, bilinear and trilinear scalar self-interactions. A general soft SUSY breaking Lagangian is written as follows:

$$\begin{aligned} \mathcal{L}_{soft} = & \left(\sum_i C_i \phi_i + \sum_{i,j} B_{ij} \mu_{ij} \phi_i \phi_j + \sum_{i,j,k} A_{ijk} f_{ijk} \phi_i \phi_j \phi_k + H.c \right) \\ & - \sum_{i,j} \phi_i^\dagger m_{ij}^2 \phi_j - \frac{1}{2} \sum_{A,\alpha} \left(M_{A\alpha} \overline{\lambda}_{A\alpha} \lambda_{A\alpha} - \frac{i}{2} M'_{A\alpha} \overline{\lambda}_{A\alpha} \gamma_5 \lambda_{A\alpha} \right) \end{aligned} \tag{159}$$

The last term is CP-violating, its form is similar to the additional term in the QCD

Lagrangian after a chiral transformation of the θ-term in (67). And in a similar way, we can remove it from the Lagrangian by a chiral transformation of the gaugino fields. Thus, a CP-violating phase enters the gaugino mass matrix. In terms of the fields, (159) gives:

$$
\begin{aligned}
\mathcal{L}_{soft} = &-\Big(\tilde{Q}^\dagger m^2_{\tilde{Q}}\tilde{Q} + \tilde{q}^\dagger_R m^2_{\tilde{q}R}\tilde{q}_R \\
&+ \tilde{L}^\dagger m^2_{\tilde{L}}\tilde{L} + \tilde{e}^\dagger_R m^2_{\tilde{e}R}\tilde{e}_R + \sum_{q=u,d} m^2_{H_q}|H_q|^2\Big) \\
&- \frac{1}{2}\Big(M_1\overline{\lambda}_0\lambda_0 + M_2\sum_A \overline{\lambda}_A\lambda_A + M_3\sum_A \overline{\tilde{g}}_A\tilde{g}_A\Big) \\
&- \frac{i}{2}\Big(M_1'\overline{\lambda}_0\gamma_5\lambda_0 + M_2'\sum_A \overline{\lambda}_A\gamma_5\lambda_A + M_3'\sum_A \overline{\tilde{g}}_A\gamma_5\tilde{g}_A\Big) \\
&+ \Big((a_u)\epsilon_{ab}\tilde{Q}^a H^b_u\tilde{u}^\dagger_R + (a_d)\tilde{Q}^a H_{da}\tilde{d}^\dagger_R + (a_e)\tilde{L}^a H_{da}\tilde{e}^\dagger_R + H.c\Big) \\
&+ \Big((c_u)\epsilon_{ab}\tilde{Q}^a H^{*b}_d\tilde{u}^\dagger_R + (c_d)\tilde{Q}^a H^*_{ua}\tilde{d}^\dagger_R + (c_e)\tilde{L}^a H^*_{ua}\tilde{e}^\dagger_R + H.c\Big) \\
&+ (bH^a_u H_{da} + H.c)
\end{aligned}
\tag{160}
$$

The sum over a, b indices is implicit in the above equation. The next to last line is allowed only in theories with no singlet gauge bosons, which is the case of the MSSM. Combining (156) and (160) leads to the full MSSM Lagrangian. The term proportional to b in the Higgs sector can be moved to the μ term by a redefinition of the Higgs field. By a field redefinition we can also remove the terms proportional to the c matrices. In the end, the MSSM has 124 free parameters, for comparison the SM has 19 free parameters.

As in the SM, in order to give mass to the particles, the ordinary ones and the superpartners, one must break the electroweak symmetry. The mechanism responsible for this is more complicated than the breaking of the EW symmetry in the SM due to the introduction of the second Higgs field. This mechanism is referred as the double Higgs mechanism. In order to break the electroweak symmetry, we use the following scalar potential:

$$V_{MSSM} = V_F + V_D + V_{soft} \tag{161}$$

With:

$$V_F = \sum_i \left| \frac{\partial \hat{W}}{\partial \hat{\phi}_i} \right|^2_{\hat{\phi}=\phi} \tag{162a}$$

$$V_D = \frac{1}{2} \sum_A \left(\sum_i \phi_i^\dagger g_A t_A \phi_i \right)^2 \tag{162b}$$

$$V_{soft} = \sum_i m_{\phi_i}^2 |\phi_i|^2 - B\mu(H_d H_u + H.c) + (a - terms) \tag{162c}$$

The t_A are the generators of the gauge group the scalar fields ϕ_i belong to and g_A are the associated coupling constants. Before turning to the search for the particle masses, one has to minimize the scalar potential (161) considering only neutral fields. So we have to minimize the Lagrangian:

$$V = (m_{H_u}^2 + |\mu|^2)|h_u^0|^2 + (m_{H_d}^2 + |\mu|^2)|h_d^0|^2 - B\mu(h_u^0 h_d^0 + H.c) + \frac{1}{8}(g^2 + g'^2)(|h_u^0|^2 - |h_d^0|^2)^2 \tag{163}$$

Where, as pointed out after (162), g and g' are respectively the $SU(2)$ and $U(1)_Y$ coupling constants, as in the SM. The minimalisation of this scalar potential involves the following quantities:

$$\tan\beta \equiv \frac{v_u}{v_d} \tag{164a}$$

$$B\mu = \frac{(m_{H_u}^2 + m_{H_d}^2 + 2\mu^2)\sin 2\beta}{2} \tag{164b}$$

$$|\mu|^2 = \frac{m_{H_u}^2 - m_{H_d}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{M_Z^2}{2} \tag{164c}$$

With $\langle h_u^0 \rangle \equiv v_u$, $\langle h_d^0 \rangle \equiv v_d$ and $v^2 = v_u^2 + v_d^2$.

Annexe B: Particle masses in the MSSM

With the same techniques as in the SM, we can get the gauge bosons masses:

$$\begin{aligned} M_W^2 &= \frac{g^2}{2} v^2 && (165a) \\ M_Z^2 &= \frac{g^2 + g'^2}{2} v^2 && (165b) \\ M_W &= M_Z \cos\theta_W && (165c) \end{aligned}$$

And we recover the weak mixing angle $\tan\theta_W \equiv g'/g$. The masses of the fermions which couple to H_d, e.g. the down quark and the electron, in the superpotential will have masses:

$$m_i = \frac{\sqrt{2} M_W f_i}{g} \cos\beta \tag{166}$$

And for the ones coupling to H_u, as the up-quark, we will have:

$$m_i = \frac{\sqrt{2} M_W f_i}{g} \sin\beta \tag{167}$$

The gluino does not couple to any fermion field in such way to give contributions to its mass term. As a consequence, the field $\tilde{g}$ is already a mass eigenstate and its mass is:

$$m_{\tilde{g}} = |M_3| \tag{168}$$

The mass matrices of the up- and down-types squaks are:

$$M_{\tilde{u}}^2 = \begin{pmatrix} m_{\tilde{q}_L}^2 + m_u^2 + M_Z^2 \left(\frac{1}{2} - Q_u \sin^2\theta_W\right) \cos 2\beta & m_u (A_u^* m_0 - \mu \cot\beta) \\ m_u (A_u m_0 - \mu^* \cot\beta) & m_{\tilde{u}_R}^2 + m_u^2 + M_Z^2 Q_u \sin^2\theta_W \cos 2\beta \end{pmatrix} \tag{169}$$

Similarly:

$$M^2_{\tilde{d}} = \begin{pmatrix} m^2_{\tilde{q}_L} + m^2_d + M^2_Z\left(-\frac{1}{2} - Q_d \sin^2\theta_W\right)\cos 2\beta & m_d(A^*_d m_0 - \mu\tan\beta) \\ m_d(A_d m_0 - \mu^*\tan\beta) & m^2_{\tilde{d}_R} + m^2_d + M^2_Z Q_d \sin^2\theta_W \cos 2\beta \end{pmatrix} \tag{170}$$

In order to explain the notations here, we have to go back to (160). To reduce the number of parameters, we can assume that the matrices $a_{u,d}$ are proportional to the Yukawa matrices $f_{u,d}$ of the SM, i.e. $a_u = A_u f_u$ and $a_d = A_d f_d$. Thus, the matrices $a_{u,d}$ can be parametrized as:

$$a_d = \frac{g m_0 m_d}{\sqrt{2} M_W \cos\beta} A_d \tag{171a}$$

$$a_u = -\frac{g m_0 m_u}{\sqrt{2} M_W \sin\beta} A_u \tag{171b}$$

The MSSM with two Higgs doublets, has the nice feature that at the Grand Unified Theory energy scale M_{GUT}, the coupling constants of the gauge group $SU(3)\times SU(2)\times U(1)_Y$ are all equal. The mass parameter m_0 is the universal scalar mass, i.e. the mass of the scalars at the GUT scale. Using the renormalisation group flows we can relate the parameters at the GUT scale to the parameters at the electroweak scale. A comprehensive derivation of the renormalisation group flows in the MSSM can be found at [19].

The mass matrices (169) and (170) are diagonalised with a matrix D_q such that:

$$\begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix} = D_q \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} \tag{172a}$$

$$D^\dagger_q M^2_{\tilde{q}} D_q = \begin{pmatrix} M^2_{\tilde{q}1} & 0 \\ 0 & M_{\tilde{q}2} \end{pmatrix} \tag{172b}$$

Where:

$$D_q = \begin{pmatrix} \cos\frac{\theta_q}{2} & -\sin\frac{\theta_q}{2}e^{-i\phi_q} \\ \sin\frac{\theta_q}{2}e^{i\phi_q} & \cos\frac{\theta_q}{2} \end{pmatrix} \tag{173}$$

In the above $q = u, d$. The phase ϕ_q is defined to be the phase of the off-diagonal terms in the mass matrices (169) and (170). Also for convenience of writing, we will write the mass matrices as $M^2_{\tilde{q}} = \begin{pmatrix} M^2_{\tilde{q}11} & M^2_{\tilde{q}12} \\ M^2_{\tilde{q}21} & M^2_{\tilde{q}22} \end{pmatrix}$. It is now straight forward to find that:

$$2M^2_{\tilde{q}1} = M^2_{\tilde{q}11} + M^2_{\tilde{q}22} + \left((M^2_{\tilde{q}11} - M^2_{\tilde{q}22})^2 + 4|M_{\tilde{q}21}|^2\right)^{\frac{1}{2}} \tag{174a}$$

$$2M^2_{\tilde{q}2} = M^2_{\tilde{q}11} + M^2_{\tilde{q}22} - \left((M^2_{\tilde{q}11} - M^2_{\tilde{q}22})^2 + 4|M_{\tilde{q}21}|^2\right)^{\frac{1}{2}} \tag{174b}$$

We chose that $M^2_{\tilde{q}1} > M^2_{\tilde{q}2}$ when $M^2_{\tilde{q}11} > M^2_{\tilde{q}22}$. The angles ϕ_q and θ_q are defined as:

$$\tan\theta_u = \frac{2|M^2_{\tilde{u}21}|}{M^2_{\tilde{u}11} - M^2_{\tilde{u}22}} = \frac{2m_u|A_u m_0 - \mu^* \cot\beta|}{M^2_{\tilde{u}11} - M^2_{\tilde{u}22}} \tag{175a}$$

$$\sin\theta_u = \pm\frac{2m_u|A_u m_0 - \mu^* \cot\beta|}{|M^2_{\tilde{u}1} - M^2_{\tilde{u}2}|} \tag{175b}$$

$$\tan\theta_d = \frac{2|M^2_{\tilde{d}21}|}{M^2_{\tilde{d}11} - M^2_{\tilde{d}22}} = \frac{2m_d|A_d m_0 - \mu^* \tan\beta|}{M^2_{\tilde{d}11} - M^2_{\tilde{d}22}} \tag{175c}$$

$$\sin\theta_d = \pm\frac{2m_d|A_d m_0 - \mu^* \tan\beta|}{|M^2_{\tilde{d}1} - M^2_{\tilde{d}2}|} \tag{175d}$$

The + sign is attributed when $M^2_{\tilde{q}1} > M^2_{\tilde{q}2}$. And:

$$\sin\phi_u = \frac{m_0|A_u|\sin\alpha_u + |\mu|\sin\theta_\mu \cot\beta}{|m_0 A_u - \mu^* \cot\beta|} \tag{176a}$$

$$\sin\phi_d = \frac{m_0|A_d|\sin\alpha_d + |\mu|\sin\theta_\mu \tan\beta}{|m_0 A_u - \mu^* \tan\beta|} \tag{176b}$$

The angles $\alpha_{u,d}$ are the phases of $A_{u,d}$ and the angle θ_μ is the phase of the μ parameter.

The neutralino mass matrix is:

$$\mathcal{L}_{neutralino\,masses} = -\frac{1}{2}(\overline{\psi}_{h_u^0}, \overline{\psi}_{h_d^0}, \overline{\lambda}_3, \overline{\lambda}_0) M^2_{neutralino} \begin{pmatrix} \psi_{h_u^0} \\ \psi_{h_d^0} \\ \lambda_3 \\ \lambda_0 \end{pmatrix} \tag{177a}$$

$$M^2_{neutralino} = \begin{pmatrix} 0 & \mu & -\frac{gv_u}{\sqrt{2}} & \frac{g'v_u}{\sqrt{2}} \\ \mu & 0 & \frac{gv_d}{\sqrt{2}} & -\frac{g'v_d}{\sqrt{2}} \\ -\frac{gv_u}{\sqrt{2}} & \frac{gv_d}{\sqrt{2}} & M_2 & 0 \\ \frac{g'v_u}{\sqrt{2}} & -\frac{g'v_d}{\sqrt{2}} & 0 & M_1 \end{pmatrix} \tag{177b}$$

The fields $\psi_{h^0_{u,d}}$ are the up- and down-type neutral Higgsinos. The neutralino mass matrix is diagonalised by an unitary matrix X as follows:

$$X^T M^2_{neutralino} X = \begin{pmatrix} m_{\widetilde{\chi}_1^0} & 0 & 0 & 0 \\ 0 & m_{\widetilde{\chi}_2^0} & 0 & 0 \\ 0 & 0 & m_{\widetilde{\chi}_3^0} & 0 \\ 0 & 0 & 0 & m_{\widetilde{\chi}_4^0} \end{pmatrix} \tag{178}$$

The diagonalisation of the neutralino quark matrix, hence the computation of its eingenvalues, is usually done numerically, we will not go into more details.

The chargino mass matrix is:

$$\mathcal{L}_{chargino\,masses} = -(\overline{\lambda}, \overline{\widetilde{\chi}}) \left(M^2_{chargino} \frac{1+\gamma_5}{2} + (M^2_{chargino})^T \frac{1-\gamma_5}{2} \right) \begin{pmatrix} \lambda \\ \widetilde{\chi} \end{pmatrix} \tag{179a}$$

$$M^2_{chargino} = \begin{pmatrix} M_2 & -gv_d \\ -gv_u & -\mu \end{pmatrix} \tag{179b}$$

The fields λ and $\widetilde{\chi}$ are not the mass eigenstates yet. In this case here, the fields λ and $\widetilde{\chi}$ are:

$$\lambda = \frac{\lambda_1 + i\lambda_2}{\sqrt{2}} \tag{180a}$$
$$\widetilde{\chi} = \frac{1+\gamma_5}{2}\psi_{h_d^-} - \frac{1-\gamma_5}{2}\psi_{h_u^+} \tag{180b}$$

The full matrix can be diagonalised using two unitary matrices conventionally called U and V. These matrices are such that:

$$\frac{1+\gamma_5}{2}\begin{pmatrix}\lambda \\ \widetilde{\chi}\end{pmatrix} = U\frac{1+\gamma_5}{2}\begin{pmatrix}\widetilde{W}_1 \\ \widetilde{\chi}_2\end{pmatrix} \tag{181a}$$
$$\frac{1-\gamma_5}{2}\begin{pmatrix}\lambda \\ \widetilde{\chi}\end{pmatrix} = V\frac{1-\gamma_5}{2}\begin{pmatrix}\widetilde{\chi}_1 \\ \widetilde{\chi}_2\end{pmatrix} \tag{181b}$$

Then:

$$\mathcal{L}_{chargino\;masses} = -(\overline{\widetilde{\chi}_1}, \overline{\widetilde{\chi}_2})\left(V^\dagger M^2_{chargino}U\frac{1+\gamma_5}{2} + U^\dagger (M^2_{chargino})^T V\frac{1-\gamma_5}{2}\right)\begin{pmatrix}\tilde{\chi}_1 \\ \widetilde{\chi}_2\end{pmatrix} \tag{182}$$

In order to write this, we must not forget that the hermitian conjugated of a left-handed field is a right handed, and vice versa. Surely we can define the matrices U and V in such way that:

$$V^\dagger M^2_{chargino}U = \begin{pmatrix} m_{\widetilde{\chi}_1} & 0 \\ 0 & m_{\widetilde{\chi}_2}\end{pmatrix} \tag{183a}$$
$$U^\dagger (M^2_{chargino})^T V = \begin{pmatrix} m_{\widetilde{\chi}_1} & 0 \\ 0 & m_{\widetilde{\chi}_2}\end{pmatrix} \tag{183b}$$

It is now obvious that:

$$V^\dagger M^2_{chargino}(M^2_{chargino})^T V = U^\dagger (M^2_{chargino})^T M^2_{chargino} U = \begin{pmatrix} m^2_{\tilde{\chi}_1} & 0 \\ 0 & m^2_{\tilde{\chi}_2} \end{pmatrix} \tag{184}$$

It is straight forward to find the eigenvalues $m_{\tilde{\chi}_{1,2}}$ of $M^2_{chargino}(M^2_{chargino})^T$:

$$\begin{aligned} m^2_{\tilde{\chi}_1} = \frac{1}{2}(|\mu|^2 + M_2^2 + 2M_W^2) + \frac{1}{2}\Big[(|\mu|^2 - M_2^2)^2 + 4M_W^2(M_W^2\cos^2 2\beta + |\mu|^2 \\ + M_2^2 - 2|\mu| M_2 \cos\theta_\mu \sin 2\beta)\Big]^{\frac{1}{2}} \end{aligned} \tag{185}$$

And similarly:

$$\begin{aligned} m^2_{\tilde{\chi}_2} = \frac{1}{2}(|\mu|^2 + M_2^2 + 2M_W^2) - \frac{1}{2}\Big[(|\mu|^2 - M_2^2)^2 + 4M_W^2(M_W^2\cos^2 2\beta + |\mu|^2 \\ + M_2^2 - 2|\mu| M_2 \cos\theta_\mu \sin 2\beta)\Big]^{\frac{1}{2}} \end{aligned} \tag{186}$$

Actually, from equation (184), one can also get the expression for U and V:

$$V = \begin{pmatrix} \cos\frac{\theta_1}{2} & -\sin\frac{\theta_1}{2} e^{-i\phi_1} \\ \sin\frac{\theta_1}{2} e^{\phi_1} & \cos\frac{\theta_1}{2} \end{pmatrix} \tag{187}$$

With:

$$\tan\theta_1 = \frac{2\sqrt{2} M_W (M_2^2\cos^2\beta + |\mu|^2\sin^2\beta + |\mu| M_2 \sin 2\beta\cos\theta_\mu)^{\frac{1}{2}}}{M_2^2 - |\mu|^2 - 2M_W^2\cos 2\beta} \tag{188a}$$

$$\tan\phi_1 = \frac{|\mu|\sin\theta_\mu\sin\beta}{M_2\cos\beta + |\mu|\cos\theta_\mu\sin\beta} \tag{188b}$$

And

$$U = \begin{pmatrix} \cos\frac{\theta_2}{2} & -\sin\frac{\theta_2}{2}e^{-i\phi_2} \\ \sin\frac{\theta_2}{2}e^{i\phi_2} & \cos\frac{\theta_2}{2} \end{pmatrix} \tag{189}$$

With:

$$\tan\theta_2 = \frac{2\sqrt{2}M_W(M_2^2\sin^2\beta + |\mu|^2\cos^2\beta + |\mu|M_2\sin 2\beta\cos\theta_\mu)^{\frac{1}{2}}}{M_2 - |\mu|^2 + 2M_W^2\cos 2\beta} \tag{190a}$$

$$\tan\phi_2 = \frac{-|\mu|\sin\theta_\mu\cos\beta}{M_2\sin\beta + |\mu|\cos\theta_\mu\cos\beta} \tag{190b}$$

References

[1] T.D Lee and C.N Yang, Phys. Rev. **104**, 254 (1956); C.S. Wu, E. Ambler, R.W. Hayward, D.D. Hoppes and R.P. Hudson, Phys. Rev. **105**, 1413 (1957).

[2] E.M. Purcell and N.F. Ramsey, Phys. Rev. **78**, 807 (1950).

[3] J.H. Christensen et al, Phys. Rev. Lett. **13** 138 (1964).

[4] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[5] F. Abe et al, CDF Collaboration, Phys. Rev. Lett. **74** 2626 (1995); S. Abachi et al, D0 Collaboration, Phys. Rev. Lett. **74** 2632 (1995).

[6] R. Jackiw and C. Rebbi, Phys. Rev. Lett. **37**, 172 (1976).

[7] Gustavo Castelo Branco, Luis Lavoura and Joao Paulo Silva, CP Violation.

[8] Marco S.Sozzi, Discrete Symmetries and CP Violation, From Experiment to Theory.

[9] Maxim Pospelov and Adam Ritz, arXiv:hep-ph/0504231v2.

[10] Steven Weinberg, The Quantum Theory of Fields II.

[11] ITEP Lectures on Particle Physics and Field Theory, Vol.1.

[12] R.J Crewther, P. Di Vecchia, G. Veneziano and E. Witten, Phys. Lett. 88B (1979) 33.

[13] S.Fubini, G.Furlan and C.Rosetti, Nuovo Cimento 40 (1965)1171; S.L. Adler and R.F Dashen, Current Algebras (Benjamin, New York, 1968).

[14] Tarek Ibrahim and Pran Nath, Phys. Rev. D **57**, 1 (1998).

[15] Janusz Rosiek, arXiv:hep-ph/9511250v3.

[16] Michael E.Peskin and Daniel V.Shroeder, An Introduction to Quantum Field Theory.

[17] Stephen P. Martin, arXiv:hep-ph/9709356.

[18] Howard Baer and Xerxes Tata, Weak Scale Supersymmetry, From Superfields to Scattering Events.

[19] L.V. Avdeev, D.I. Kazakov and I.N. Kondrashuk, arXiv:hep-ph/9709397v2.