How Universal can Turbulence be ?

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content

- comments on universality
- scaling properties
- characterization without scaling assumption

- how universal is turbulence?

How Universal can Turbulence be ?

WIKIPEDIA
The Free Encyclopedia

Unsolved problems in physics

Is it possible to make a theoretical model to describe the behavior of a turbulent flow — in particular, its internal structures?	?

concept of universality

- only one model is required

concept of universality

- Kolmogorov 41
  inertial range

$$\eta << r << L$$

- Kolmogorov Obukhov 62
  self-similar energy statistics

$$< (ln\epsilon_r)^2 >= \Lambda_0^2 - \mu\, ln(r)$$

scaling law

$$\begin{aligned} < u_r^n > &\propto < \epsilon_r^{n/3} > r^{n/3} \\ &\propto r^{\xi_n} \end{aligned}$$

concept of universality

- Kolmogorov 41
  inertial range

$$\eta << r << L$$

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  self-similar energy statistics

$$< (ln\epsilon_r)^2 >= \Lambda_0^2 - \mu\, ln(r)$$

scaling law ??

$$< u_r^n > \quad \propto \quad < \epsilon_r^{n/3} > r^{n/3}$$
$$\propto \quad r^{\xi_n}$$

content

- comments on
  - universality is based on what?
- scaling properties

scaling properties

hints for scaling behaviour

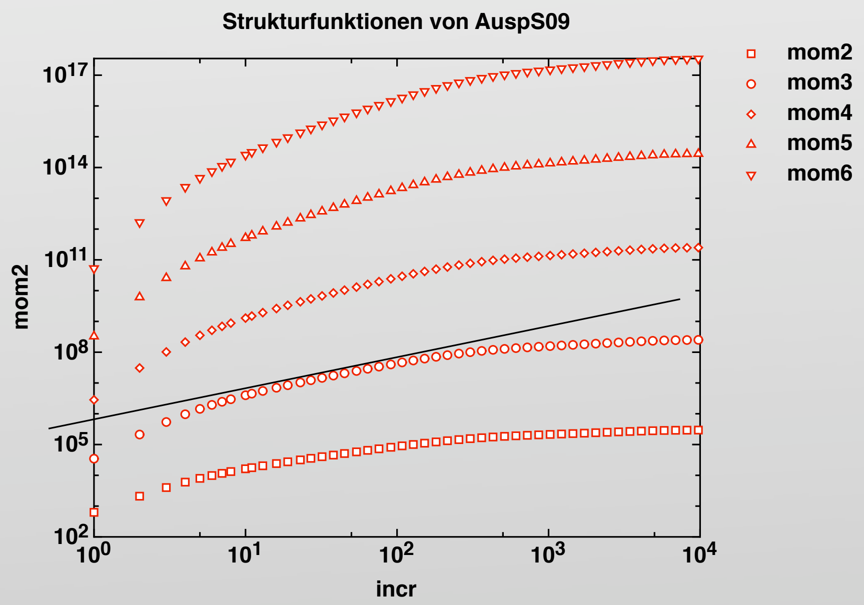
- 4/5th law $$< u_r^3 > = -\frac{4}{5} < \epsilon_r > r + 6\nu \frac{\partial}{\partial r} < u_r^2 >$$

scaling properties

hints for scaling behaviour

- 4/5th law
- structure functions

$$< u_r^n > \quad \propto \quad r^{\xi_n}$$

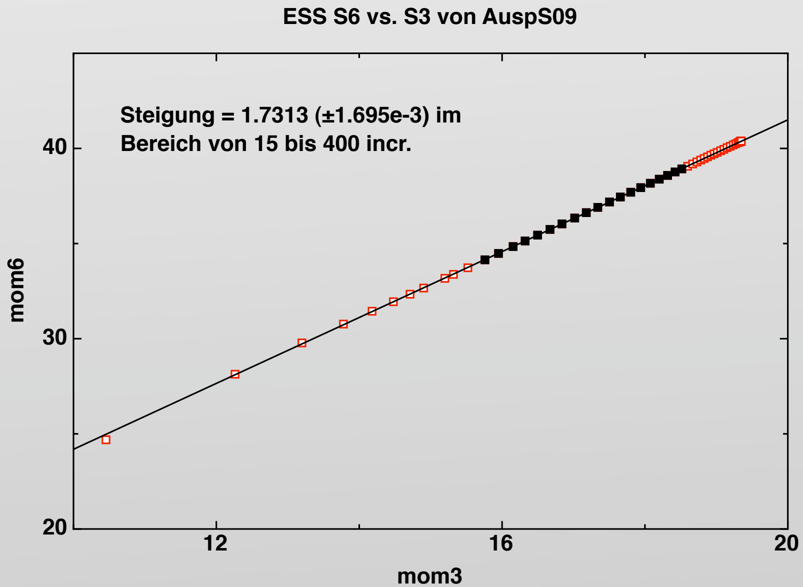


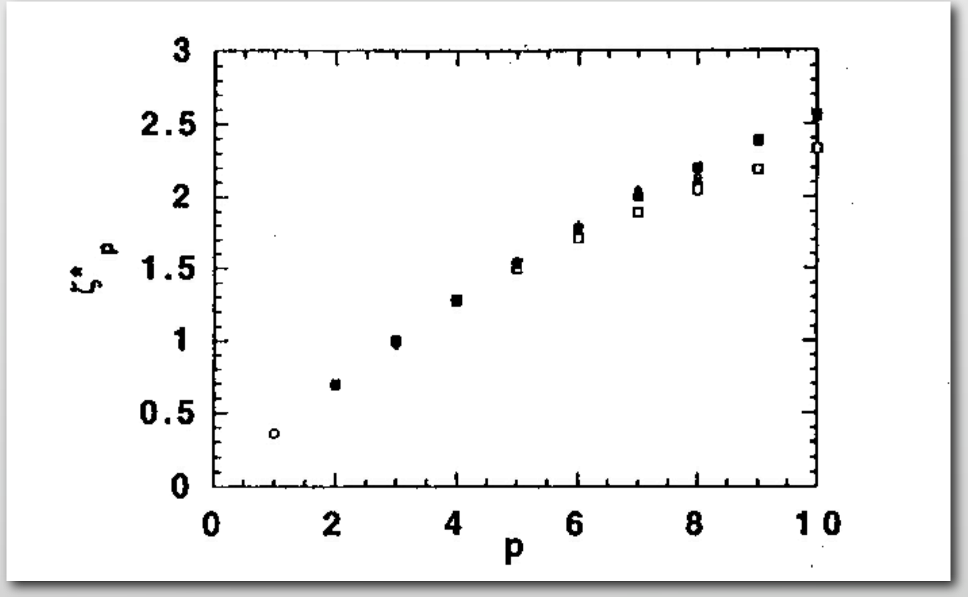
scaling properties

hints for scaling behaviour

- 4/5th law
- structure functions
- ESS

$$< u_r^n > \quad \propto \quad < u_r^3 >^{\xi_n}$$





(Benzi et.al. (1993))

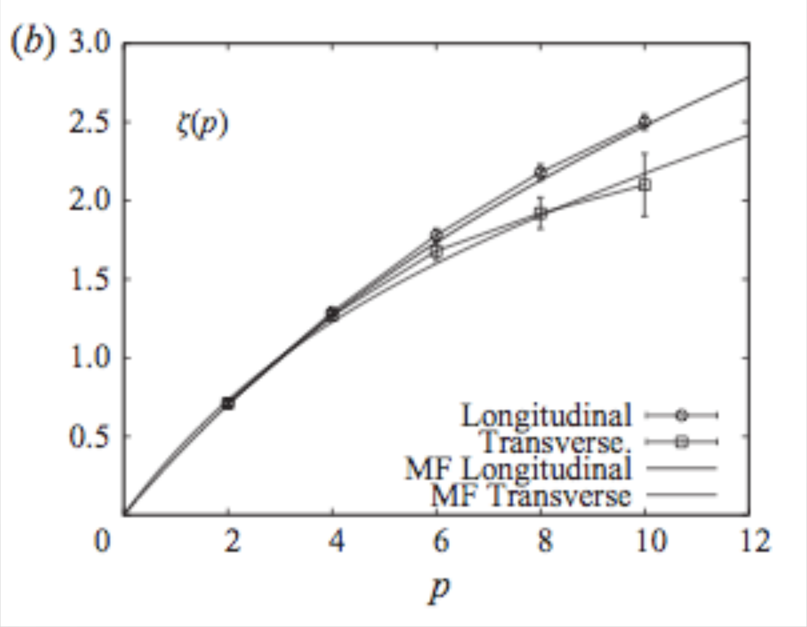
A. Arneodo et.al. Europhys. Lett (1996)

scaling properties

hints for scaling behaviour

- 4/5th law
- structure functions
- ESS
- long / transversal

$$< u_r^n > \quad \propto \quad r^{\xi_n}$$



Benzi et.al. JFM (2010)

scaling properties

are there hints against scaling behaviour ?

- Castaing

$$< (ln\epsilon_r)^2 > \propto r^\beta$$

Castaing et.al. PRL (1994)

- Kolmogorov Obukhov 62

  self-similar energy statistics

$$< (ln\epsilon_r)^2 > = \Lambda_0^2 - \mu\, ln(r)$$

scaling properties

are there hints against scaling behaviour ?

- Castaing
- Re dependencies
- ...

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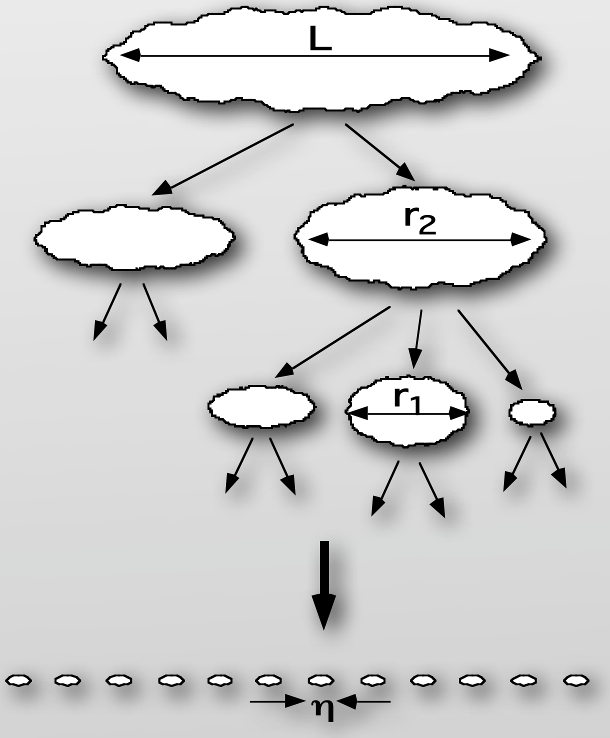
stochastic cascade process

- idea of a turbulent cascade:

  large vortices are generating small ones



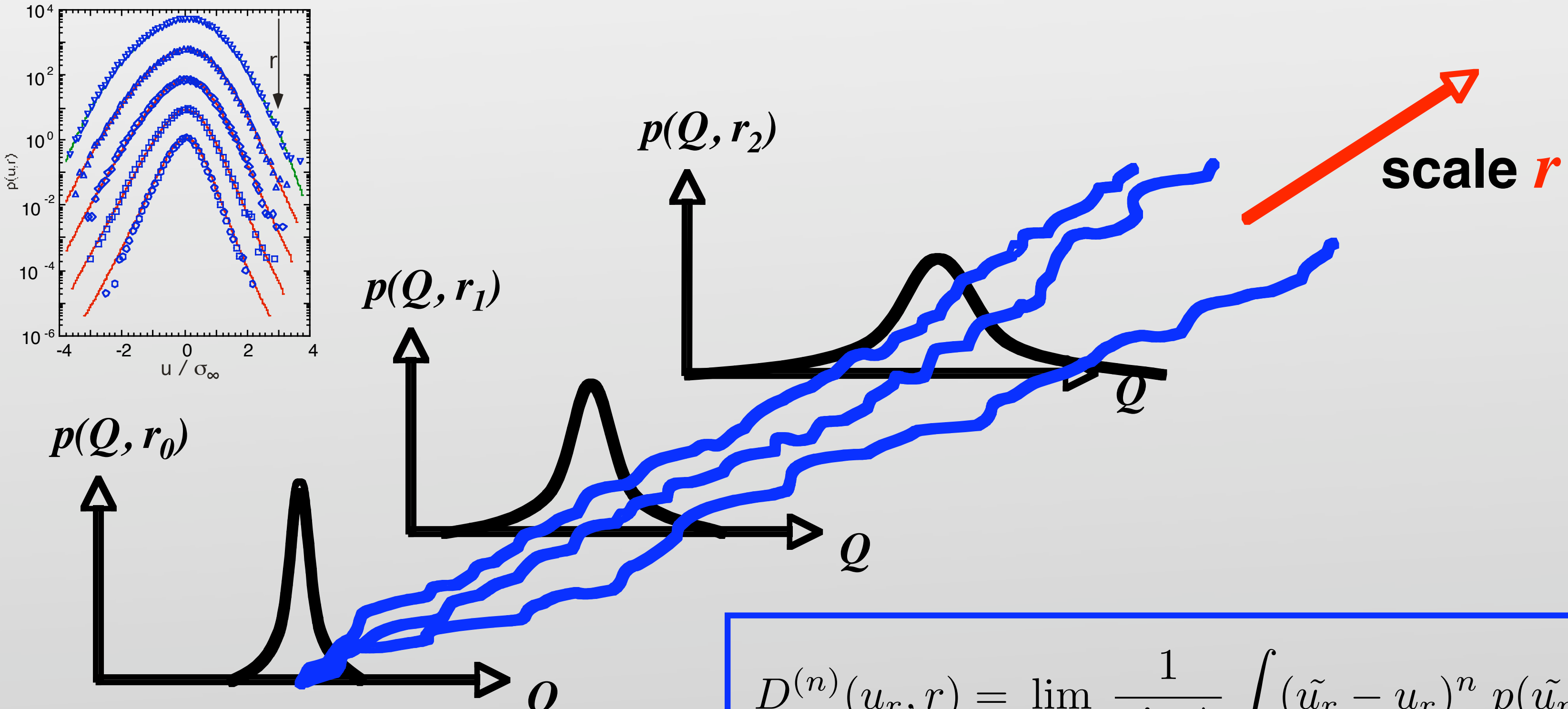
$$\partial_r u_r$$

$$\partial_r p_r(u_r)$$

=> stochastic cascade process evolving in r

stochastic cascade the idea

- the reconstruction of stochastic equation goes back to Kolmogorov work from 1931



$$D^{(n)}(u_r, r) = \lim_{\Delta \to 0} \frac{1}{n! \cdot \Delta} \int (\tilde{u_r} - u_r)^n \, p(\tilde{u_r}, r + \Delta | u_r, r) d\tilde{u_r}$$

$$-\frac{\partial}{\partial r} p(u_r, r | u_0 r_0) = [-\frac{\partial}{\partial u_r} D^{(1)}(u_r, r) + \frac{\partial^2}{\partial u_r^2} D^{(2)}(u_r, r)] p(u_r, r | u_0, r_0)$$

multi-scale statistics -3-

measured Fokker-Planck equation further results

$$-\frac{\partial}{\partial r}p(u_r, r|u_0 r_0) = [-\frac{\partial}{\partial u_r}D^{(1)}(u_r, r) + \frac{\partial^2}{\partial u_r^2}D^{(2)}(u_r, r)]p(u_r, r|u_0, r_0)$$

differential equations for the structure function

$$-\frac{\partial}{\partial r}\left\langle u_r{}^n\right\rangle = n \cdot \left\langle u_r{}^{n-1}D^{(1)}(u_r,r)\right\rangle + n \cdot (n-1)\left\langle u_r{}^{n-2}D^{(2)}(u_r,r)\right\rangle$$

multi-scale statistics -3-

measured Fokker-Planck equation further results

$$-\frac{\partial}{\partial r} p(u_r, r|u_0 r_0) = [-\frac{\partial}{\partial u_r} D^{(1)}(u_r, r) + \frac{\partial^2}{\partial u_r^2} D^{(2)}(u_r, r)] p(u_r, r|u_0, r_0)$$

differential equations for the structure function

$$-\frac{\partial}{\partial r}\langle u_r{}^n \rangle = n \cdot \langle u_r{}^{n-1} D^{(1)}(u_r, r) \rangle + n \cdot (n-1) \langle u_r{}^{n-2} D^{(2)}(u_r, r) \rangle$$

- closed equation for structure functions and scaling behaviour if

$$\begin{aligned} D^{(1)}(u_r, r) &= d_1^u(r) u_r \\ D^{(2)}(u_r, r) &= d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2 \end{aligned}$$

- and $d_1^u(r) \propto 1/r$, $d_2^{uu}(r) \propto 1/r$ other terms zero => K62

multi-scale statistics -4-

differential equations for the structure function

$$-\frac{\partial}{\partial r}\left\langle {u_r}^n \right\rangle = n \cdot \left\langle {u_r}^{n-1} D^{(1)}(u_r,r) \right\rangle + n \cdot (n-1) \left\langle {u_r}^{n-2} D^{(2)}(u_r,r) \right\rangle$$

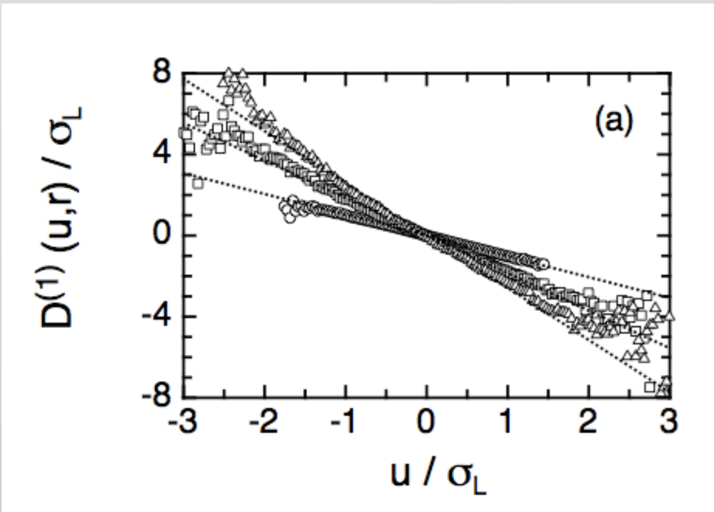
- violation of proper scaling

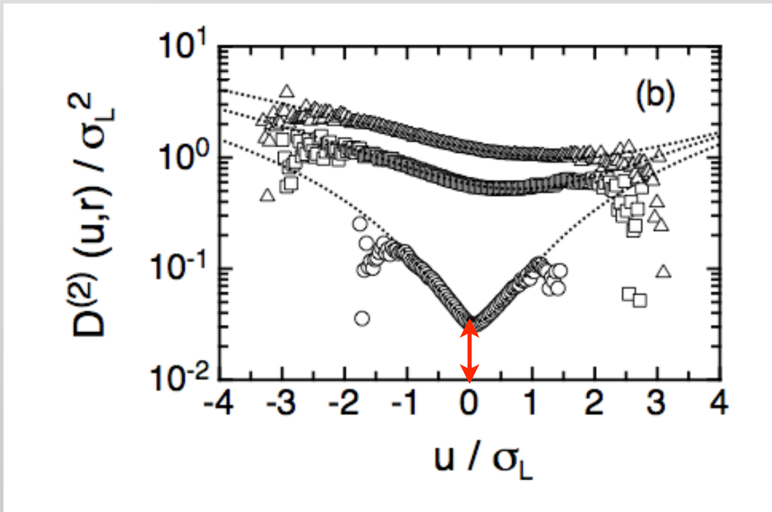
$$D_1^{(1)}(u_r,r) = d_1^u(r) u_r$$

$$D^{(2)}(u_r,r) = d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2$$

$d_1^u(r) \propto 1/r$

$d_2^{uu}(r) \propto 1/r$



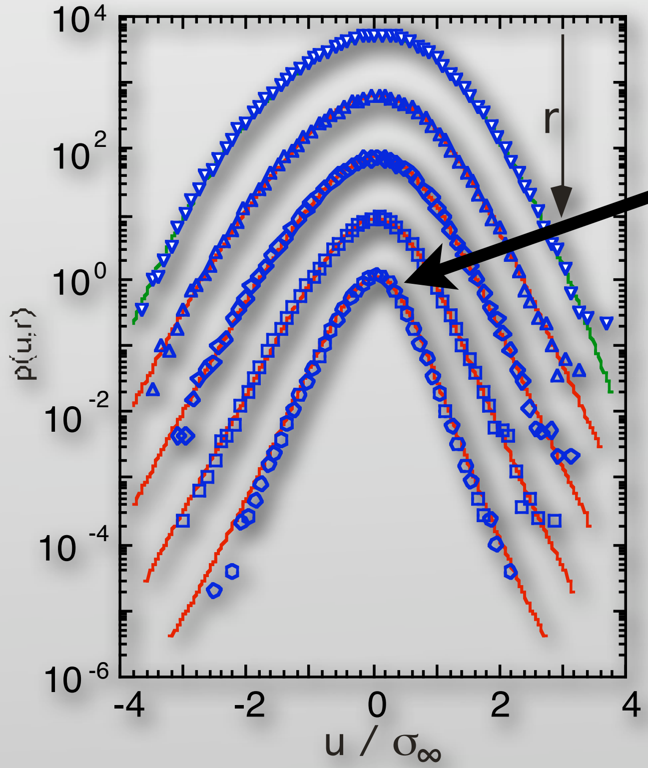


multi-scale statistics -5-

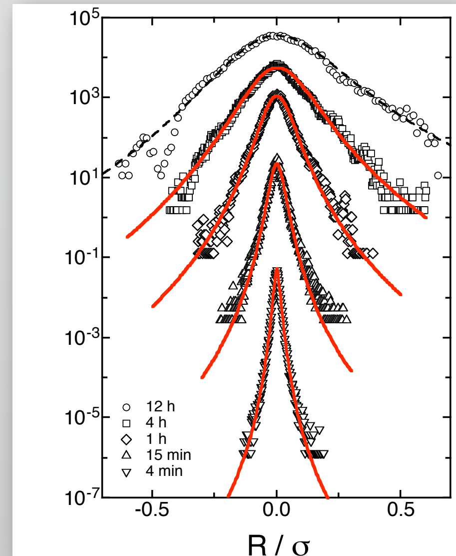
CARL VON OSSIETZKY universität OLDENBURG

additive term in the diffusion term: -> additive noise

$$
\begin{aligned}
D_1^{(1)}(u_r, r) &= d_1^u(r) u_r \\
D^{(2)}(u_r, r) &= d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2
\end{aligned}
$$



Gaussian tip



comparison of data with numerical solution of the Kolmogorov equation

content

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  - multi-scale statistics - additive noise

content

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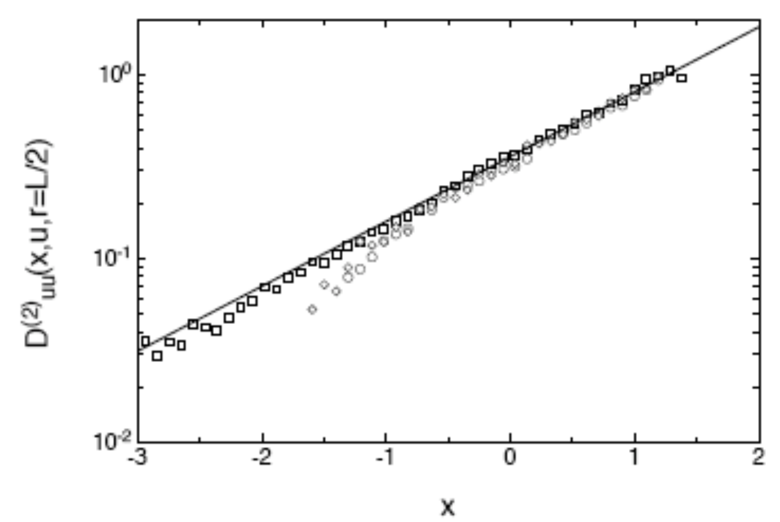
  - energy cascade

energy cascade e_r

2-dim Fokker-Planck analysis

$$\mathbf{q}(r) = \begin{pmatrix} u(r) \\ x(r) \end{pmatrix}. \qquad x(r) = \ln(\epsilon_r/\bar{\epsilon}).$$



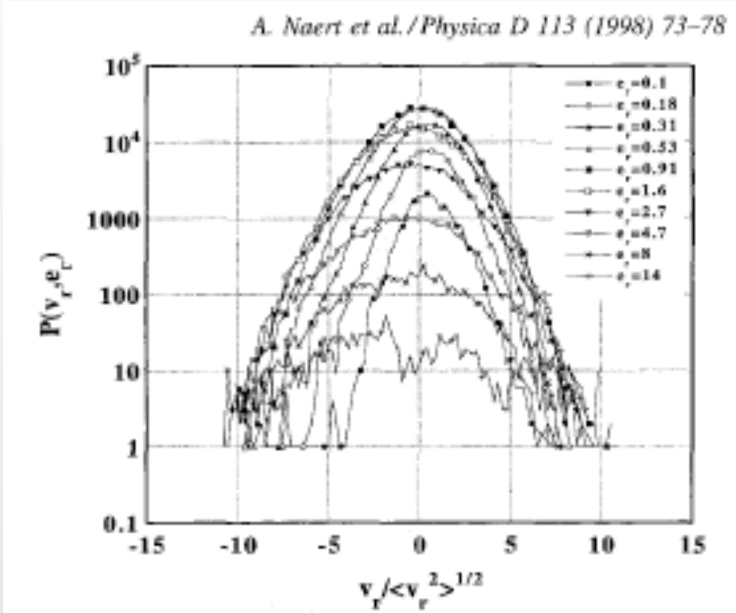
$u = -\sigma_\infty$ *(circles)*, $u = 0$ *(squares) and* $u = +\sigma_\infty$ *(diamonds).*

result in form of a
Langevin equation for the cascade

$$\begin{aligned} -\frac{\partial}{\partial r} u(r) &= -\frac{1}{r}\gamma(r)\, u(r) + m \exp\left(\frac{a_1}{2} x(r)\right) \Gamma_u(r), \\ -\frac{\partial}{\partial r} x(r) &= +\frac{1}{r} G(r)\, x(r) + \frac{1}{r} F(r) + \sqrt{\frac{1}{r} D^{(2)}_{xx}(u,x,r)}\; \Gamma_x(r). \end{aligned}$$

no multiplicative
noise anymore !!
No interittency!!

energy cascade -2-



result in form of a
Langevin equation for the cascade

$$
\begin{aligned}
-\frac{\partial}{\partial r} u(r) &= -\frac{1}{r}\gamma(r)\, u(r) + m \exp\left(\frac{a_1}{2} x(r)\right) \Gamma_u(r), \\
-\frac{\partial}{\partial r} x(r) &= +\frac{1}{r} G(r)\, x(r) \boxed{+ \frac{1}{r} F(r)} + \sqrt{\frac{1}{r} D^{(2)}_{xx}(u, x, r)}\ \Gamma_x(r) .
\end{aligned}
$$

scaling violating term - nessesary for energy conservation

Naert et.al. PRE (1997)
Renner et.al. unpublished

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  - energy cascade and energy conservation

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  - longitudinal-transversal structure functions

turbulence: long/transversal

spatial correlation in different directions

Quantities

- longitudinal increment

$$u_r(x) = \left[\vec{u}(\vec{x} + \vec{r}) - \vec{u}(\vec{x})\right] \cdot \hat{r}$$

- transversal increment

$$v_r(x) = \left|\left[\vec{u}(\vec{x} + \vec{r}) - \vec{u}(\vec{x})\right] \times \hat{r}\right|$$

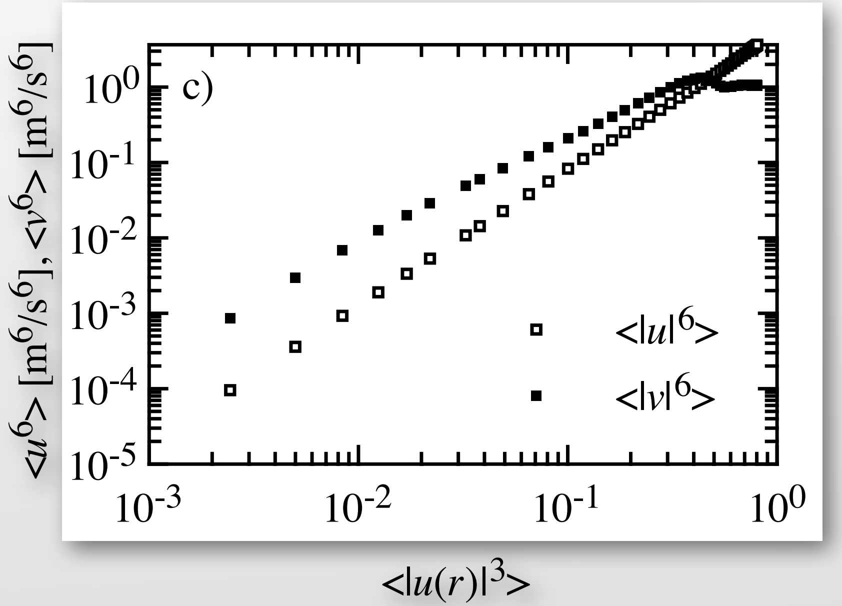
turbulence: long/transversal -2-

extended selfsimilartiy ESS

supposed scaling laws

$$\langle |u_r|^n \rangle \ \propto \ \langle |u_r|^3 \rangle^{\xi_n^l}$$

$$\langle |v_r|^n \rangle \ \propto \ \langle |u_r|^3 \rangle^{\xi_n^t}$$



turbulence: long/transversal -2-

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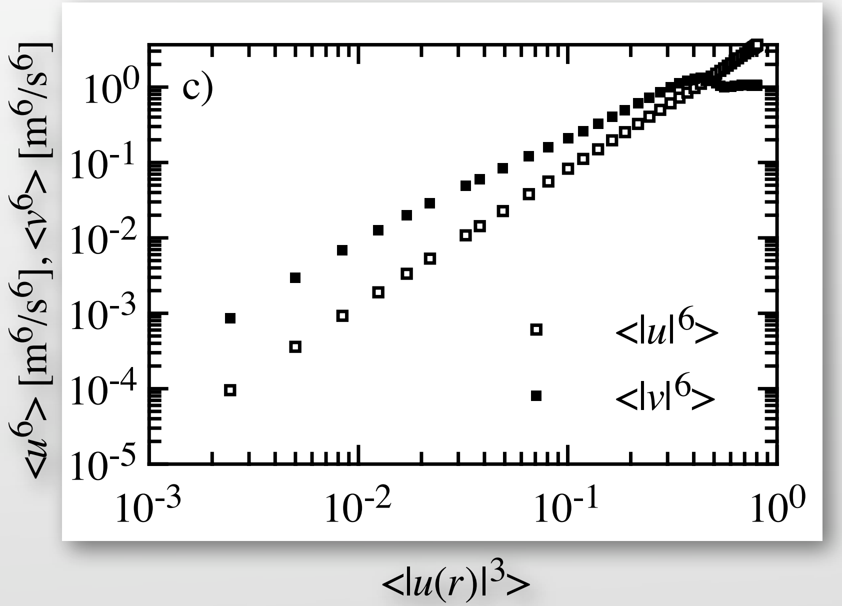
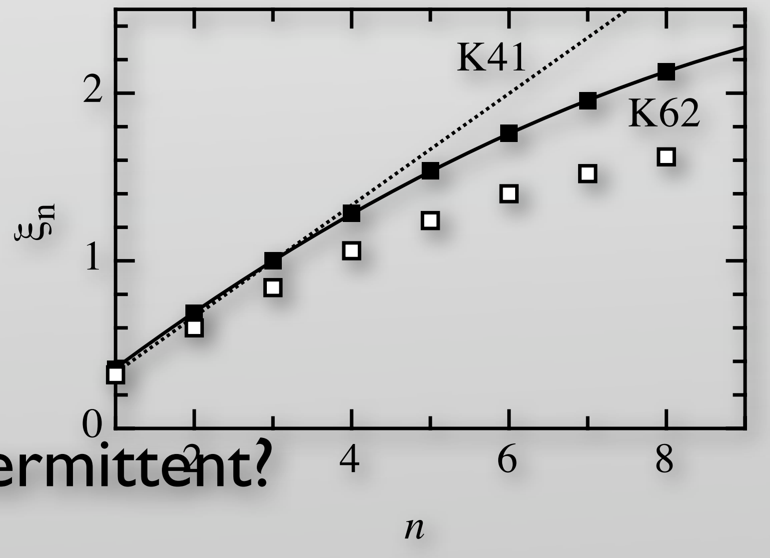


open problem: (Antonia 97, Benzi 97, van der Water 99, Grossman et.al. 97....)

$$\xi_n^t > \xi_n^l$$

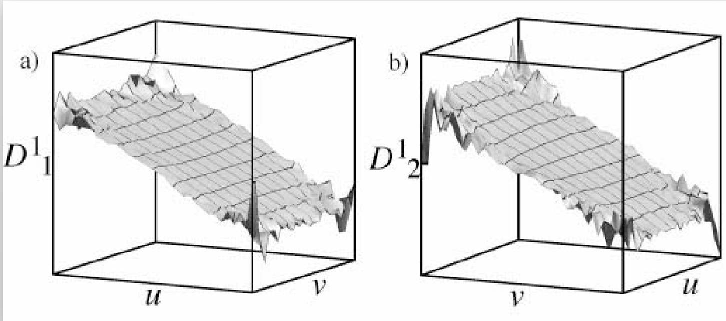


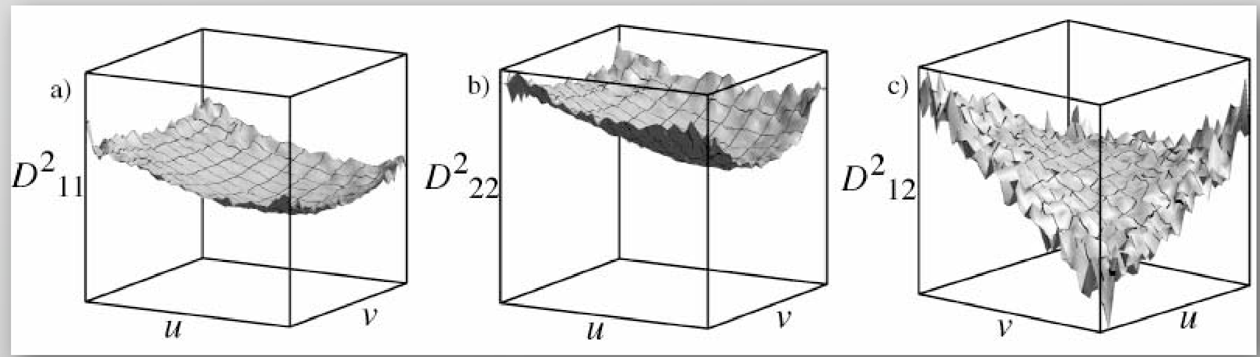
are transversal structures more intermittent?

turbulence: long/transversal -3-

$$-r\frac{\partial}{\partial r}p(\mathbf{u},r|\mathbf{u_0},r_0) =$$

$$\left(-\sum_{i=1}^{n}\frac{\partial}{\partial u_i}D_i^{(1)} + \sum_{i,j=1}^{n}\frac{\partial^2}{\partial u_i \partial u_j}D_{ij}^{(2)}\right)p(\mathbf{u},r|\mathbf{u_0},r_0)$$





turbulence: long/transversal -3-

$$-r\frac{\partial}{\partial r}p(\mathbf{u},r|\mathbf{u_0},r_0) =$$

$$\left(-\sum_{i=1}^{n}\frac{\partial}{\partial u_i}D_i^{(1)}+\sum_{i,j=1}^{n}\frac{\partial^2}{\partial u_i \partial u_j}D_{ij}^{(2)}\right)p(\mathbf{u},r|\mathbf{u_0},r_0)$$

$$\begin{aligned}-\frac{\partial}{\partial r}\langle u^m v^n\rangle = & \\ + \quad & m\langle u^{m-1}v^n D_1\rangle + n\langle u^m v^{n-1} D_2\rangle \\ + \quad & \frac{m(m-1)}{2}\langle u^{m-2}v^n D_{11}\rangle + \frac{n(n-1)}{2}\langle u^m v^{n-2} D_{22}\rangle \\ + \quad & mn\langle u^{m-1}v^{n-1} D_{12}\rangle .\end{aligned}$$

turbulence: long/transversal -3-

$$-r\frac{\partial}{\partial r}p(\mathbf{u},r|\mathbf{u_0},r_0) =$$

$$\left(-\sum_{i=1}^{n}\frac{\partial}{\partial u_i}D_i^{(1)} + \sum_{i,j=1}^{n}\frac{\partial^2}{\partial u_i \partial u_j}D_{ij}^{(2)}\right)p(\mathbf{u},r|\mathbf{u_0},r_0)$$

$$-\frac{\partial\langle u^2\rangle}{\partial r} = \left(2d_1^u + d_{11}^{uu}\right)\langle u^2\rangle + d_{11} + d_{11}^{vv}\langle v^2\rangle$$

$$-\frac{\partial\langle u^3\rangle}{\partial r} = \left(3d_1^u + 3d_{11}^{uu}\right)\langle u^3\rangle + 3d_{11}^{u}\langle u^2\rangle + 3d_{11}^{vv}\langle uv^2\rangle$$

Compare with the Karman/Horwarth equations:

$$-\frac{\partial\langle u^2\rangle}{\partial r} = -2\frac{\langle u^2\rangle}{r} + 2\frac{\langle v^2\rangle}{r}$$

First Kármán-

$$-\frac{\partial\langle u^3\rangle}{\partial r} = \frac{\langle u^3\rangle}{r} - \frac{6\langle uv^2\rangle}{r}$$

Second Kármán-

turbulence: long/transversal -4-

rescaling symmetry: *r => 3r/2*

Siefert Journal of Turbulence **7**, (No 50) 1-35 (2006)

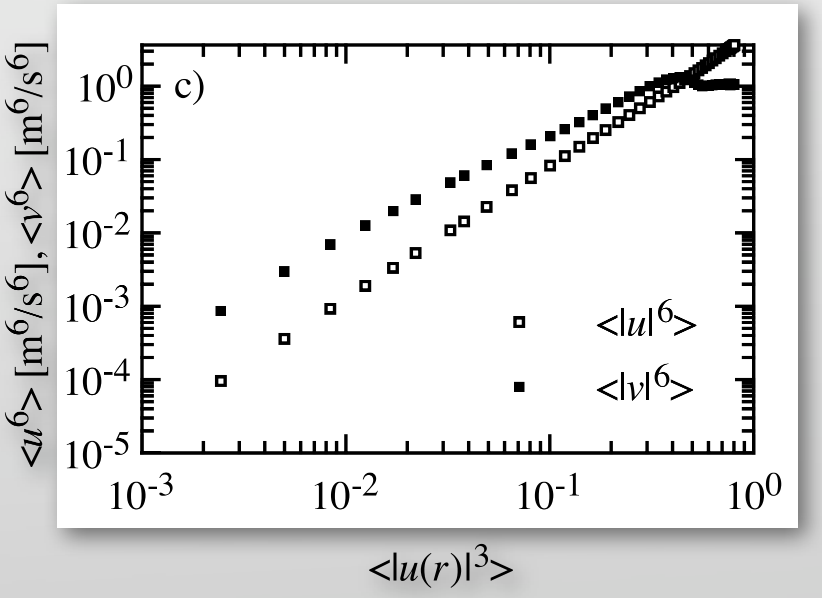
turbulence: long/transversal -4-

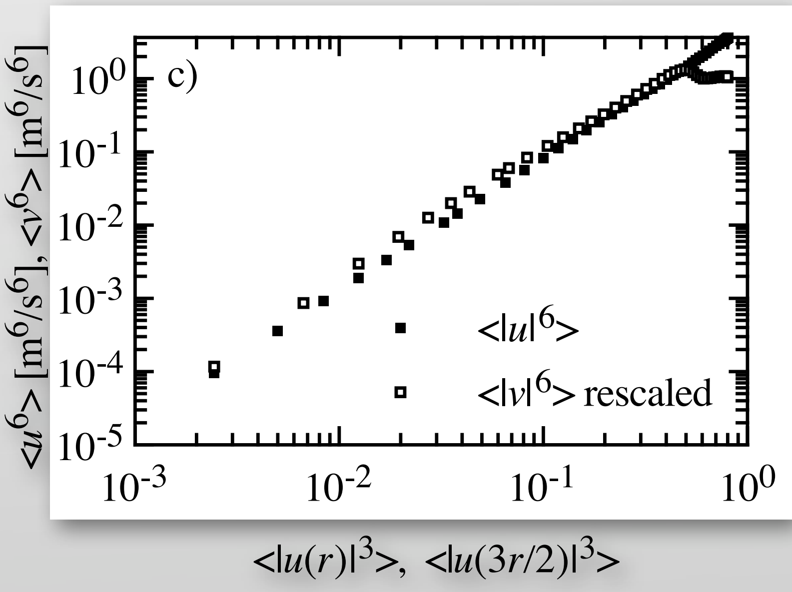
rescaling symmetry: *r => 3r/2*

$$< |v(r)|^n > \propto < |u(r)|^3 >^{\xi_n^t}$$

new ESST :

$$< |v(r)|^n > \propto < |u(3r/2)|^3 >^{\xi_n}$$





Siefert Journal of Turbulence **7**, (No 50) 1-35 (2006)

turbulence: long/transversal -5-

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Assume • pure scaling: $\langle u^n(r)\rangle = c_l^n r^{\xi_l^n}$ and $\langle v^n(r)\rangle = c_t^n r^{\xi_t^n}$

• and same intermittency $d_{11}^{uu}(r) \equiv d_{22}^{vv}(r)$

$$\Longrightarrow \langle v^n(r)\rangle = \langle u^n(\tfrac{3}{2}r)\rangle = c_t^n r^{\xi_t^n} = c_l^n (\tfrac{3}{2}r)^{\xi_l^n} \qquad (\Rightarrow \xi_l^n = \xi_t^n)$$

$n = 2$ $\quad c_t^2/c_l^2 \approx 1.33$ $\quad$ 2% deviation from literature

$n = 4$ $\quad c_t^4/c_l^4 \approx 1.72$ $\quad$ 3% deviation from literature

R. Antonia, M. Ould-Rouis, Y. Zhu, and F. Anselmet, Europhys. Lett. 37, 85 (1997)

turbulence: long/transversal -4-

rescaling symmetry: *r => 3r/2*

$$< |v(r)|^n > \propto < |u(r)|^3 >^{\xi_n^t}$$

$$< |v(r)|^n > \propto < |u(3r/2)|^3 >^{\xi_n}$$

consistent with Karman equation:

$$-r \frac{\partial}{\partial r} \langle u_r{}^2 \rangle = 2 \langle u_r{}^2 \rangle - 2 \langle v_r{}^2 \rangle$$

or

$$\langle v_r{}^2 \rangle = \langle u_r{}^2 \rangle + \frac{r}{2} \frac{\partial}{\partial r} \langle u_r{}^2 \rangle$$

taken as Taylor series

$$\langle v_r{}^2 \rangle \approx \langle u_{3/2r}{}^2 \rangle$$

Phys. Rev. E 70(R) (2004)

turbulence: long/transversal -6-

rescaling symmetry: *r => 3r/2*

$$< |v(r)|^n > \propto < |u(r)|^3 >^{\xi_n^t}$$

$$< |v(r)|^n > \propto < |u(3r/2)|^3 >^{\xi_n}$$

striking result

$$< u_{3r/2}^n > \propto \ (3r/2)^{\xi_n} \ \propto \ r^{\xi_n}$$

- this is only possible if the scaling laws does not hold

Phys. Rev. E 70(R) (2004)

content

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  - energy cascade and energy conservation
  - longitudinal-transversal structure functions

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- universal turbulence?
  - Reynolds dependencies

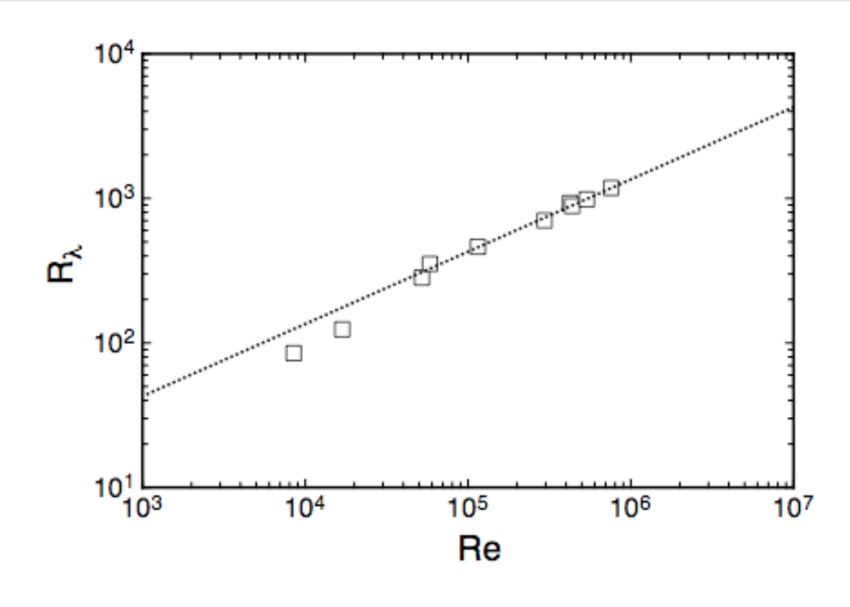
Re dependence -I-

- universality of turbulence: Re dependence?

$$D^{(1)}(u_r, r) = d_1^u(r) u_r$$
$$D^{(2)}(u_r, r) = d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2$$

Phys. Rev. Lett. **89**, (2002)

experimental data sets



Re dependence -2-

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- universality of turbulence: Re dependence?

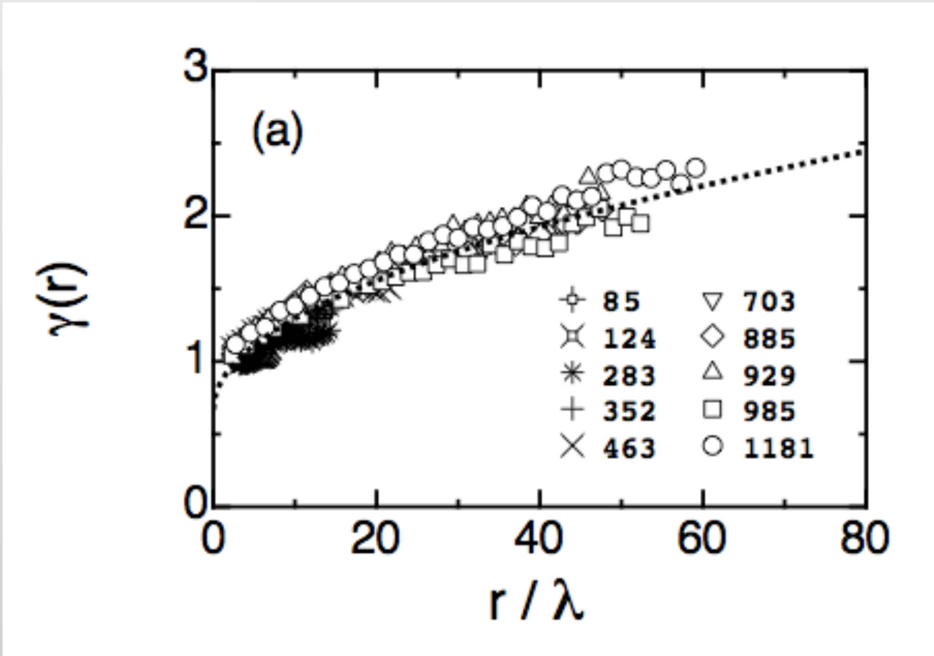
$$D^{(1)}(u_r, r) = d_1^u(r) u_r$$
$$D^{(2)}(u_r, r) = d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2$$

Phys. Rev. Lett. **89**, (2002)

$$d_1^u(r) = -\frac{1}{r}\gamma(r)$$

$$\gamma(r) = \frac{2}{3} + c\sqrt{\frac{r}{\lambda}}$$

$D^{(l)}$ universal



Re dependence -3-

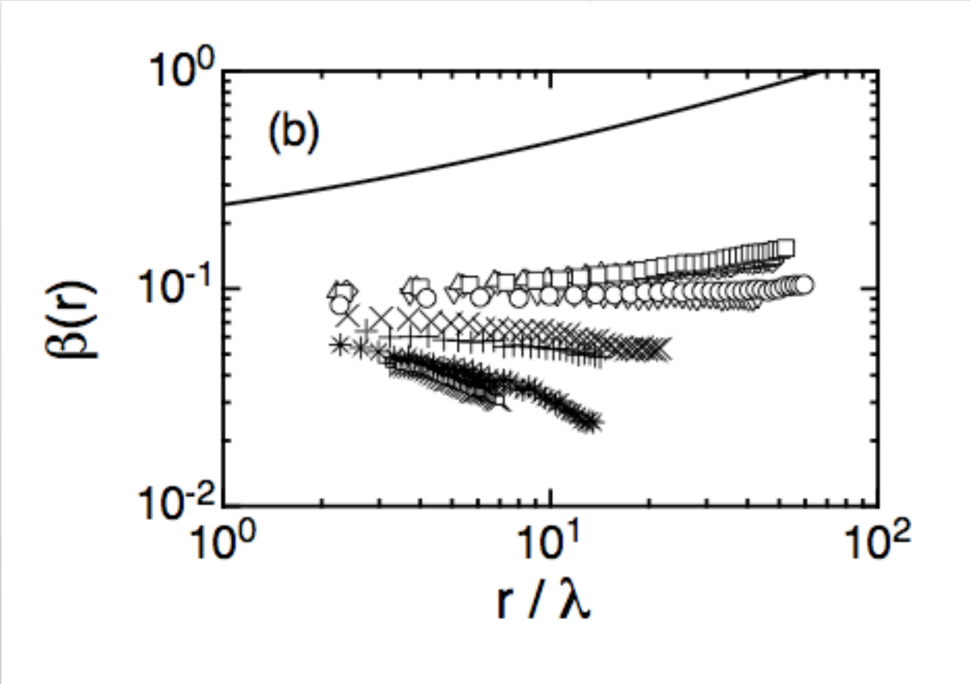
- universality of turbulence: Re dependence?

$$D^{(1)}(u_r, r) = d_1^u(r) u_r$$
$$D^{(2)}(u_r, r) = d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2$$

Phys. Rev. Lett. **89**, (2002)

$$d_2^{uu}(r) = f(Re)$$

$$d_2(r) \propto Re^{-3/8}/r$$



Re dependence -3-

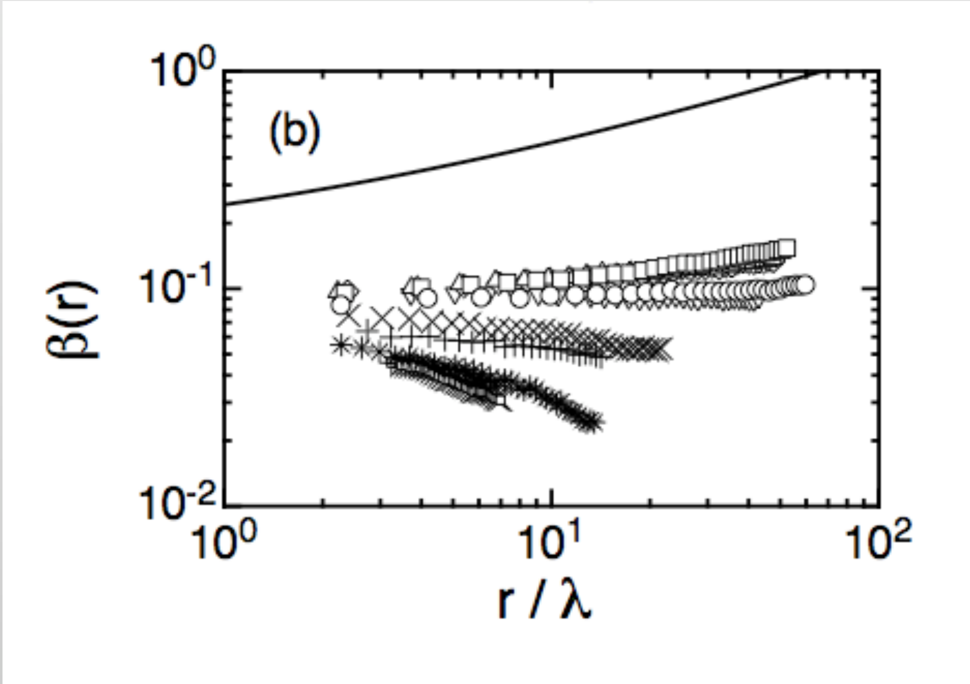
- universality of turbulence: Re dependence?

$$D^{(1)}(u_r, r) = d_1^u(r) u_r$$
$$D^{(2)}(u_r, r, Re) = d_2(r, Re) + d_2^u(r, Re) u_r + d_2^{uu}(r, Re) u_r^2$$

Phys. Rev. Lett. **89**, (2002)

$$d_2^{uu}(r) = f(Re)$$

$$d_2(r) \propto Re^{-3/8}/r$$



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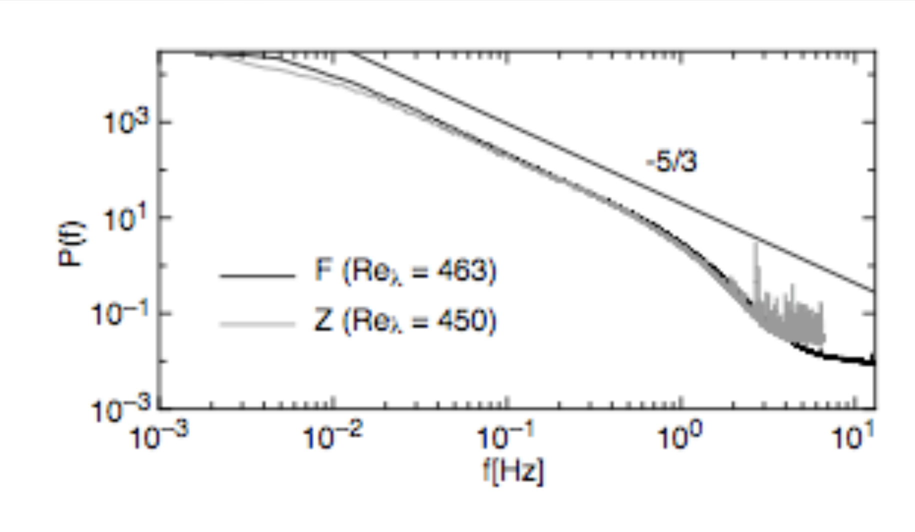
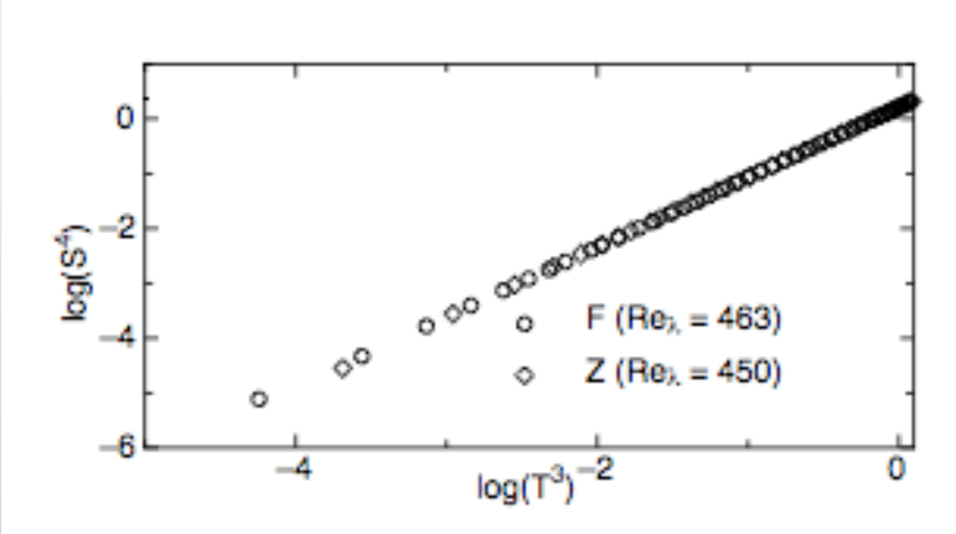
  - energy cascade and energy conservation
  - longitudinal-transversal structure functions
- universal turbulence?
  - Reynolds dependencies
  - flow dependencies

universality of turbulence -2-

- different type of flows

wake behind cylinder and free jet





Laupichler thesis. Ol (2007)

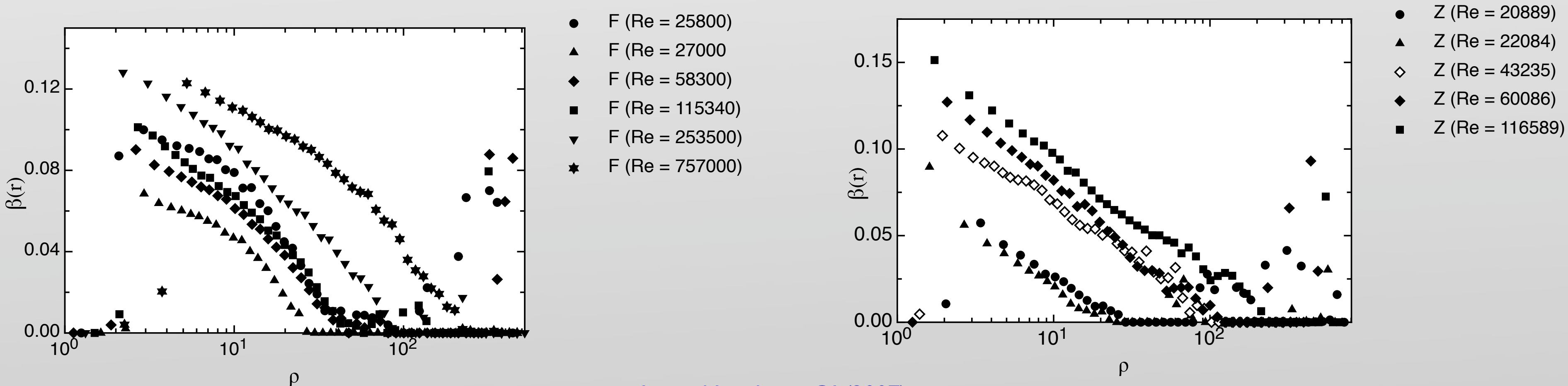
universality of turbulence -2-

- different type of flows

wake behind cylinder and free jet

$$-\frac{\partial}{\partial r}p(u_r,r|u_0 r_0) = [-\frac{\partial}{\partial u_r}D^{(1)}(u_r,r) + \frac{\partial^2}{\partial u_r^2}D^{(2)}(u_r,r)]p(u_r,r|u_0,r_0)$$

$$\begin{aligned} D^{(1)}(u_r,r) &= d_1^u(r)u_r \\ D^{(2)}(u_r,r) &= d_2(r) + d_2^u(r)u_r + d_2^{uu}(r)u_r^2 \end{aligned}$$

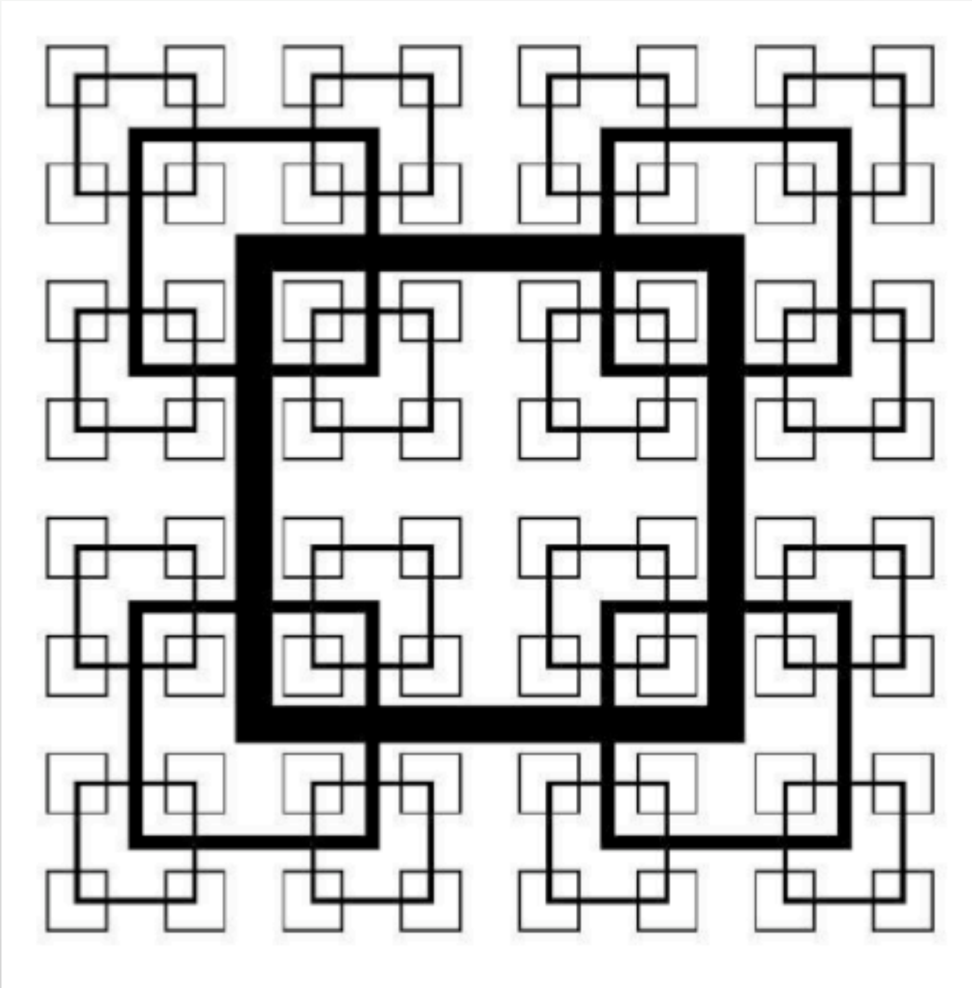


Laupichler thesis. Ol (2007)

content

- comments on
  - universality is based on what?
- scaling properties
- characterization without scaling assumption
  - multi-scale statistics - additive noise
  - energy cascade and energy conservation
  - longitudinal-transversal structure functions
- universal turbulence?
  - Reynolds dependencies
  - flow dependencies

fractal grid turbulence - Christos Vassilicos

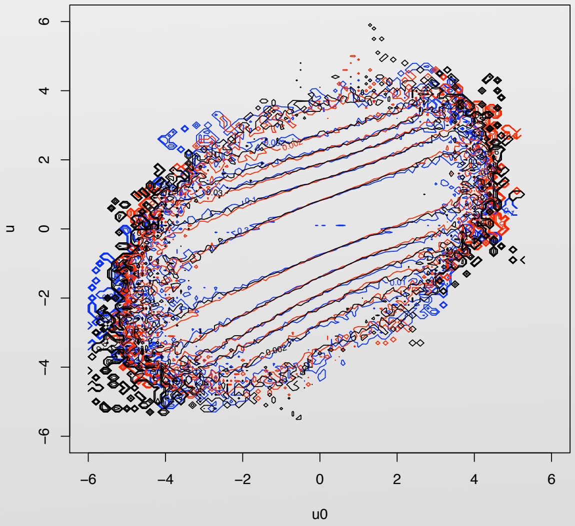


fractal grid turbulence - Christos Vassilicos

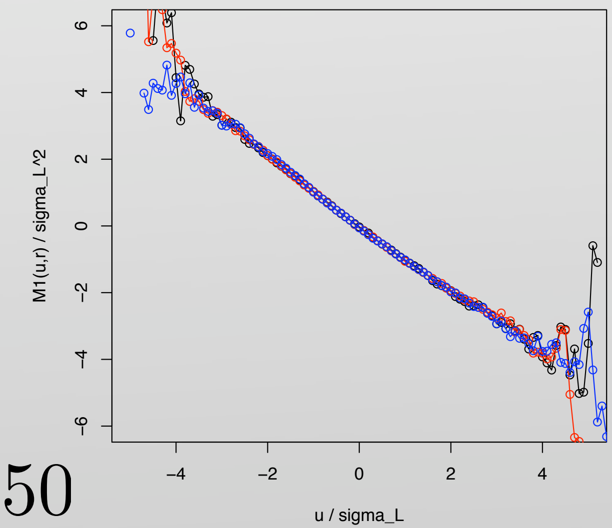
- properties do not change with Re

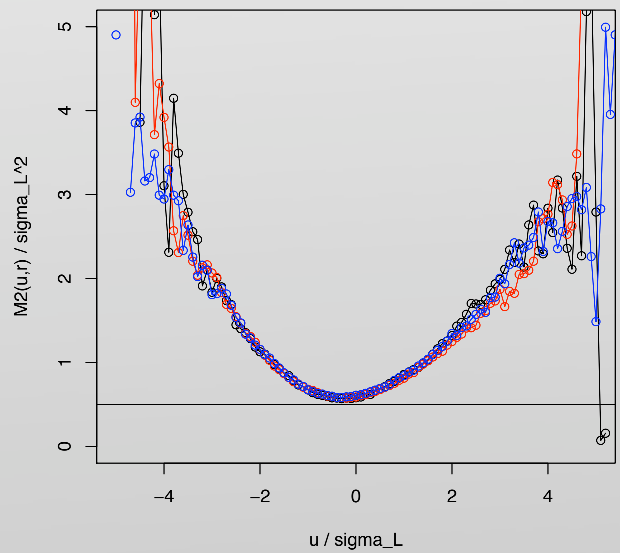
at L and L/2:
- conditional pdf
- Drift $D^{(1)}$ and diffusion $D^{(2)}$

do not depend on Re



$R_\lambda = 150 \ldots 750$



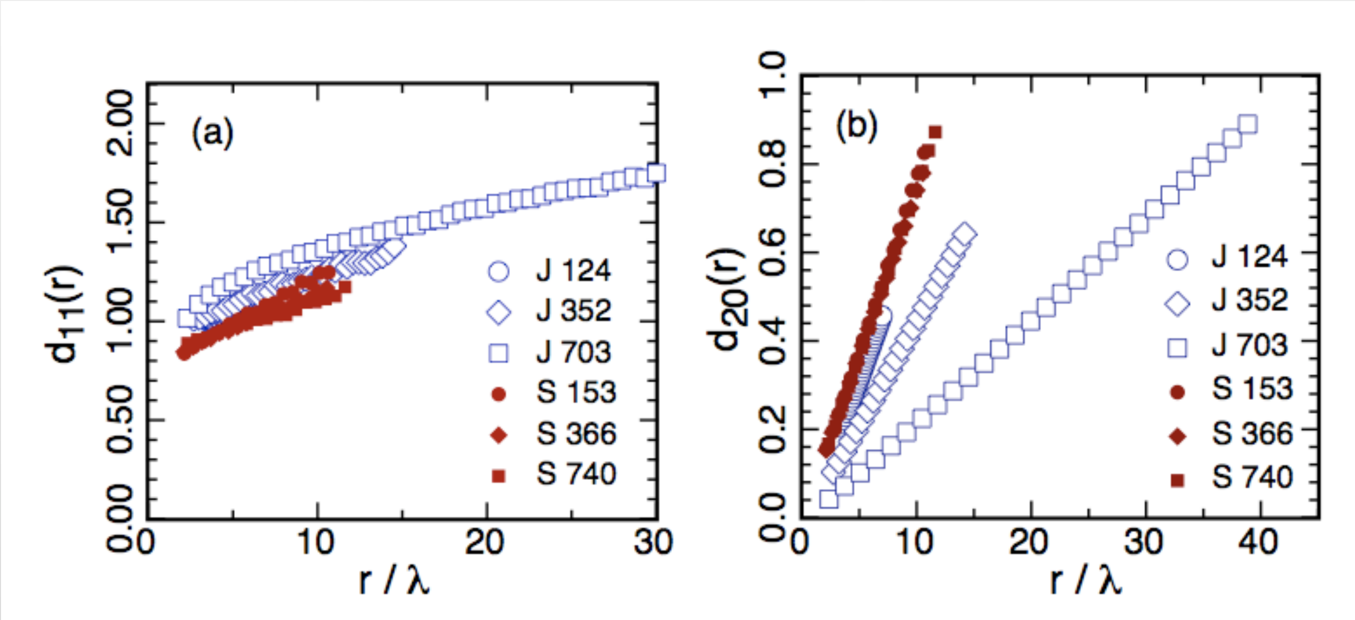


Phys. Rev. Lett. (2010) in press

fractal grid turbulence - Christos Vassilicos

- Re dependency

$$D^{(1)}(u_r, r) = d_1^u(r) u_r$$
$$D^{(2)}(u_r, r) = d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2$$

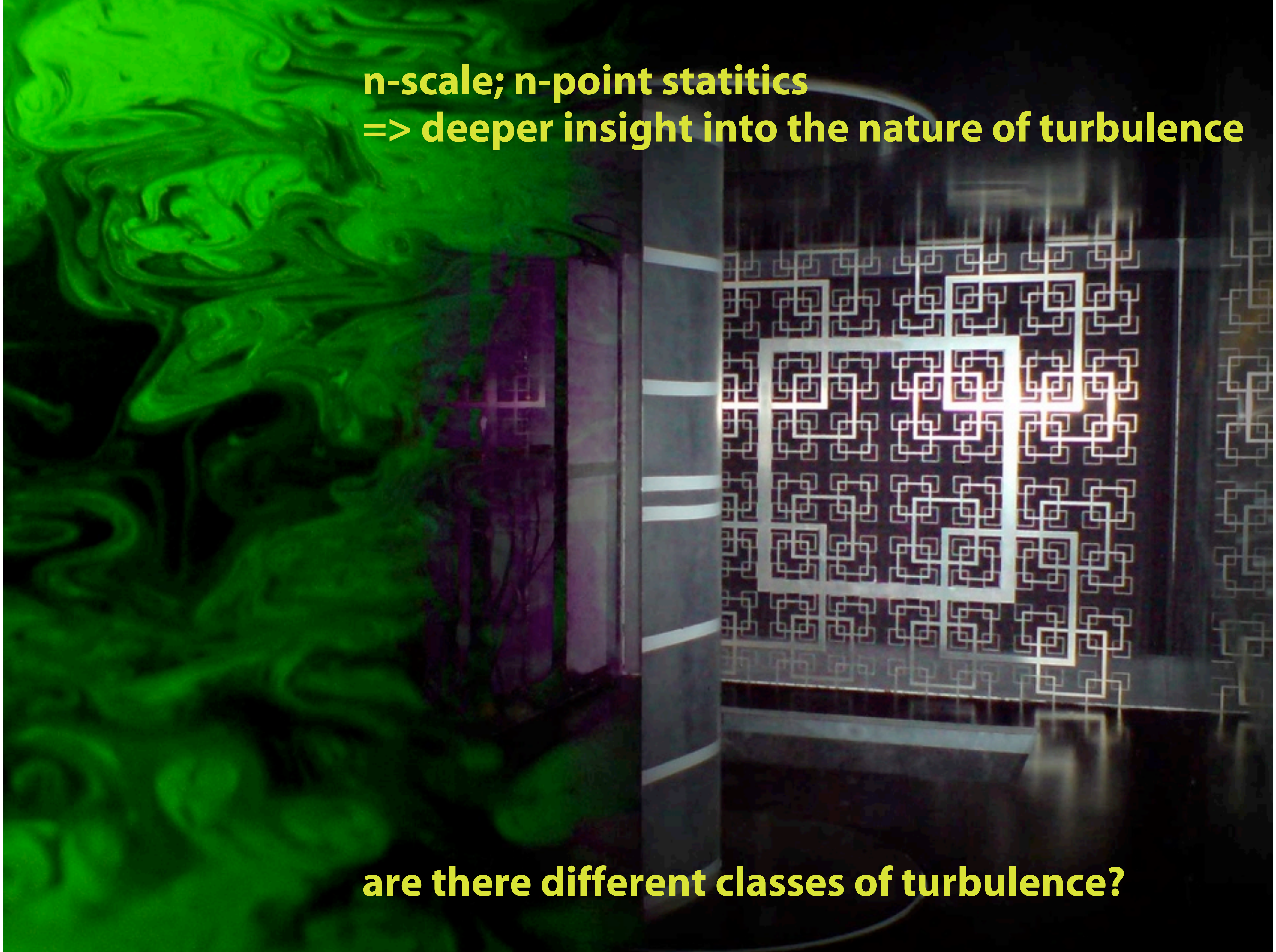


=> Exp: cascade process depends on Re

Phys. Rev. Lett. (2010) in press

summary

- universality is based on what?
- scaling properties one scale property
- characterization with multi-scale statistics
  - additive noise
  - energy conservation
  - longitudinal-transversal structure functions
- universal turbulence?
  - Reynolds dependencies
  - flow dependencies
  - new class of fractal generated turbulence


n-scale; n-point statitics
=> deeper insight into the nature of turbulence
are there different classes of turbulence?